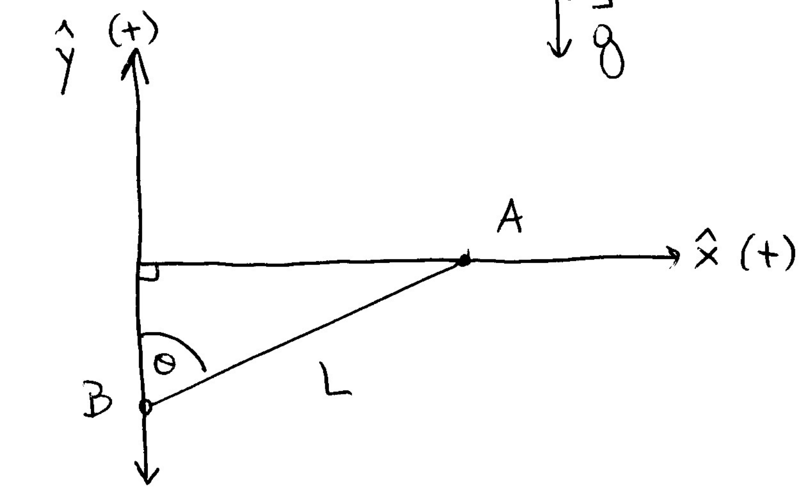
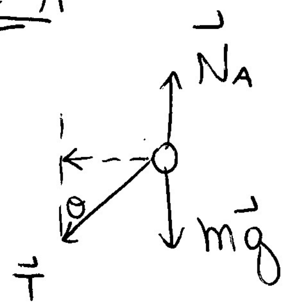
Ejercicio 1.4.7



$\downarrow \vec{g}$

Para calcular $N(\theta)$ y $T(\theta)$ es necesario empezar haciendo DCL de cada cuerpo.

DCL A



$$\sum \vec{F} = m \cdot \vec{a}$$

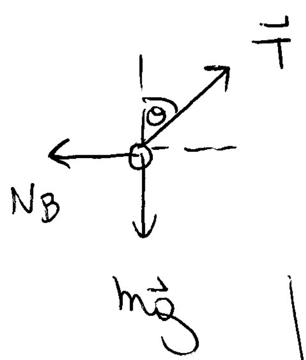
$\hat{x}]$ $\quad -T\operatorname{sen}\theta = m\ddot{x}_A$

$\hat{y}]$ $\quad N_A - mg - T\cos\theta = 0 = m\cancel{\ddot{y}_A}^{\,0}$

↓ ya que está obligado a moverse sobre $y=0$

$$\Rightarrow \boxed{m\ddot{x}_A = -T\operatorname{sen}\theta} \quad (1)$$

DCL B



$\hat{x}]$ $\quad -N_B + T\operatorname{sen}\theta = m\cancel{\ddot{x}_B}^{\,0} = 0$

↓ obligado a $x=0$

$$\hat{y}] \quad \boxed{T\cos\theta - mg = m\ddot{y}_B} \quad (2)$$

$$\boxed{\text{Notación: } \ddot{x}_A = \ddot{x}, \quad \ddot{y}_B = \ddot{y}}$$

Notar que debido a la geometría del problema podemos escribir:

$$X = L\,\text{sen}\theta \Rightarrow \dot{X} = L\cos\theta\,\dot{\theta} \Rightarrow \ddot{X} = -L\,\text{sen}\theta\,\dot{\theta}^2 + L\cos\theta\,\ddot{\theta} \quad (3)$$

$$y = -L\cos\theta \Rightarrow \dot{y} = L\,\text{sen}\theta\,\dot{\theta} \Rightarrow \ddot{y} = L\cos\theta\,\dot{\theta}^2 + L\,\text{sen}\theta\,\ddot{\theta} \quad (4)$$

Si tomamos $(1)\cdot\text{sen}\theta - (2)\cdot\cos\theta$; y reemplazamos antes (3) en (1) y (4) en (2):

$$-T\,\text{sen}^2\theta - T\cos^2\theta + mg\cos\theta = m\left[-L\,\text{sen}^2\theta\,\dot{\theta}^2 - L\cos^2\theta\,\dot{\theta}^2\right]$$

$$-T + mg\cos\theta = -mL\dot{\theta}^2$$

$$\Rightarrow \boxed{mL\dot{\theta}^2 = T - mg\cos\theta.} \quad (*)$$

y $\dot{\theta}$?

Energía!! Nada de integrar...

$$E = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\dot{y}^2 - mgL\cos\theta$$

$$= \frac{1}{2}m\left(L^2\cos^2\theta\,\dot{\theta}^2 + L^2\,\text{sen}^2\theta\,\dot{\theta}^2\right) - mgL\cos\theta \qquad \forall t$$

Inicialmente: $\dot{x} = 0$; $\dot{y} = 0 \Rightarrow \boxed{\dot{\theta} = 0}$ $\boxed{\theta = \frac{\pi}{2}}$

$$\Rightarrow \boxed{E_0 = 0}$$

$$\Rightarrow 0 = \frac{1}{2}mL^2\dot{\theta}^2 - mgL\cos\theta$$

$$\Rightarrow mL^2\dot{\theta}^2 = 2mgL\cos\theta$$

$$\Rightarrow \boxed{mL\dot{\theta}^2 = 2mg\cos\theta}$$

Reemplazamos en (*)

$$\Rightarrow 2mg\cos\theta = T - mg\cos\theta$$

$$\Rightarrow \boxed{T(\theta) = 3mg\cos\theta}$$

y con esto:

$$N_A = mg + \underbrace{3mg\cos\theta}_{T}\cdot\cos\theta = mg + 3mg\cos^2\theta$$

$$\Rightarrow \boxed{N_A = mg(1+3\cos^2\theta)}$$

y:

$$\boxed{N_B = \underbrace{3mg\cos\theta}_{T}\sin\theta}$$