CAP. 4 DINAMICA DE FLUIDOS

PRINCIPIO DE CONSERVACION DEL MOMENTUM (ENFOQUE DIFERENCIAL)

2da LEY DE NEWTON $$d\vec{F} = dm \frac{D\vec{v}}{Dt}$$

$$dm = \rho \, dV$$

$$\frac{d\vec{F}}{dV} = \rho \frac{D\vec{v}}{Dt} = \rho \left(\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} \right)$$

PRINCIPIO DE CONSERVACION DEL MOMENTUM
(ENFOQUE DIFERENCIAL)

FUERZAS
- MASICAS
- SUPERFICIALES
  - NORMALES
  - TANGENCIALES

FUERZAS MASICAS:

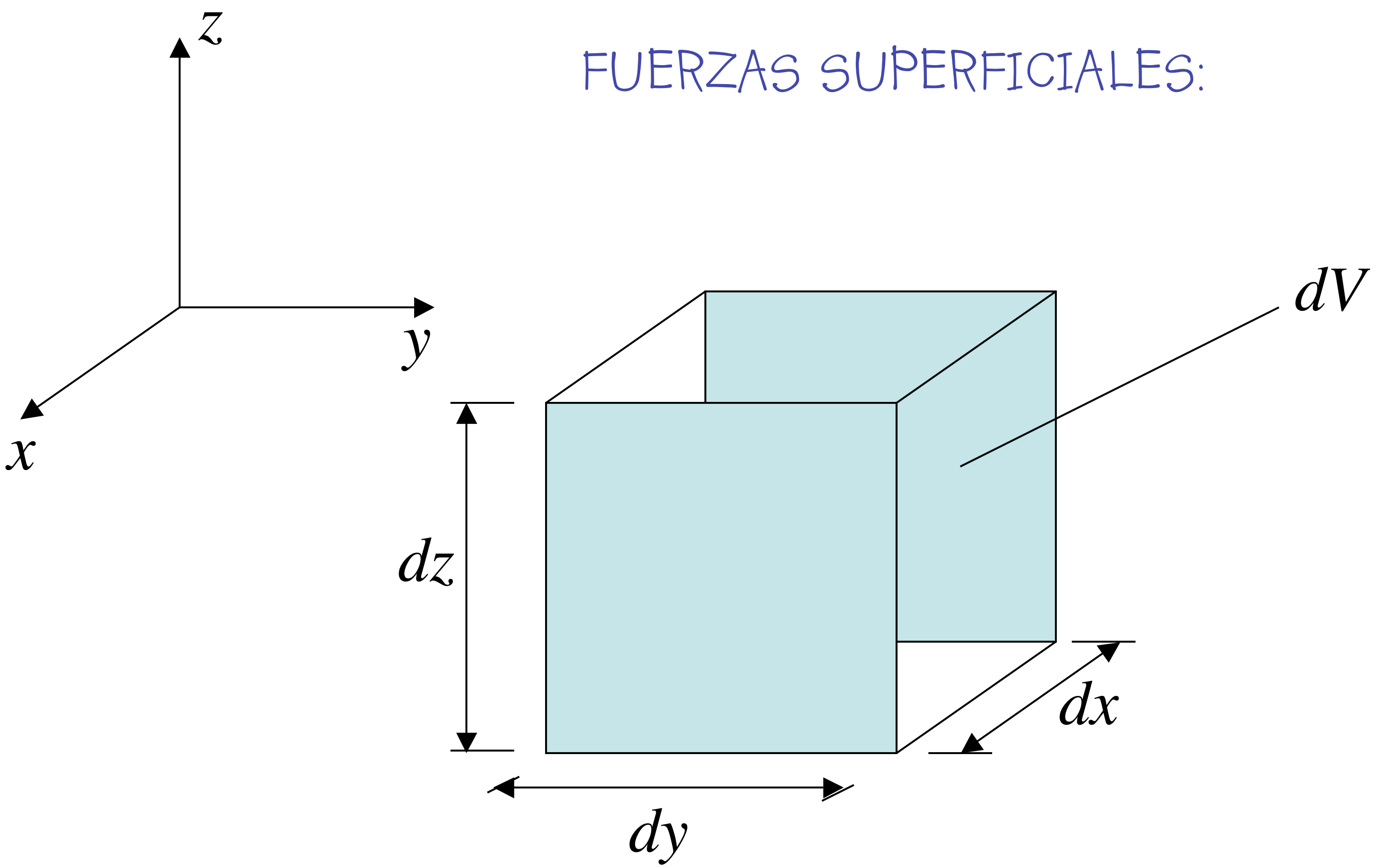
CAMPO GRAVITACIONAL:

$$d\vec{F}_m = dm\,\vec{g}$$

$$\frac{d\vec{F}_m}{dV} = \rho\,\vec{g}$$

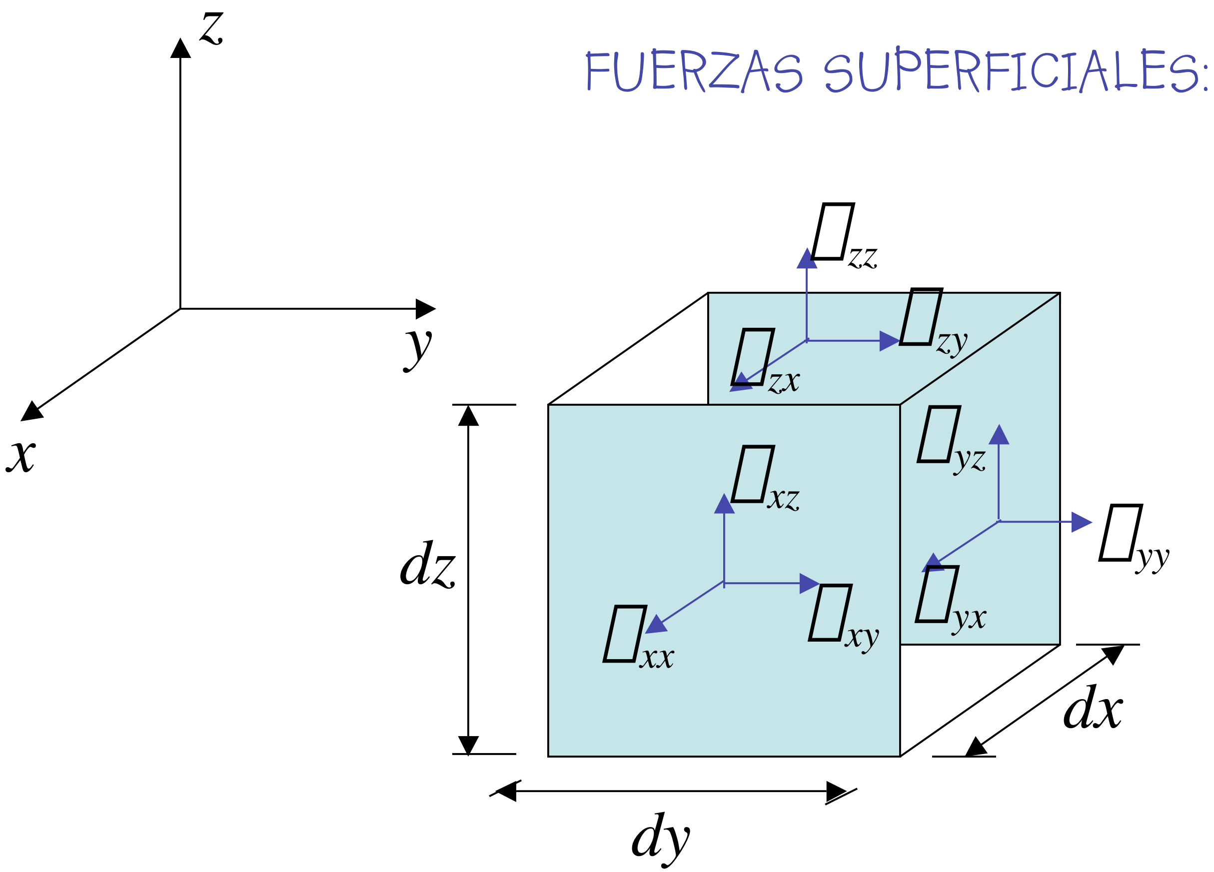
PRINCIPIO DE CONSERVACION DEL MOMENTUM (ENFOQUE DIFERENCIAL)

FUERZAS SUPERFICIALES:



PRINCIPIO DE CONSERVACION DEL MOMENTUM (ENFOQUE DIFERENCIAL)

FUERZAS SUPERFICIALES:



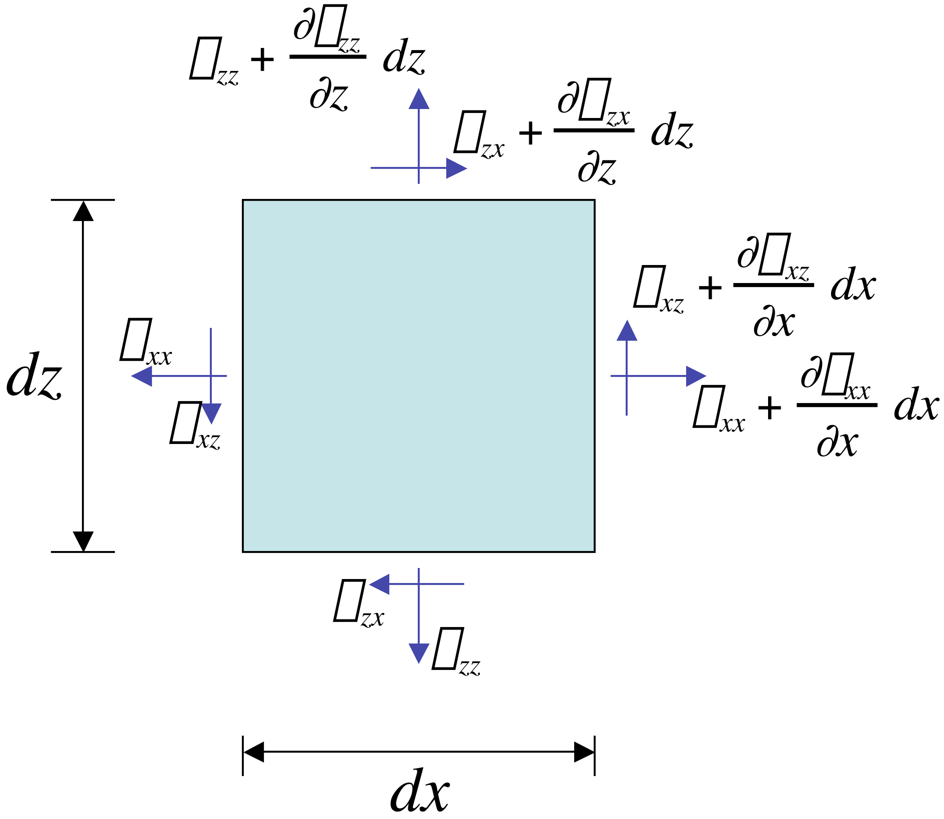
TENSOR DE ESFUERZOS:

$$\square_{ij}$$

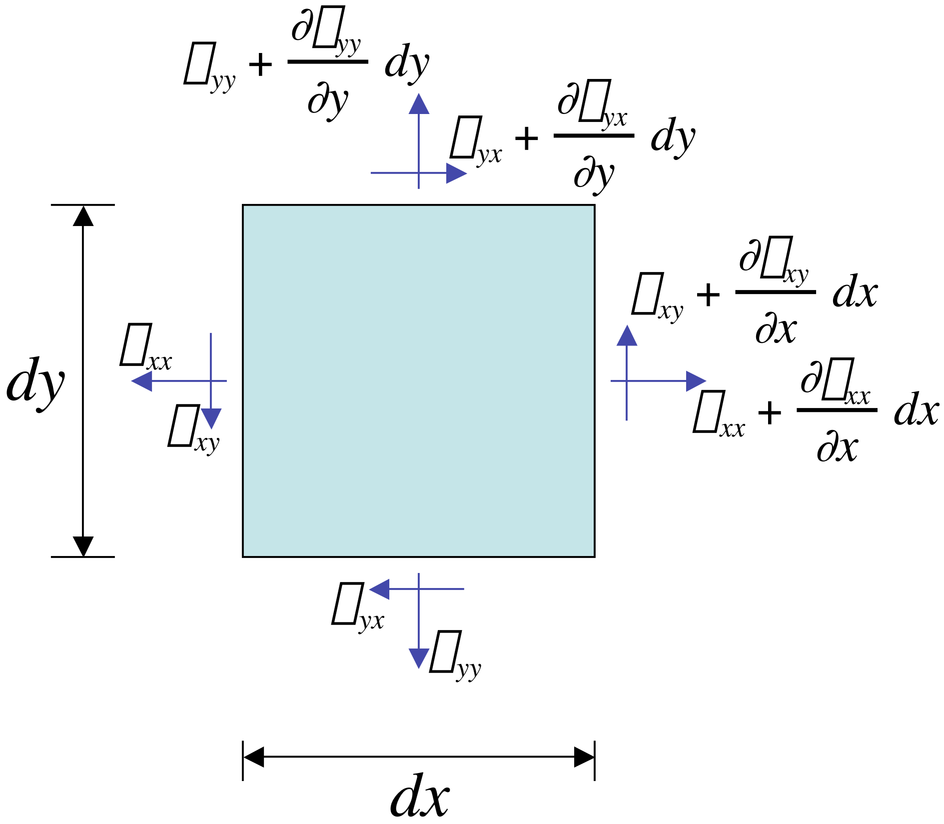
APUNTA EN DIRECCION j

EN UNA CARA CUYA NORMAL ES i

PRINCIPIO DE CONSERVACION DEL MOMENTUM (ENFOQUE DIFERENCIAL)



PRINCIPIO DE CONSERVACION DEL MOMENTUM (ENFOQUE DIFERENCIAL)



$$dF_{sx} = \left\{ \left(\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x}\, dx\right) - \sigma_{xx} \right\} dy\, dz +$$

$$\left\{ \left(\tau_{zx} + \frac{\partial \tau_{zx}}{\partial z}\, dz\right) - \tau_{zx} \right\} dx\, dy +$$

$$\left\{ \left(\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y}\, dy\right) - \tau_{yx} \right\} dx\, dz$$

$$dF_{sx} = \left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right) dx\, dy\, dz$$

$$dF_{sy} = \left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}\right) dx\, dy\, dz$$

$$dF_{sz} = \left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z}\right) dx\, dy\, dz$$

$$\frac{dF_{sj}}{dV} = \frac{\partial \tau_{xj}}{\partial x} + \frac{\partial \tau_{yj}}{\partial y} + \frac{\partial \tau_{zj}}{\partial z}$$

NOTACION TENSORIAL $$\frac{dF_{sj}}{dV} = \frac{\partial \tau_{ij}}{\partial x_i}$$

SUBINDICE REPETIDO → SUMATORIA SOBRE ESE INDICE

PRINCIPIO DE CONSERVACION DEL MOMENTUM (ENFOQUE DIFERENCIAL)

$$\frac{d\vec{F}}{dV} = \frac{d\vec{F}_m}{dV} + \frac{d\vec{F}_s}{dV} = \rho \frac{D\vec{v}}{Dt} = \rho \left(\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v}\right)$$

$$\rho \vec{g}$$

$$\left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right)\hat{i} + \left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}\right)\hat{j} + \left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}\right)\hat{k}$$

$$u\frac{\partial \vec{v}}{\partial x} + v\frac{\partial \vec{v}}{\partial y} + w\frac{\partial \vec{v}}{\partial z}$$

PRINCIPIO DE CONSERVACION DEL MOMENTUM (ENFOQUE DIFERENCIAL)

$$S/x: \quad \left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{yx}}{\partial y} + \frac{\partial \sigma_{zx}}{\partial z}\right) + \rho g_x = \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right)$$

$$S/y: \quad \left(\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{zy}}{\partial z}\right) + \rho g_y = \rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right)$$

$$S/z: \quad \left(\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z}\right) + \rho g_z = \rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right)$$

PRINCIPIO DE CONSERVACION DEL MOMENTUM (ENFOQUE DIFERENCIAL)

NOTACION TENSORIAL

$$\frac{\partial \sigma_{ij}}{\partial x_i} + \rho g_j = \rho \left(\frac{\partial u_j}{\partial t} + u_i \frac{\partial u_j}{\partial x_i} \right)$$

TENSOR DE ESFUERZOS

PRESION

ESFUERZOS VISCOSOS

PRINCIPIO DE CONSERVACION DEL MOMENTUM (ENFOQUE DIFERENCIAL)

LEY CONSTITUTIVA

RELACIONAR TENSOR DE ESFUERZOS CON TENSOR DE DEFORMACION

LEY CONSTITUTIVA

HIPOTESIS DE STOKES

1. EL FLUIDO ES CONTINUO

2. EXISTE UNA RELACION LINEAL ENTRE ESFUERZOS TANGENCIALES Y DEFORMACIONES ANGULARES

3. EL FLUIDO ES ISOTROPICO

4. CUANDO LAS DEFORMACIONES ANGULARES SE ANULAN, LOS ESFUERZOS TANGENCIALES SE ANULAN Y LOS ESFUERZOS NORMALES SE IGUALAN A LA PRESION TERMODINAMICA

LEY CONSTITUTIVA

$$\tau_{ij} = f(\varepsilon_{ij})$$

TENSOR DE ESFUERZOS

TENSOR DE DEFORMACION

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j}\right)$$

$$\tau_{ij} = 2\mu\varepsilon_{ij} = \mu\left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j}\right) \qquad i \neq j$$

VISCOSIDAD

$$\sigma_{ij} = -p + 2\mu\varepsilon_{ij} + \lambda(\nabla \cdot \vec{v}) \qquad i = j$$

2do COEFICIENTE DE VISCOSIDAD

LEY CONSTITUTIVA

ESFUERZOS NORMALES PROMEDIO

$$\bar{\sigma}_n = \frac{1}{3}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz}) = \frac{1}{3}\{\ 3\,(-p + \lambda\,\nabla \cdot \vec{v}\,) + 2\,\mu\frac{1}{2}((\frac{\partial u}{\partial x} + \frac{\partial u}{\partial x}) + (\frac{\partial v}{\partial y} + \frac{\partial v}{\partial y}) + (\frac{\partial w}{\partial z} + \frac{\partial w}{\partial z}))\ \}$$

$$\bar{\sigma}_n = \frac{1}{3}\{\ (3\,(-p + \lambda\,\nabla \cdot \vec{v}\,) + 2\,\mu\,\underbrace{(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z})}_{\nabla \cdot \vec{v}})\ \}$$

$$\bar{\sigma}_n = -p + (\lambda + \frac{2}{3}\,\mu)\,\nabla \cdot \vec{v}$$

FLUIDO INCOMPRESIBLE

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FLUIDO INCOMPRESIBLE

$$\vec{\sigma}_n = -p$$

$$\sigma_{ij} = 2\,\mu\, e_{ij} = \mu\left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j}\right) \qquad i \neq j$$

$$\sigma_{ij} = -p + 2\,\mu\, e_{ij} + \lambda\,\cancel{(\nabla \cdot \vec{v})} \qquad i = j$$

$$\sigma_{ij} = -p\,\delta_{ij} + 2\,\mu\, e_{ij}$$

DELTA DE KROENECKER

PRINCIPIO DE CONSERVACION DEL MOMENTUM (ENFOQUE DIFERENCIAL)

$$\frac{\partial \tau_{ij}}{\partial x_i} + \rho g_j = \rho\left(\frac{\partial u_j}{\partial t} + u_i \frac{\partial u_j}{\partial x_i}\right)$$

$$\frac{\partial}{\partial x_i}\left(-p\,\delta_{ij} + 2\mu\,\varepsilon_{ij}\right) + \rho g_j = \rho\left(\frac{\partial u_j}{\partial t} + u_i \frac{\partial u_j}{\partial x_i}\right)$$

$$\frac{\partial}{\partial x_i}\left(-p\,\delta_{ij} + \mu\left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j}\right)\right) + \rho g_j = \rho\left(\frac{\partial u_j}{\partial t} + u_i \frac{\partial u_j}{\partial x_i}\right)$$

PRINCIPIO DE CONSERVACION DEL MOMENTUM (ENFOQUE DIFERENCIAL)

$$\frac{\partial}{\partial x_i}(-p\,\delta_{ij}) = -(\frac{\partial p}{\partial x_1}\delta_{1j} + \frac{\partial p}{\partial x_2}\delta_{2j} + \frac{\partial p}{\partial x_3}\delta_{3j}) = -(\frac{\partial p}{\partial x_j}\underbrace{\cancel{\delta_{jj}}}_{1}) = -\frac{\partial p}{\partial x_j}$$

$$\frac{\partial}{\partial x_i}(\mu(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j})) = \mu(\frac{\partial^2 u_j}{\partial x_i^2} + \frac{\partial}{\partial x_i}\frac{\partial u_i}{\partial x_j}) = \mu(\frac{\partial^2 u_j}{\partial x_i^2} + \frac{\partial}{\partial x_j}\frac{\partial u_i}{\partial x_i})$$

$$\frac{\partial}{\partial x_j}(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z})$$

$$\nabla \cdot \vec{v} = 0$$

PRINCIPIO DE CONSERVACION DEL MOMENTUM (ENFOQUE DIFERENCIAL)

$$-\frac{\partial p}{\partial x_j} + \mu \frac{\partial^2 u_j}{\partial x_i\, \partial x_i} + \rho g_j = \rho \left(\frac{\partial u_j}{\partial t} + u_i \frac{\partial u_j}{\partial x_i} \right)$$

$$\nabla p \qquad \nabla^2 \vec{v} \qquad \vec{g} \qquad \frac{D\vec{v}}{Dt}$$

$$\rho \frac{D\vec{v}}{Dt} = -\nabla p + \mu \nabla^2 \vec{v} + \rho \vec{g}$$

ECUACION DE NAVIER-STOKES

(FLUIDO INCOMPRESIBLE Y NEWTONIANO)

ECUACION DE NAVIER-STOKES
(FLUIDO INCOMPRESIBLE Y NEWTONIANO)

$$\rho \frac{D\vec{v}}{Dt} = -\nabla p + \mu \nabla^2 \vec{v} + \rho \vec{g}$$

COORDENADAS CARTESIANAS:

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \rho g_x + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = -\frac{\partial p}{\partial y} + \rho g_y + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} + \rho g_z + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$

ECUACION DE NAVIER-STOKES
(FLUIDO INCOMPRESIBLE Y NEWTONIANO)

$$\rho \frac{D\vec{v}}{Dt} = -\nabla p + \mu \nabla^2 \vec{v} + \rho \vec{g}$$

3 ECUACIONES

4 INCOGNITAS:

$$u,\ v,\ w,\ p$$

4ta ECUACION:

$$\nabla \cdot \vec{v} = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

ECUACION DE NAVIER-STOKES
(FLUIDO INCOMPRESIBLE Y NEWTONIANO)

$$\frac{\partial \vec{v}}{\partial t} + \underbrace{u\frac{\partial \vec{v}}{\partial x} + v\frac{\partial \vec{v}}{\partial y} + w\frac{\partial \vec{v}}{\partial z}}_{\text{ACELERACION CONVECTIVA}} = -\frac{1}{\rho}\nabla p + \vec{g} + \frac{\mu}{\rho}\nabla^2 \vec{v}$$

ACELERACION LOCAL

PRESION

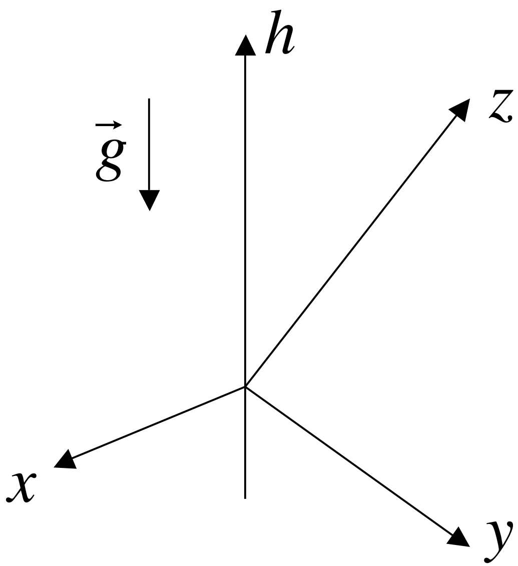
PESO

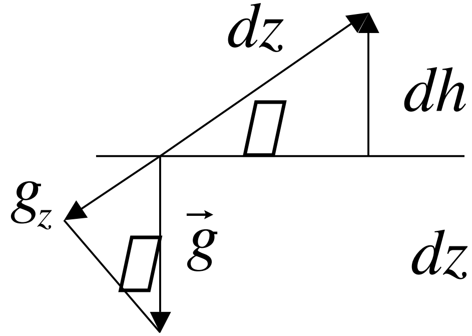
FUERZAS VISCOSAS

ν

$$\nabla \cdot \vec{v} = 0$$

ACELERACION DE GRAVEDAD





$$dz \sin\theta = dh$$

$$\sin\theta = \frac{dh}{dz}$$

$$g_z = -g \sin\theta$$

$$g_z = -g \frac{\partial h}{\partial z} \qquad g_x = -g \frac{\partial h}{\partial x} \qquad g_y = -g \frac{\partial h}{\partial y}$$

$$\vec{g} = -g \nabla h$$

$$\frac{\partial \vec{v}}{\partial t} + u\frac{\partial \vec{v}}{\partial x} + v\frac{\partial \vec{v}}{\partial y} + w\frac{\partial \vec{v}}{\partial z} = -\frac{1}{\rho}\nabla p + \vec{g} + \nu\nabla^2\vec{v}$$

$$\vec{g} = -g\,\nabla h$$

$$\frac{\partial \vec{v}}{\partial t} + u\frac{\partial \vec{v}}{\partial x} + v\frac{\partial \vec{v}}{\partial y} + w\frac{\partial \vec{v}}{\partial z} = -\frac{1}{\rho}\nabla p - g\,\nabla h + \nu\nabla^2\vec{v}$$

$$-\nabla\left(\frac{p}{\rho} + g\,h\right)$$

$$-\nabla\frac{\hat{p}}{\rho}$$

PRESION MOTRIZ: $\hat{p} = p + \rho\, g\, h$

ECUACION DE NAVIER-STOKES
(FLUIDO INCOMPRESIBLE Y NEWTONIANO)

$$\frac{D\vec{v}}{Dt} = -\frac{1}{\rho}\nabla\hat{p} + \nu\nabla^2\vec{v}$$

$$\nabla \cdot \vec{v} = 0$$