Name:

MWF9 Ivan Petrakiev
MWF10 Oliver Knill
MWF10 Thomas Lam
MWF10 Michael Schein
MWF10 Teru Yoshida
MWF11 Andrew Dittmer
MWF11 Chen-Yu Chi
MWF12 Kathy Paur
TTh10 Valentino Tosatti
TTh11.5 Kai-Wen Lan
TTh11.5 Jeng-Daw Yu

- Please mark the box to the left which lists your section.
- Do not detach pages from this exam packet or unstaple the packet.
- Show your work. Answers without reasoning can not be given credit except for the True/False and multiple choice problems.
- Please write neatly.
- Do not use notes, books, calculators, computers, or other electronic aids.
- Unspecified functions are assumed to be smooth and defined everywhere unless stated otherwise.
- You have 180 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10

11A		10
12A		10
13A		10

11B		10
12B		10
13B		10

Total:		140

Problem 1) True/False questions (20 points)

1) T F The projection vector $\text{proj}_{\vec{v}}(\vec{w})$ is parallel to $\vec{w}$.

2) T F Any parametrized surface S is a graph of a function $f(x,y)$.

3) T F If the directional derivatives $D_{\vec{v}}(f)(1,1)$ and $D_{\vec{w}}(f)(1,1)$ are both 0 for $\vec{v} = \langle 1,1\rangle/\sqrt{2}$ and $\vec{w} = \langle 1,-1\rangle/\sqrt{2}$, then $(1,1)$ is a critical point.

4) T F The linearization $L(x,y)$ of $f(x,y) = x+y+4$ at $(0,0)$ satisfies $L(x,y) = f(x,y)$.

5) T F For any function $f(x,y)$ of two variables, the line integral of the vector field $\vec{F} = \nabla f$ on a level curve $\{f = c\}$ is always zero.

6) T F If $\vec{F}$ is a vector field of unit vectors defined in $1/2 \leq x^2+y^2 \leq 2$ and $\vec{F}$ is tangent to the unit circle C, then $\int_C \vec{F}\cdot d\vec{r}$ is either equal to 2π or -2π.

7) T F If a curve C intersects a surface S at a right angle, then at the point of intersection, the tangent vector to the curve is parallel to the normal vector of the surface.

8) T F The curvature of the curve $\vec{r}(t) = \langle \cos(3t), \sin(6t)\rangle$ at the point $\vec{r}(0)$ is smaller than the curvature of the curve $\vec{r}(t) = \langle \cos(30t), \sin(60t)\rangle$ at the point $\vec{r}(0)$.

9) T F At every point (x,y,z) on the hyperboloid $x^2+y^2-z^2 = 1$, the vector $\langle x, y, -z\rangle$ is tangent to the hyperboloid.

10) T F The set $\{\phi = \pi/2, \theta = \pi\}$ in spherical coordinates is the negative x axis.

11) T F The integral $\int_0^1 \int_0^{2\pi} \int_0^{\pi} \rho^2 \sin^2(\phi)\ \ d\phi\ d\theta\ d\rho$ is equal to the volume of the unit ball.

12) T F Four points A, B, C, D are located in a single common plane if $(B-A)\cdot((C-A)\times(D-A)) = 0$.

13) T F If a function $f(x,y)$ has a local maximum at $(0,0)$, then the discriminant D is negative.

14) T F The integral $\int_0^x \int_y^1 f(x,y)\ dx\ dy$ represents a double integral over a bounded region in the plane.

15) T F The following identity is true: $\int_0^3 \int_0^{2x} x^2\ dydx = \int_0^6 \int_{y/2}^3 x^2\ dx\ dy$

TF problems 16-20 are for regular and physics sections only:

16) T F The integral $\iint_S \mathrm{curl}(\vec{\mathrm{F}}) \cdot d\vec{\mathrm{S}}$ over the surface S of a cube is zero for all vector fields $\vec{F}$.

17) T F A vector field $\vec{F}$ defined on three space which is incompressible ($\mathrm{div}(\vec{F}) = 0$) and irrotational ($\mathrm{curl}(\vec{F}) = 0$) can be written as $\vec{F} = \nabla f$ with $\Delta f = \nabla^2 f = 0$.

18) T F If a vector field $\vec{F}$ is defined at all points of three-space except the origin, and $\mathrm{curl}(\vec{F}) = \vec{0}$ everywhere, then the line integral of $\vec{F}$ around the circle $x^2 + y^2 = 1$ in the xy-plane is equal to zero.

19) T F The identity $\mathrm{curl}(\mathrm{grad}(\mathrm{div}(\vec{F}))) = \vec{0}$ is true for all vector fields $\vec{F}(x, y, z)$.

20) T F If $\vec{F} = \mathrm{curl}(\vec{G})$, where $\vec{G} = \langle e^{e^x}, 5^x z^5, \sin y \rangle$, then $\mathrm{div}(\vec{F}(x, y, z)) > 0$ for all (x, y, z).

TF problems 21-25 are for biochem sections only:

21) T F The expected value of the sum of two random variables is the sum of their expected values.

22) T F Let X be a random variable. Suppose that we know both the expectation $E(X)$ and and variance $\mathrm{D}(X)$. Does this information determine $\mathrm{E}(X^2 - 5X + 4)$?

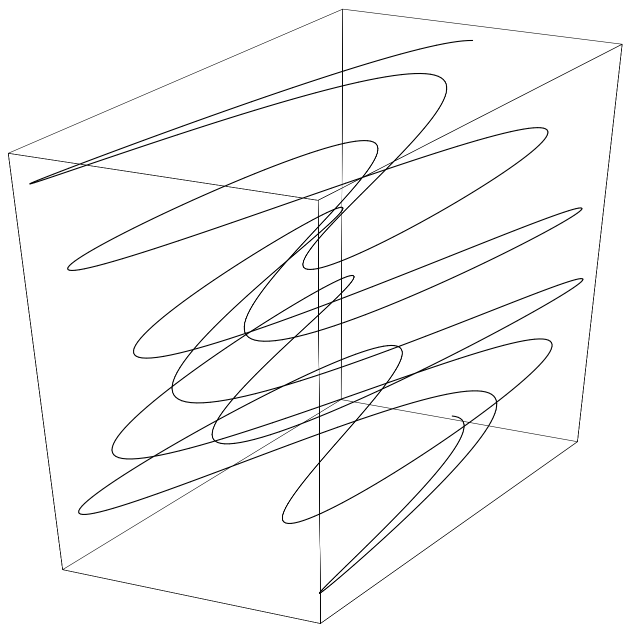
23) T F If A and B are two events and B has positive probability, then $P(A|B)$ is always less than or equal to $P(A)$.

24) T F The function $\Phi_\xi = \frac{1}{1+x^2}$ is the distribution function of some random variable ξ.

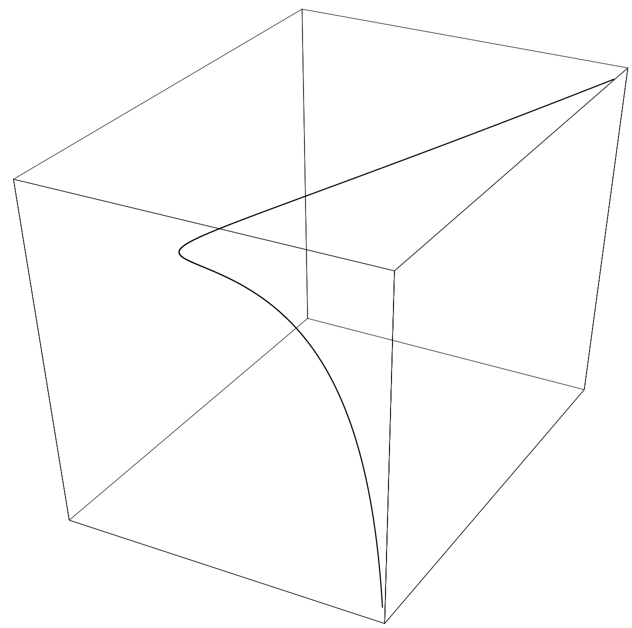
25) T F Suppose you throw two fair dice. The probability that the sum of their upturned faces is 11 is $2/36$.

Problem 2) (10 points)

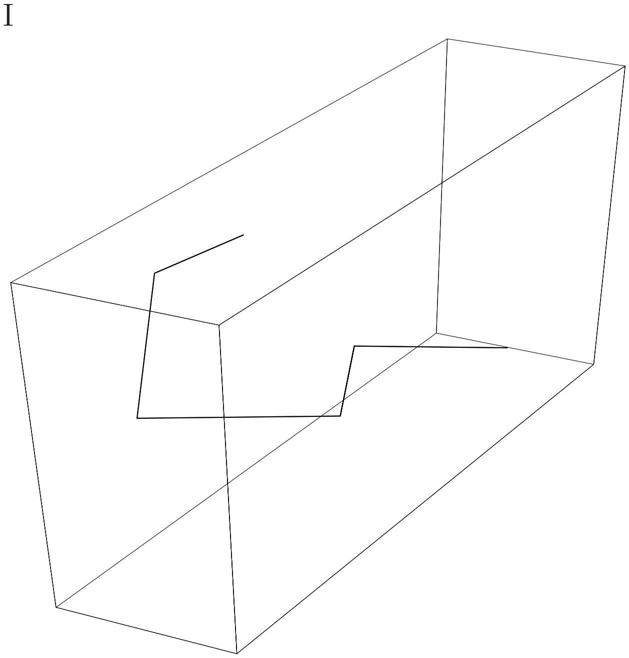
Match the equations with the space curves. No justifications are needed.



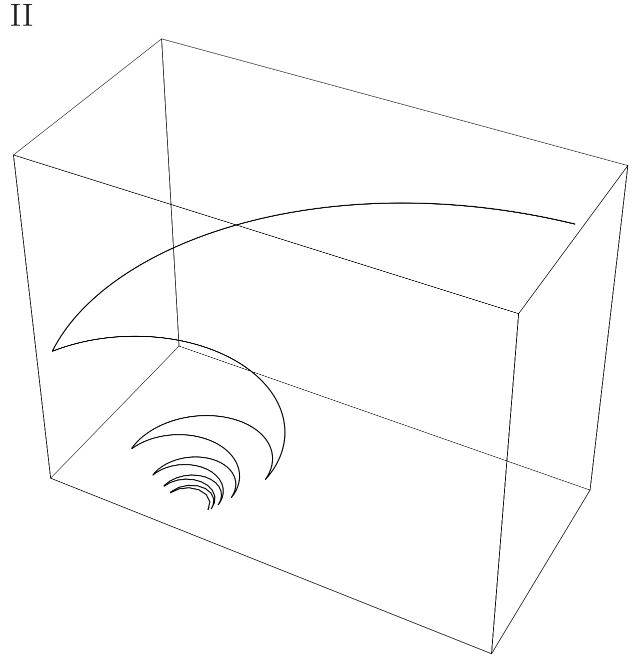
I



II



III

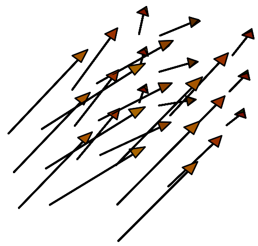


IV

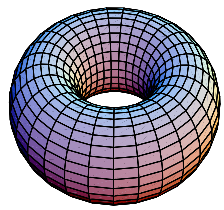
Enter I,II,III,IV here	Equation
	$\vec{r}(t) = \langle t^2, t^3 - t, t\rangle$
	$\vec{r}(t) = \langle \lvert 1 - \lvert t\rvert\rvert, \lvert t - \lvert t-1\rvert\rvert, t\rangle$
	$\vec{r}(t) = \langle 2\sin(5t), \cos(11t), t\rangle$
	$\vec{r}(t) = \langle t\sin(1/t), t\lvert\cos(1/t)\rvert, t\rangle$

Problem 3) (10 points)

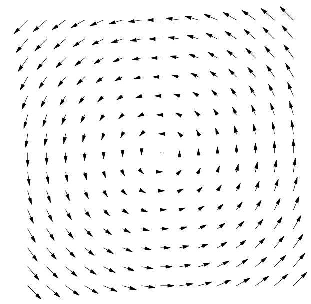
Match the equations with the objects. No justifications are needed.



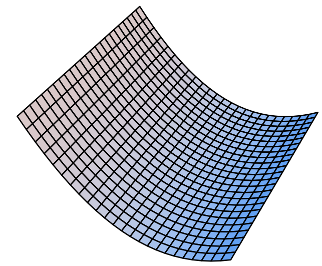
I



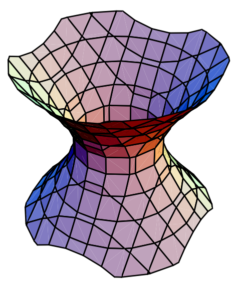
II



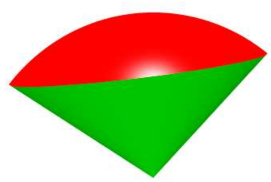
III



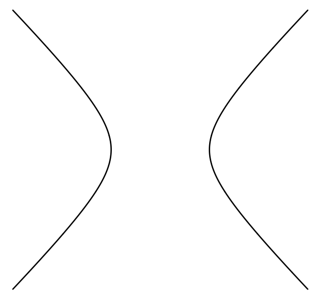
IV



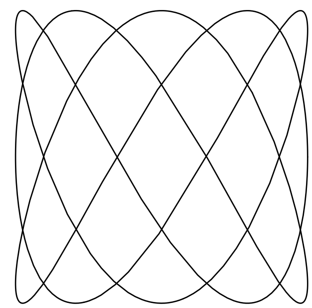
V



VI



VII



VIII

Enter I,II,III,IV,V,VI,VII,VIII here	Equation
	$\vec{r}(s,t) = \langle (2+\cos(s))\cos(t), (2+\cos(s))\sin(t), \sin(s) \rangle$
	$\vec{r}(t) = \langle \cos(3t), \sin(5t) \rangle$
	$x^2 + y^2 - z^2 = 1$
	$\vec{F}(x,y,z) = \langle -y, x, 1 \rangle$
	$x^2 + y^2 + z^2 \leq 1, x^2 + y^2 \leq z^2, z \geq 0$
	$z = f(x,y) = x^2 - y$
	$g(x,y) = x^2 - y^2 = 1$
	$\vec{F}(x,y) = \langle -y, x \rangle$

Problem 4) (10 points)

a) Find an equation for the plane Σ passing through the points $\vec{r}(0), \vec{r}(1), \vec{r}(2)$, where $\vec{r}(t) = \langle t^2, t^4, t \rangle$.

b) Find the distance between the point $\vec{r}(-1)$ and the plane Σ found in a).

Problem 5) (10 points)

A vector field $\vec{F}(x, y)$ in the plane is given by $\vec{F}(x, y) = \langle x^2+5, y^2-1 \rangle$. Find all the critical points of $|\vec{F}(x, y)|$ and classify them. At which point or points is $|\vec{F}(x, y)|$ minimal?

Hint: Extremize $f(x, y) = |\vec{F}(x, y)|^2$.

Problem 6) (10 points)

A house is situated at the point $(0, 0)$ in the middle of a mountainous region. The altitude at each point (x, y) is given by the equation $f(x, y) = 4x^2y + y^3$. There is a pathway in the shape of an ellipse around the house, on which the (x, y) coordinates satisfy $2x^2 + y^2 = 6$. Find the highest and lowest points in the closed region bounded by the path.

Problem 7) (10 points)

a) (4 points) Where does the tangent plane at $(1, 1, 1)$ to the surface $z = e^{x-y}$ intersect the z axis?

b) (4 points) Estimate $f(x, y, z) = 1 + \log(1 + x + 2y + z) + 2\sqrt{1+z}$ at the point $(0.02, -0.001, 0.01)$.

c) (2 points) $f(x, y, z) = 0$ defines z as a function $g(x, y)$ of x and y. Find the partial derivative $g_x(x, y)$ at the point $(x, y) = (0, 0)$.

Problem 8) (10 points)

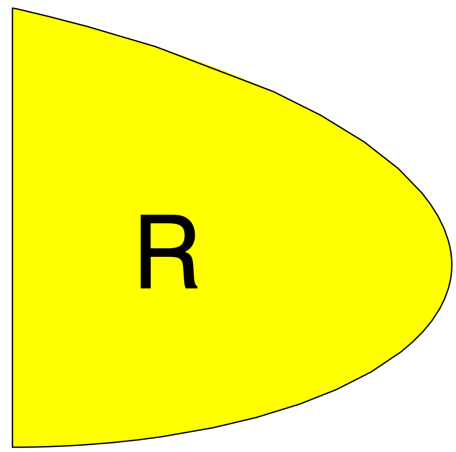
For each of the following quantities, set up a double or triple integral using any coordinate system you like. You do not have to evaluate the integrals, but the bounds of each single integral must be specified explicitly.

1. (3 points) The volume of the tetrahedron with vertices $(0,0,0)$, $(3,0,0)$, $(0,3,0)$ and $(0,0,3)$.
2. (4 points) The surface area of the piece of the paraboloid $z = x^2 + y^2$ lying in the region $z = x^2 + y^2$, where $0 \leq z \leq 1$.
3. (3 points) The volume of the solid bounded by the planes $z = -1$, $z = 1$ and the one-sheeted hyperboloid $x^2 + y^2 - z^2 = 1$.

Problem 9) (10 points)

A region R in the xy-plane is given in polar coordinates by $r(\theta) \leq \theta$ for $\theta \in [0, \pi/2]$. Find the double integral

$$\iint_R \frac{\sin(\sqrt{x^2+y^2})}{\sqrt{x^2+y^2}(\pi/2 - \sqrt{x^2+y^2})} \, dx \, dy \, .$$



Problem 10) (10 points)

A car drives up a freeway ramp which is parametrized by

$$\vec{r}(t) = \langle \cos(t), 2\sin(t), t \rangle, \qquad 0 \leq t \leq 3\pi \, .$$

a) (3 points) Set up an integral which gives the length of the ramp. You do need not need to evaluate it.

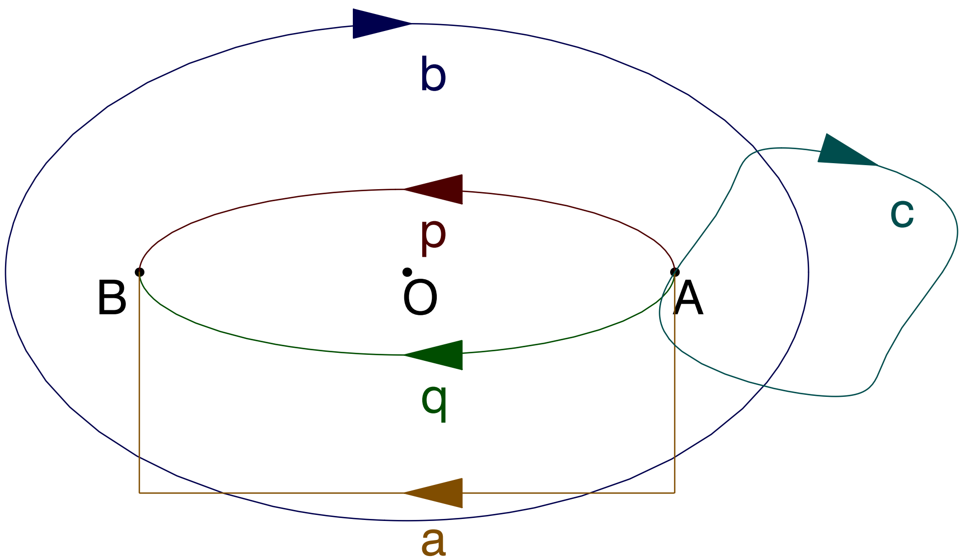
b) (3 points) Find the unit tangent vector $\vec{T}$ to the curve at the point where $t = 0$.

c) (4 points) Suppose the wind pattern in the area is such that the wind exerts a force $\vec{F} = \langle 4x^2, y, 0 \rangle$ on the car at the position (x, y, z). What is the total work done by the car against the wind as it drives up the ramp?

The following problem 11A is for regular and physics sections only:

Problem 11 A) (10 points)

Suppose $\vec{F}$ is an irrotational vector field in the plane (that is, its curl is everywhere zero) that is not defined at the origin $O = (0,0)$. Suppose the line integral of $\vec{F}$ along the path p from A to B is 5 and the line integral of $\vec{F}$ along the path q from A to B is -4. Find the line integral of $\vec{F}$ along the following three paths:



a) (3 points) The path a from A to B going clockwise below the origin.

b) (4 points) The closed path b encircling the origin in a clockwise direction.

c) (3 points) The closed path c starting at A and ending in A without encircling the origin.

The following problem 12A is for regular and physics sections only:

Problem 12 A) (10 points)

Let S be the surface which bounds the region enclosed by the paraboloid $z = x^2 + y^2 - 1$ and the xy plane. Let $\vec{F}$ be the vector field $\vec{F}(x,y,z) = \langle x + e^{\sin(z)}, z, -y \rangle$.

a) (5 points) Find the flux of $\vec{F}$ through the surface S.

b) (5 points) Find the flux of $\vec{F}$ through the part of the surface S that belongs to the paraboloid, oriented so that the normal vector points downwards.

The following problem 13A is regular and physics sections only:

Problem 13 A) (10 points)

Let $\vec{F}$ be the vector field $\vec{F}(x, y, z) = \langle 4z + \cos(\cos x), y^2, x + 2y \rangle$.

a) (4 points) Let C be the curve given by the parameterization $\vec{r}(t) = \langle \cos t, 0, \sin t \rangle$, for $0 \leq t \leq 2\pi$. Find the line integral of $\vec{F}$ along C.

b) (6 points) Let S be the hemisphere of the unit sphere defined by $y \leq 0$. Find the flux of the curl of $\vec{F}$ out of S. In other words, find

$$\iint_S \operatorname{curl}(\vec{F}) \cdot d\vec{S}\ .$$

For part b), the surface S is oriented so that the normal vector has a positive y-component.

The following problem 11 B is for biochem sections only:

Problem 11 B) (10 points)

A laboratory uses a certain test for HIV. If a patient is infected, the test will give a positive result with probability 95% and a negative result with probability 5%. If the patient is not infected, the test gives a negative result with probability 99%.

a) (3 points) A person who is not infected is tested ten times. What is the probability that at least two of the tests will be positive?

b) (4 points) The Centers for Disease Control estimate that 0.4 % of Americans are HIV-positive. An American is chosen randomly and tested for HIV. The test gives a positive result. What is the probability that this person does not have HIV?

c) (3 points) The person is tested again, and again the test gives a positive result. What is the probability that the person does not have HIV?

The following problem 12 B is for biochem sections only:

Problem 12 B) (10 points)

Adam has to wait for his sister Sally and mother Mary to be ready before he can head to the football game. Let S be the random variable equal to the waiting time for Sally, which has density function $p_S(x) = \begin{cases} e^{-x} & if\ x \geq 0 \\ 0 & \text{otherwise.} \end{cases}$

Let M be the random variable equal to the waiting time for Mary, which has density function $p_M(x) = \begin{cases} 2e^{-2x} & \text{if } x \geq 0 \\ 0 & \text{otherwise.} \end{cases}$

a) (3 points) What is the expectation and variance of of S?

b) (3 points) Calculate the distribution functions $\Phi_S(x)$ and $\Phi_M(x)$.

c) (4 points) Let X be the random variable which is equal to the time when Adam can leave, that is, until both Sally and Mary are ready. Assume that S and M are independent random variables. What is the probability density function $p_X(x)$ of X?

Hint: First calculate the distribution function $\Phi_X(x)$ and observe that $\Phi_X(x)$ is the probability that Adam can leave before time x.

The following problem 13 B is for biochem sections only:

Problem 13 B) (10 points)

There are 2 white balls and 3 blue balls in a bag. We select two balls from the bag, without replacing the first ball after selecting it. Let A be the event that the first selected ball is white, and let B be the event that exactly one of the selected balls is blue.

a) (4 points) Compute $P(A)$, $P(B)$ and $P(A|B)$.

b) (3 points) Are A and B independent events?

c) (3 points) What is the expectation of the random variable X giving the total number of blue balls selected?