TRIGONOMETRIA

UNIDADES :

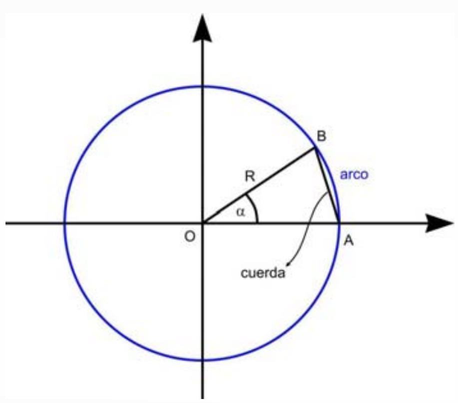
- GRADOS (°) , MINUTOS (')
- RADIANES

EQUIVALENCIA :

GRADOS	RADIANES	
360°	2π	GIRO COMPLETO
180°	π	$\frac{1}{2}$ GIRO
90°	$\frac{\pi}{2}$	$\frac{1}{4}$ GIRO
1°	$\frac{\pi}{180}$	$\frac{1}{360}$ GIRO

Introducción a la Física FI10A-02
Andrés Meza
Semestre Otoño 2006

RADIANES



$$\frac{\text{longitud}}{\text{radio}} = \frac{2\pi R}{R} = 2\pi$$

independiente del radio de la circunf.

DEFINICIÓN

La magnitud de un ángulo en RADIANES es igual a la razón entre la longitud del arco de circunferencia que subtiende y el valor del radio de dicha circunferencia

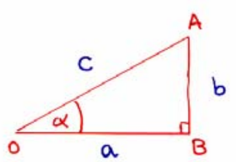
Ej.

$$\theta = 360° \Rightarrow \theta = \frac{2\pi R}{R} = 2\pi \text{ rad}$$

$$1 \text{ rad} = 57{,}3°$$

$$\boxed{\text{LONGITUD ARCO} = \alpha R}$$

Funciones seno, coseno y tangente



$$\text{sen}\,\alpha \equiv \frac{|AB|}{|OA|} = \frac{b}{c}$$

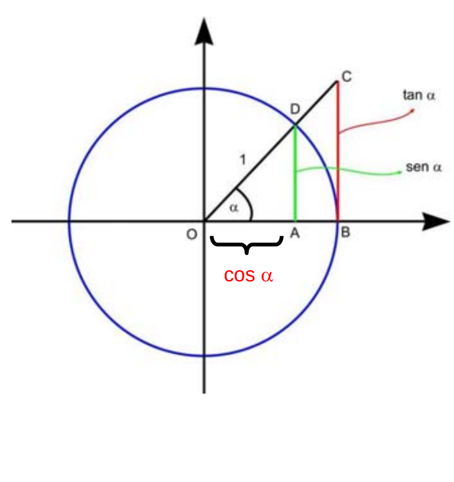
$$\cos\alpha \equiv \frac{|OB|}{|OA|} = \frac{a}{c}$$

$$\tan\alpha \equiv \frac{\text{sen}\,\alpha}{\cos\alpha} = \frac{b}{a}$$

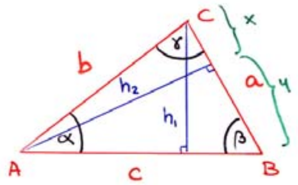
PROPIEDADES

- $\text{sen}^2\alpha + \cos^2\alpha = 1$
- $\text{sen}\left(\frac{\pi}{2} - \alpha\right) = \cos\alpha$
- $\cos\left(\frac{\pi}{2} - \alpha\right) = \text{sen}\,\alpha$

Otra manera…



Teoremas del seno y del coseno



$$h_1 = b \operatorname{sen} \alpha$$

$$h_1 = a \operatorname{sen} \beta$$

$$\Rightarrow \quad b \operatorname{sen} \alpha = a \operatorname{sen} \beta$$

$$\frac{b}{\operatorname{sen} \beta} = \frac{a}{\operatorname{sen} \alpha}$$

POR OTRO LADO

$$h_2 = c \operatorname{sen} \beta$$

$$h_2 = b \operatorname{sen} \gamma \qquad \Rightarrow \quad \frac{b}{\operatorname{sen} \beta} = \frac{c}{\operatorname{sen} \gamma}$$

Teorema del seno

$$\boxed{\frac{a}{\operatorname{sen} \alpha} = \frac{b}{\operatorname{sen} \beta} = \frac{c}{\operatorname{sen} \gamma}}$$

TEOREMA DEL COSENO

$$\boxed{c^2 = a^2 + b^2 - 2ab \cos \gamma}$$

USANDO TEOREMA DE PITÁGORAS

$$h_2^2 + x^2 = b^2 \Rightarrow h_2^2 = b^2 - x^2$$

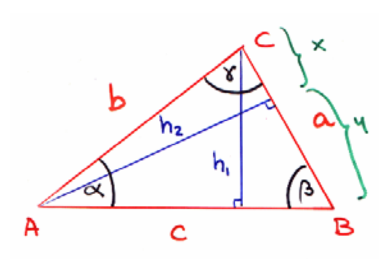
$$h_2^2 + y^2 = c^2 \Rightarrow h_2^2 = c^2 - y^2$$

PERO $\quad x + y = a \Rightarrow y = a - x$

ENTONCES

$$b^2 - x^2 = c^2 - (a-x)^2$$

$$b^2 - \cancel{x^2} = c^2 - a^2 + 2ax - \cancel{x^2}$$



$$b^2 = c^2 - a^2 + 2ax$$

PERO $\quad x = b\cos\gamma$

$$\therefore \quad c^2 = a^2 + b^2 - 2ab\cos\gamma$$

ANALOGAMENTE

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$b^2 = a^2 + c^2 - 2ac\cos\beta$$

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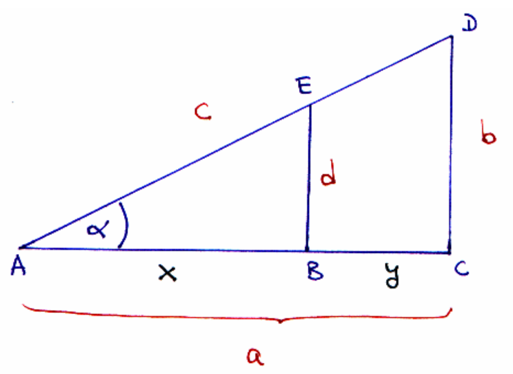


Teorema de Tales de Mileto

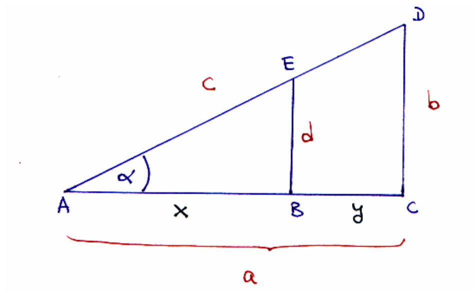
$$\frac{\overline{BE}}{\overline{CD}} = \frac{d}{b} = \frac{x}{a} = \frac{\overline{AB}}{\overline{AC}}$$

Dem:

$$\tan\alpha \equiv \frac{\operatorname{sen}\alpha}{\cos\alpha} = \frac{\overline{BE}}{\overline{AB}} = \frac{d}{x}$$

$$\tan\alpha = \frac{\overline{CD}}{\overline{AC}} = \frac{b}{a}$$

$$\Rightarrow \frac{d}{x} = \frac{b}{a} \Rightarrow \boxed{\frac{d}{b} = \frac{x}{a}}$$



SERIES

$$(1+x)^{\alpha} \equiv 1 + \frac{\alpha}{1!}x + \frac{\alpha(\alpha-1)}{2!}x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!}x^3 + \ldots\ldots$$

Factorial:

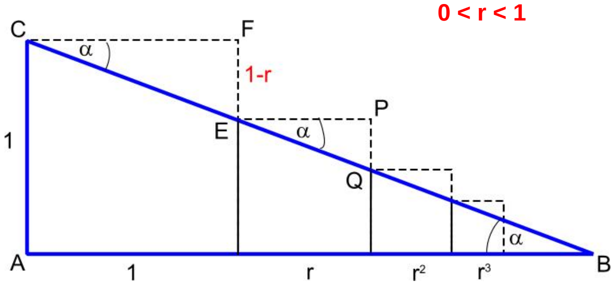
$$0! = 1$$
$$1! = 1$$
$$2! = 1.2 = 2$$
$$3! = 1.2\cdot 3 = 6$$
$$4! = 1\cdot 2\cdot 3\cdot 4 = 24$$
$$\vdots$$

Demuestre que

$$\sum_{n=0}^{\infty} r^n = 1 + r + r^2 + r^3 + \ldots = \frac{1}{1-r}$$

Dem:

0 < r < 1



$\overline{EP} = r \qquad \overline{CF} = 1 \qquad \overline{EF} = 1 - r$

$\overline{PQ} = \,?$

En el ΔCEF :

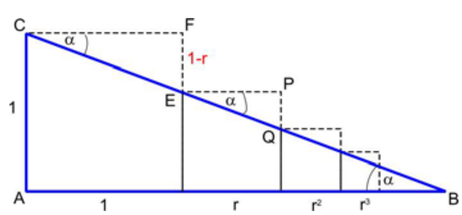
$$\tan\alpha = \frac{\overline{EF}}{\overline{CF}} = \frac{1-r}{1} = 1-r$$

En el ΔEPQ :

$$\tan\alpha = \frac{\overline{PQ}}{\overline{EP}} = \frac{\overline{PQ}}{r}$$

$$\Rightarrow \qquad 1 - r = \frac{\overline{PQ}}{r}$$

$$\overline{PQ} = r(1-r)$$



En el ΔABC :

$$\tan\alpha = \frac{\overline{AC}}{\overline{AB}} = \frac{1}{\overline{AB}}$$

pero

$$\tan\alpha = \frac{\overline{PQ}}{r} = 1 - r$$

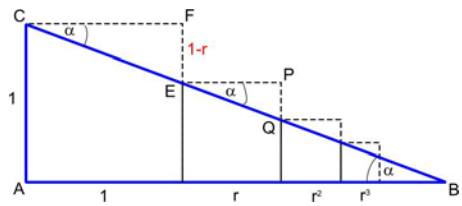
$$\Rightarrow \quad 1 - r = \frac{1}{\overline{AB}}$$

$$\underbrace{\overline{AB}} = \frac{1}{1-r}$$

$$\underbrace{1 + r + r^2 + r^3 + \ldots} = \frac{1}{1-r}$$

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$$

QED



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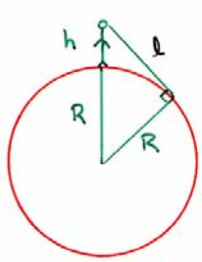
ALCANCE VISUAL SOBRE EL HORIZONTE



$R = 6400$ km

$$l^2 + R^2 = (R+h)^2$$

$$l^2 + \cancel{R^2} = \cancel{R^2} + 2Rh + h^2$$

$$l = \sqrt{2Rh + h^2}$$

$$l = \sqrt{2Rh}\left(1 + \frac{h}{2R}\right)^{1/2}$$

APROXIMACIÓN

Si $x << 1$ $\quad (1+x)^{\alpha} \approx 1 + \alpha x$

EN NUESTRO CASO $h \approx 2\ m$

$$\Rightarrow \quad \frac{h}{2R} = \frac{1}{6.4\times10^{6}} \approx 10^{-7} << 1$$

POR LO TANTO

$$l \approx \sqrt{2Rh}\left(1 + \frac{h}{4R}\right)$$

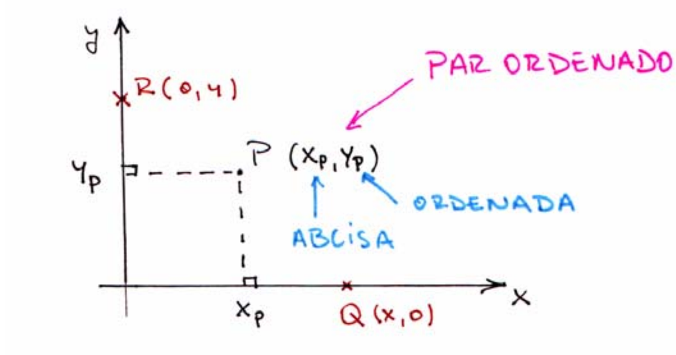
EVALUANDO

$$l \approx \sqrt{2\times 6.4\times10^{6}\times 2} \times 1 \quad m$$

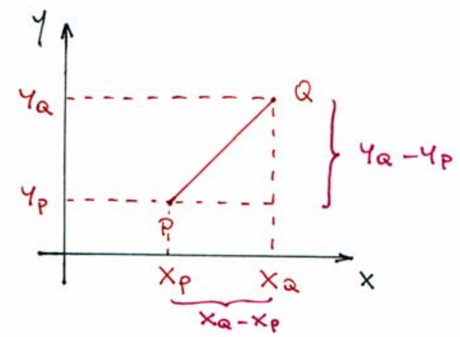
$$l \approx 2\times10^{3}\sqrt{6.4} \quad m$$

$$l \approx 5 \quad km$$

COORDENADAS



DISTANCIA ENTRE 2 PUNTOS



$$|PQ| = \sqrt{(x_Q - x_P)^2 + (y_Q - y_P)^2}$$

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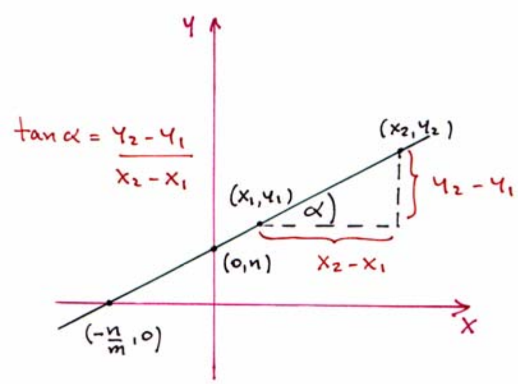
ECUACIÓN DE LA RECTA



$$y = mx + n$$

m : PENDIENTE $(m = \tan\alpha)$

n : VALOR DE LA COORDENADA DONDE LA RECTA CORTA EL EJE y