Then,

$$\sigma_1 = \sigma_{avg} + R = 16.20 + 24.39 = 40.6 \text{ ksi}$$
$$\sigma_3 = \sigma_{avg} - R = 16.20 - 24.39 = -8.2 \text{ ksi} \qquad (5)$$

The in-plane principal stresses are labeled σ_1 and σ_3, since the out-of-plane principal stress, $\sigma_r = 0$, is the intermediate principal stress. Then, from Eq. 8.32,

$$\tau_{\substack{abs\\max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{40.6 \text{ ksi} - (-8.2 \text{ ksi})}{2} \qquad (6a)$$

or

$$\tau_{\substack{abs\\max}} = 24.4 \text{ ksi} \qquad \textbf{Ans.} \quad (6b)$$

Review the Solution The calculations in Eqs. (2) and (3) should be rechecked. Points X and Y are plotted correctly, so σ_1 and σ_3 appear to be correct. Finally, since the working stresses in this example should not produce yielding of the drill pipe, the absolute maximum shear stress should be much less than half the tensile yield strength. Therefore, the answer in Eq. (6b) seems reasonable.

MDS9.4 **Shaft Subjected to Combined Axial Loading and Torsion**

Many interesting applications of deformable-body mechanics in the field of oilwell drilling engineering are presented in *Oilwell Drilling Engineering—Principles and Practice,* by H. Rabia, [Ref. 9-5].

General Combined Loading. In the final example problem on stresses due to combined loading, we consider a problem that involves all types of stress resultants: F, T, M, and V.

EXAMPLE 9.5

Wind blowing on a sign produces a pressure whose resultant, P, acts in the $-y$ direction at point C, as shown in Fig. 1. The weight of the sign, W_s, acts vertically through point C, and the thin-wall pipe that supports the sign has a weight W_p.

Following the procedure outlined at the beginning of Section 9.4, determine the principal stresses at points A and B, where the pipe column is attached to its base. Use the following numerical data.[7]

$$\text{Pipe } OD = 3.50 \text{ in.}, A = 2.23 \text{ in}^2, I_y = I_z = 3.02 \text{ in}^4, I_p = 6.03 \text{ in}^4,$$
$$W_s = 125 \text{ lb}, W_p = 160 \text{ lb}, P = 75 \text{ lb}, b = 40 \text{ in.}, L = 220 \text{ in.}$$

[7]Cross-sectional properties of the 3.50-in.-OD pipe are from Table D.7.

Plan the Solution It will be a good idea to tabulate the stress resultants, stress formulas, and so forth, so that no stress contribution will be missed. The weight W_s contributes to the axial force, and it also produces a moment about the y axis. The wind force P produces a transverse shear force in the y direction, and it also causes a torque about the x axis and a moment about the z axis. A correct free-body diagram is essential.

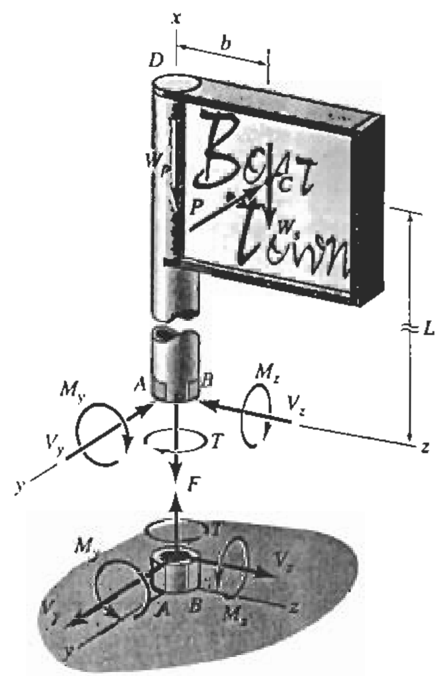

Fig. 1 A cantilevered sign.

Solution

Stress Resultants: All six stress resultants on the cross section at the base of the pipe are shown in Fig. 1. The upper portion of Fig. 1 can serve as a free-body diagram for determining these six stress resultants. The sign convention is the one introduced in Fig. 2.40. Let us tabulate the equilibrium equations and indicate what stress is produced by each stress resultant and label each individual stress.

Individual Stresses: Using the formulas from Table 9.1, we can compute the numerical value of each of the nonzero stresses listed in Table 1.

$$\sigma_{A1} = \sigma_{B1} = \frac{F}{A} = \frac{-(125 \text{ lb}) - (160 \text{ lb})}{2.23 \text{ in}^2} = -128 \text{ psi} \tag{7}$$

The shear stress τ_{B2} is due to the transverse shear force V_y. The basic shear stress formula is

$$\tau_{B2} = \frac{V_y Q}{I_z t} \tag{8}$$

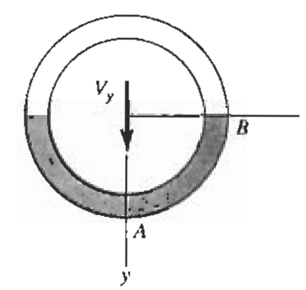

Fig. 2

where Q has to be calculated for the shaded area in Fig. 2. In Example Problem 6.16, it was shown that the shear stress in this case (stress at the neutral axis of a thin-wall pipe) is given by

$$\tau = \frac{2V}{A} \tag{9a}$$

TABLE 1 A Table of Stress Resultants and the Stresses Produced

Eq. No.	Equilibrium Equation		Stress at A	Stress at B
(1)	$\sum F_x = 0$	$F = -W_s - W_p$	σ_{A1}	σ_{B1}
(2)	$\sum F_y = 0$	$V_y = -P$	—	τ_{B2}
(3)	$\sum F_z = 0$	$V_z = 0$	—	—
(4)	$\sum M_x = 0$	$T = Pb$	τ_{A4}	τ_{B4}
(5)	$\sum M_y = 0$	$M_y = -W_s b$	—	σ_{B5}
(6)	$\sum M_z = 0$	$M_z = -PL$	σ_{A6}	—

Therefore,

$$\tau_{B2} = \frac{2(75\text{ lb})}{2.23\text{ in}^2} = 67\text{ psi} \tag{9b}$$

$$\tau_{A4} = \tau_{B4} = \frac{Tr_o}{I_p} = \frac{(Pb)r_o}{I_p} \tag{10a}$$

so

$$\tau_{A4} = \tau_{B4} = \frac{(75\text{ lb})(40\text{ in.})(1.75\text{ in.})}{6.03\text{ in}^4} = 871\text{ psi} \tag{10b}$$

The flexural stresses due to M_y and M_z are given by Eq. 6.30.

$$\sigma_{B5} = \frac{M_y r_o}{I_y} = \frac{(-W_t b)r_o}{I_y} \tag{11a}$$

$$\sigma_{B5} = \frac{-(125\text{ lb})(40\text{ in.})(1.75\text{ in.})}{3.02\text{ in}^4} = -2897\text{ psi} \tag{11b}$$

$$\sigma_{A6} = \frac{-M_z r_o}{I_z} = \frac{-(-PL)r_o}{I_z} \tag{12a}$$

$$\sigma_{A6} = \frac{(75\text{ lb})(220\text{ in.})(1.75\text{ in.})}{3.02\text{ in}^4} = 9561\text{ psi} \tag{12b}$$

Superposition of Stresses: Using the above values, and taking proper note of the physical significance of the sign of each term by referring to Fig. 1, we get the stresses shown in Fig. 3.

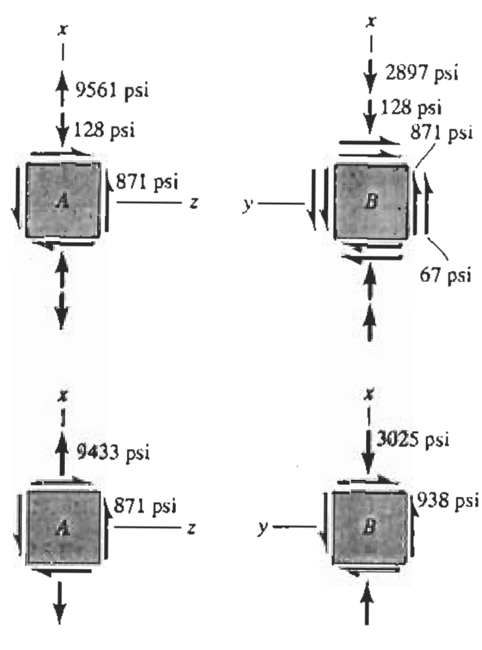

Fig. 3 The states of stress at points A and B.

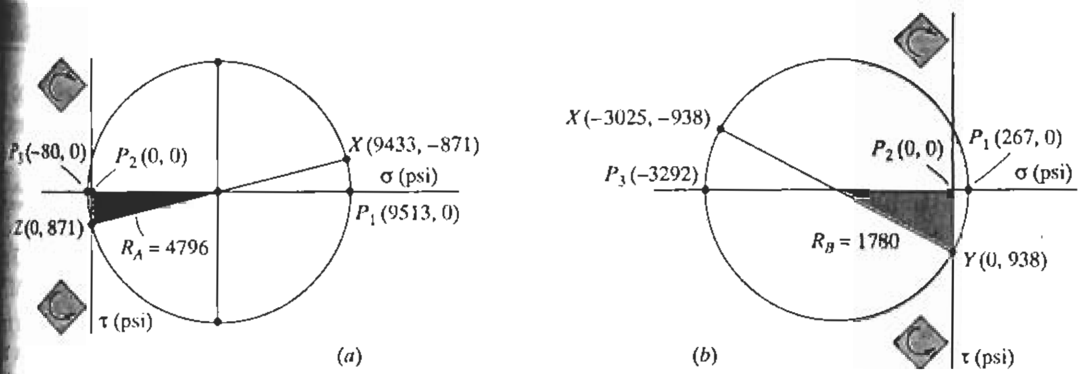

Fig. 4 Mohr's circles for in-plane stresses at points A and B.

Using the stresses shown in Fig. 3, we can construct a Mohr's circle for the states of plane stress at points A and B on the pipe surface. The radial normal stress is $\sigma_r = 0$ at both places. From Fig. 4*a*.

$$R_A = \sqrt{(9433/2)^2 + (871)^2} = 4796 \text{ psi} \tag{13}$$

$$\begin{aligned}(\sigma_1)_A &= (9433/2) + 4796 = 9513 \text{ psi} \\ (\sigma_3)_A &= (9433/2) - 4796 = -80 \text{ psi}\end{aligned} \tag{14}$$

and, from Fig. 4*b*,

$$R_B = \sqrt{(-3025/2)^2 + (938)^2} = 1780 \text{ psi} \tag{15}$$

$$\begin{aligned}(\sigma_1)_B &= (-3025/2) + 1780 = 267 \text{ psi} \\ (\sigma_3)_B &= (-3025/2) - 1780 = -3292 \text{ psi}\end{aligned} \tag{16}$$

In summary, the principal stresses at points A and B, rounded to three significant figures, are:

$$\begin{aligned}&(\sigma_1)_A = 9510 \text{ psi}, \quad (\sigma_2)_A = 0, \quad (\sigma_3)_A = -80 \text{ psi} \\ &(\sigma_1)_B = 267 \text{ psi}, \quad (\sigma_2)_B = 0, \quad (\sigma_3)_B = -3290 \text{ psi}\end{aligned} \qquad \textbf{Ans.}$$

Review the Solution By showing all six possible internal resultants at the cross section where stresses are to be calculated, by writing down and solving all six possible equilibrium equations, and by carefully considering what stress(es) is (are) produced by each stress resultant, we have accounted for the effects of all loads on the structure. As noted earlier, we have been careful to make sure that each stress component acts in the direction that "makes sense." For example, the force P bends the pipe in the direction that produces tension at point A, and so forth.

The maximum flexural stress at the base occurs at neither A nor B. Equation 6.30 could be used to combine the flexural stresses due to M_y and M_z, and we would also have to consider the effect of shear stress. (See Homework Problem 9.4-26.)

MDS9.5 **Member Subjected to Combined Axial, Shear, and Bending Stresses**

shear stress. When both in-plane principal stresses are negative (Fig. 8.24*a*) or when both are positive (Fig. 8.24*c*), the absolute maximum shear stress acts on planes at 45° to the free surface, and the maximum in-plane shear stress is not the absolute maximum shear stress. **Even though the *z* faces are stress free, they must be taken into account in determining the absolute maximum shear stress!** But, in every case, from Eqs. 8.32 and 8.33 we have

$$\boxed{\begin{aligned} \tau_{\substack{abs\\max}} &= \frac{\sigma_{max} - \sigma_{min}}{2} \\ \sigma_s &= \frac{\sigma_{max} + \sigma_{min}}{2} \end{aligned}} \qquad (8.35)$$

The stresses σ_{max} and σ_{min} are signed quantities (i.e., tension positive, compression negative); they are not just magnitudes.

EXAMPLE 8.5

An element in plane stress has the stresses shown in Fig. 1. (a) Determine the three principal stresses. Use a Mohr's circle to determine in-plane stresses. (b) Determine the maximum in-plane shear stress. (c) Determine the orientation of the principal planes, and sketch the principal-stress element. (d) Determine the absolute maximum shear stress. Show an element oriented so that the absolute maximum shear stress acts on the element.

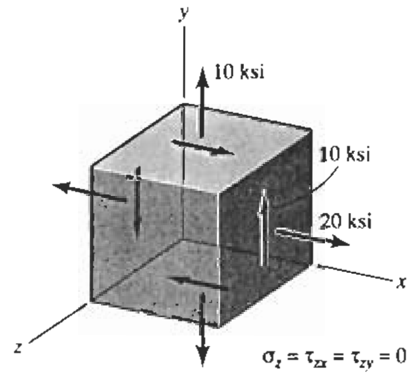

Fig. 1 An element in plane stress.

Plan the Solution We need to determine the principal directions and in-plane principal stresses for the *xy* plane using the Mohr's circle technique of Section 8.5. From Mohr's circle we can also get the maximum in-plane shear stress. In Part (c) we will have to order the principal stresses $\sigma_1 \geq \sigma_2 \geq \sigma_3$, and compare the three principal stresses in this problem (the two in-plane principal stresses plus $\sigma_z = 0$) with the three cases depicted in Fig. 8.24. The maximum absolute shear stress is calculated using Eq. 8.32.

Solution

(a) Principal Stresses: One of the principal stresses is $\sigma_z = 0$, since $\tau_{zx} = \tau_{zy} = 0$. The other two principal stresses are obtained from the Mohr's circle in Fig. 2.

From triangle *XCA* we get

$$R = \sqrt{(\overline{CA})^2 + (\overline{XA})^2} = \sqrt{(5\text{ ksi})^2 + (10\text{ ksi})^2}$$

So,

$$R = \sqrt{125}\text{ ksi} = 11.18\text{ ksi} \qquad (1)$$

Since all points on the Mohr's circle in Fig. 2 have $\sigma > 0$, $\sigma_z = 0$ is the minimum principal stress. Therefore, the intersections of Mohr's

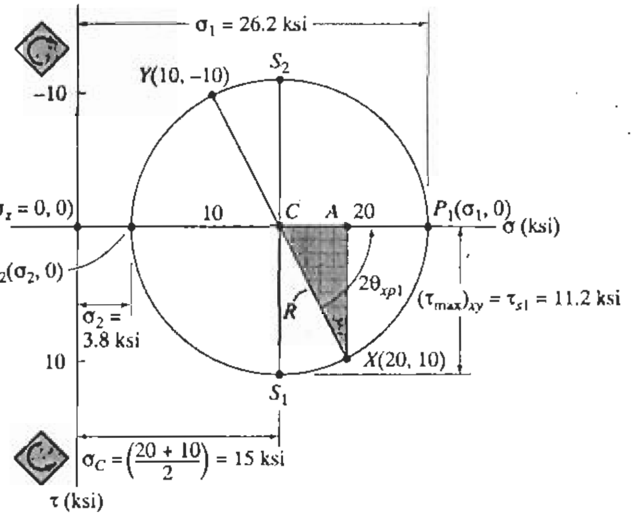

Fig. 2 Mohr's circle for the xy plane, with P_3 shown for reference.

circle with the σ axis are labeled p_1 and p_2. From the circle in Fig. 2,

$$\begin{aligned} \sigma_1 &= \sigma_C + R = 15 \text{ ksi} + 11.2 \text{ ksi} = 26.2 \text{ ksi} \\ \sigma_2 &= \sigma_C - R = 15 \text{ ksi} - 11.2 \text{ ksi} = 3.8 \text{ ksi} \end{aligned} \quad (2)$$

Therefore, the three principal stresses are

$$\sigma_1 = 26.2 \text{ ksi}, \quad \sigma_2 = 3.8 \text{ ksi}, \quad \sigma_3 = 0 \qquad \textbf{Ans. (a)} \quad (3)$$

(b) Maximum In-Plane Shear Stress: The maximum shear stress in the xy plane is the shear stress at point S_1 in Fig. 2, or

$$(\tau_{max})_{xy} = R = 11.2 \text{ ksi} \qquad \textbf{Ans. (b)} \quad (4)$$

(c) Principal-Stress Element: To orient the principal-stress element ($p_1p_2p_3$ axes) relative to the xyz axes we only need to relate p_1 and p_2 to x and y, since we already know that $p_3 \equiv z$ (since $\sigma_z < \sigma_2 < \sigma_1$). From Fig. 2 we can determine the angle $2\theta_{xp1}$. From triangle XCA we get

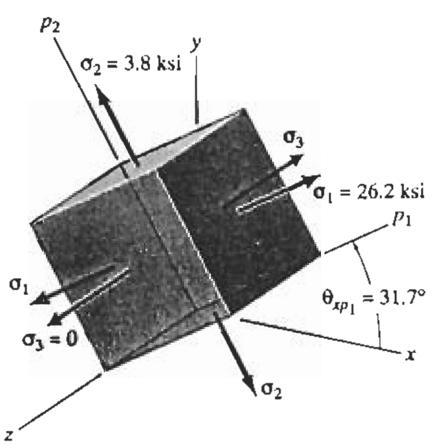

Fig. 3 The principal-stress element.

$$2\theta_{xp1} = \tan^{-1}\left(\frac{10}{5}\right) = 63.43° \quad (5a)$$

$$\theta_{xp1} = 31.7° \qquad \textbf{Ans. (c)} \quad (5b)$$

A properly oriented principal-stress element is shown in Fig. 3.

(d) Absolute Maximum Shear Stress: The plane-stress Mohr's circle in Fig. 2 corresponds to Case III (Fig. 8.24c). Therefore, we need to construct a p_1p_3 Mohr's circle. For clarity, we will draw another figure, Fig.

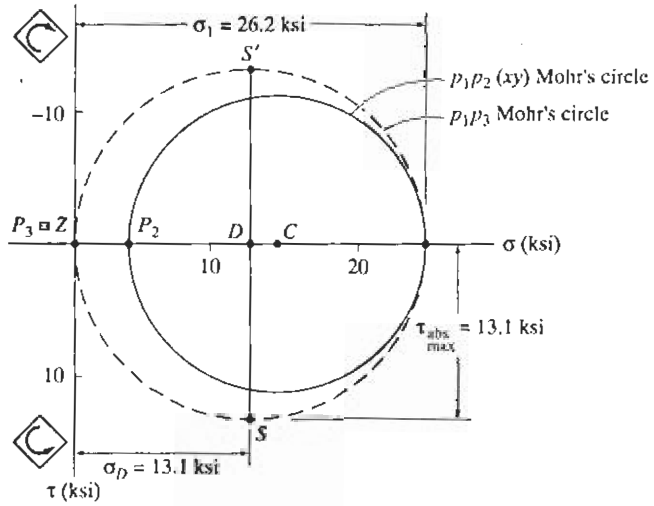

Fig. 4 Mohr's circles for determining $\tau_{\substack{abs\\max}}$.

4, repeating part of Fig. 2. From the dashed-line p_1p_3 Mohr's circle in Fig. 4 we get

$$\tau_{\substack{abs\\max}} = \frac{\sigma_1}{2} = 13.1 \text{ ksi} \qquad \textbf{Ans. (d)} \qquad (6)$$

Figures 5*a* through 5*c* depict the planes of absolute maximum shear stress. First, in Figs. 5*a* and 5*b* the orientations of the planes of maximum shear stress at 45° to the p_1 and p_3 axes (faces) are illustrated. Finally, in Fig. 5*c* a two-dimensional view of the p_1p_3 plane is shown, looking "down" the p_2 axis.

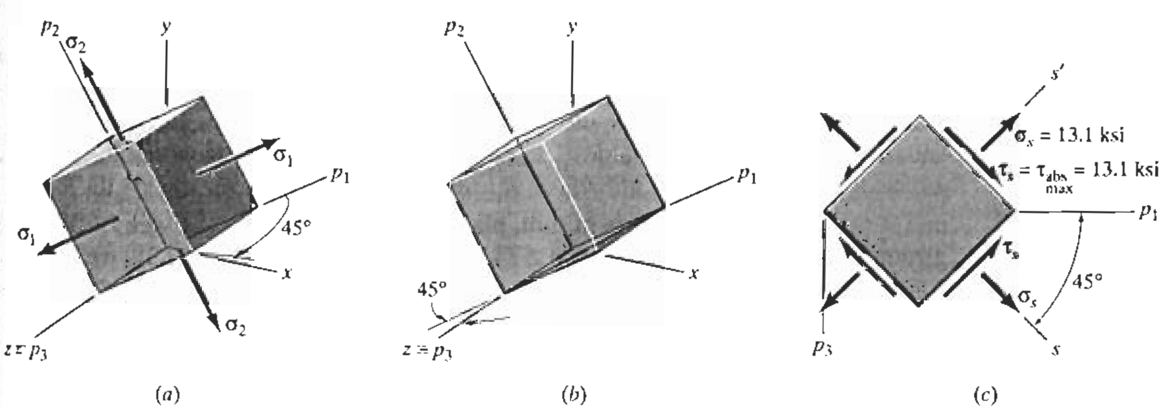

Fig. 5 Planes of absolute maximum shear stress.

Review the Solution We should first check to make sure the points X and Y in Fig. 2 correctly represent the stresses on the x and y faces in Fig. 1, especially making sure that the sign of the shear stress is correct at X and Y. Since the answers in Eqs. (3), (4), (5), and (6) came directly from the Mohr's circles in Figs. 2 and 3, we can visually check to see if they are reasonable.

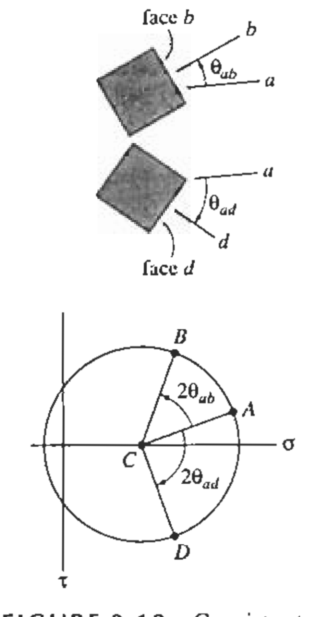

FIGURE 8.18 Consistent angles.

- Two planes that are 90° apart on the physical body are represented by th two points at the extremities of a diameter, such as points X and Y or and P_2 in Fig. 8.16.
- If we rotate counterclockwise by an angle θ_{ab} to go from face a to face b the physical body, we must rotate in that same direction through the angl $2\theta_{ab}$ to get from point A on Mohr's circle to point B. Figure 8.18 illustrat this property of the Mohr's circle sign convention. In equations, a positi angle is always counterclockwise.
- The *principal planes* are represented by points P_1 and P_2 at the intersectio of Mohr's circle with the σ axis (Fig. 8.16). The corresponding *princip stresses* are $\sigma_1 = \sigma_{avg} + R$ and $\sigma_2 = \sigma_{avg} - R$.
- The planes of maximum shear stress are represented by points S_1 and that lie directly below and above the center of the Mohr's circle (Fig. 8.16 The corresponding stresses are: (σ_{avg}, R) on face s_1, and $(\sigma_{avg}, -R)$ on face
- Since the stresses on orthogonal planes n and t are represented by the poi at each end of a diameter of Mohr's circle,

$$\sigma_n + \sigma_t = \sigma_x + \sigma_y \quad (8$$

repeat

EXAMPLE 8.4

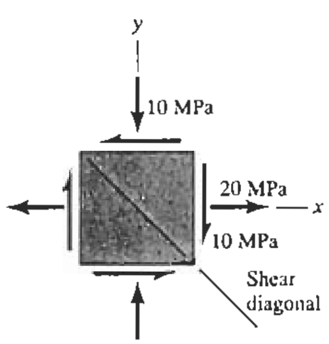

Fig. 1 A state of plane stress.

For the plane-stress state in Example Problems 8.1 and 8.2 (Fig. 1), do the following: (a) Draw Mohr's circle. (b) Determine the stresses on all faces of an element that is rotated 30° counterclockwise from th orientation of the stress element in Fig. 1. (c) Determine the orientatio of the principal planes; determine the principal stresses. (d) Determin the orientation of the planes of maximum shear stress; determine th value of the maximum shear stress.

Solution We can just follow the procedure outlined on page 543.

(a) Mohr's Circle: On grid paper (Fig. 2), plot point X at (20 MPa, −1 MPa) and plot point Y, at (−10 MPa, 10 MPa). The center of the circ is obtained by connecting X and Y. The diameter crosses the σ axis C: $(\sigma_{avg}, 0)$ where

$$\sigma_{avg} = \frac{20\,\text{MPa} - 10\,\text{MPa}}{2} = 5\,\text{MPa} \quad (1$$

The circle is drawn with center at C and passing through points and Y. The radius R is calculated from the shaded triangle XCB in Fig

$$R = \sqrt{(15\,\text{MPa})^2 + (10\,\text{MPa})^2} = \sqrt{325}\,\text{MPa} = 18.03\,\text{MPa} \quad (2$$

(b) Stresses on x′ and y′ Faces: Locate the points on Mohr's circle th correspond to rotating the stress element by 30°. This means rotati 60° counterclockwise from the XY diameter on Mohr's circle. We lab these two points X' and Y'.

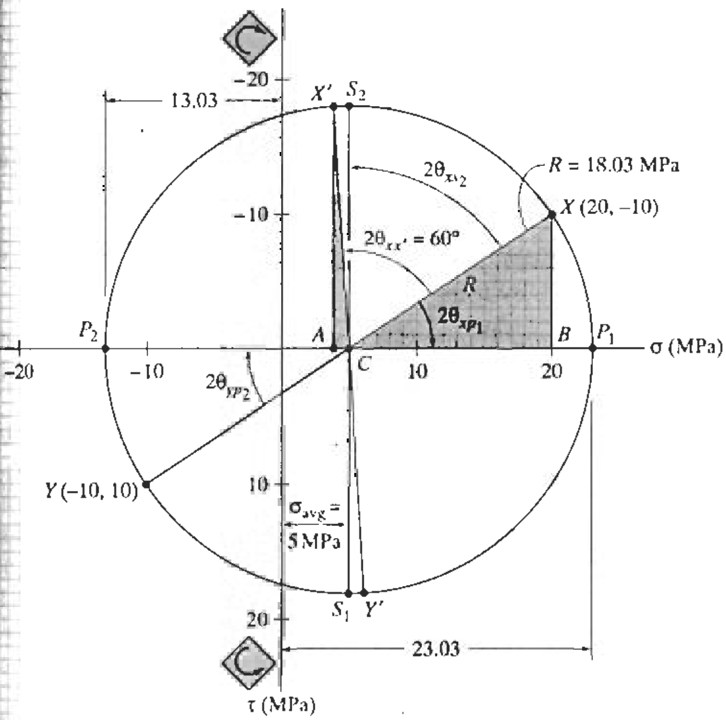

Fig. 2 Mohr's circle.

To determine the stresses at points X' and Y', we need to establish the geometry and trigonometry of the triangle $X'CA$. To determine $\angle X'CA$ we need to first determine the (clockwise) angle $2\theta_{xp1}$, in Fig. 2. Using the triangle, XCB, we get

$$2\theta_{xp1} = \tan^{-1}\left(\frac{10}{15}\right) = 33.69° \qquad (3a)$$

$$\theta_{xp1} = 16.8° \circlearrowright \qquad (3b)$$

Therefore,

$$\angle X'CA = 180° - 60° - 2\theta_{xp1} = 86.31° \qquad (4)$$

From the triangle $X'CA$,

$$\overline{AC} = R\cos(\angle X'CA) = 18.03\cos(86.31°) \qquad (5a)$$

or

$$\overline{AC} = 1.16\ \text{MPa} \qquad (5b)$$

Therefore,

$$\left.\begin{aligned} \sigma_{x'} &= \sigma_{avg} - \overline{AC} = 3.84\ \text{MPa} \\ \sigma_{y'} &= \sigma_{avg} + \overline{AC} = 6.16\ \text{MPa} \end{aligned}\right\} \qquad \textbf{Ans. (b)} \quad (6)$$

Also, from triangle $X'CA$ we get

$$\tau_{x'y'} = -R\sin(\angle X'CA) = -18.03\sin(86.31°)$$

or

$$\tau_{x'y'} = -18.0\ \text{MPa} \qquad \textbf{Ans. (b)} \quad (7)$$

Equations (6) and (7) are the same answers that we obtained in Example Problem 8.1 by using formulas directly.

(c) Principal Planes and Principal Stresses: We have already calculated θ_{sp1} in Eq. (3). From Fig. 2,

$$2\theta_{sp2} = 2\theta_{sp1} = 33.69° \qquad (8a)$$
$$\theta_{sp2} = 16.8° \qquad (8b)$$

Also, from Fig. 2,

$$\sigma_1 = \sigma_{avg} + R = 5\ \text{MPa} + 18.03\ \text{MPa} = 23.0\ \text{MPa}$$
$$\sigma_2 = \sigma_{avg} - R = 5\ \text{MPa} - 18.03\ \text{MPa} = -13.0\ \text{MPa}$$

or

$$\sigma_1 = 23.0\ \text{MPa}, \qquad \sigma_2 = -13.0\ \text{MPa} \qquad \textbf{Ans. (c)} \quad (9)$$

(d) Maximum In-Plane Shear Stress: The planes of maximum in-plane shear stress are represented by the points S_1 and S_2 on Mohr's circle. From Fig. 2,

$$2\theta_{s1} = 90° + 2\theta_{sp1} = 90° + 33.69° = 123.69°$$

so

$$\theta_{s1} = 61.8°$$

Also, by referring to Fig. 2, we see that

$$2\theta_{s2} = 90° - 2\theta_{sp1} = 90° - 33.69° = 56.31°$$
$$\theta_{s2} = 28.2°$$

The maximum in-plane shear stress occurs on plane s_1 and on plane and is given by

$$\tau_{s1,s2} = R = 18.0\ \text{MPa} \qquad \textbf{Ans.}$$

On the planes of maximum shear stress, the normal stress is

$$\sigma_{s1} = \sigma_{s2} = \sigma_{avg} = 5\ \text{MPa} \qquad \textbf{Ans.}$$

Review the Solution It is very important to construct the Mohr's circle properly. Once that is done, subsequent results can be checked visually by (roughly) estimating the values of normal stresses and shear stresses that are required and the angles that are required.

The comments in the *Review the Solution* section of Example Problem 8.3 apply to the results that we obtained above by using Mohr's circle.

MDS8.1 – 8.3 Mohr's Circle—Stress Transformations

8.6 TRIAXIAL STRESS; ABSOLUTE MAXIMUM SHEAR STRESS

Figure 8.2 depicts a general *three-dimensional state of stress*, referred to cartesian (x, y, z) axes, but in Sections 8.2 through 8.5 we dealt only with plane stress—formulating stress transformation equations, determining expressions for principal stresses and maximum in-plane shear stresses, and establishing a graphical representation of the plane-stress transformation equations, called Mohr's circle. We now need to look further at three-dimensional stress states. In particular, we will briefly consider *principal stresses* for a general state of stress, and will then examine *absolute maximum shear stresses* in greater detail.

Principal Stresses and Principal Directions. For a general three-dimensional state of stress at a point, it can be shown that: **there are three principal stresses,** and **the corresponding principal planes are mutually perpendicular.**[9] There is no shear stress on the principal planes. The three **principal stresses** are labeled in the order—maximum, intermediate, and minimum:

$$\boxed{\sigma_1 = \sigma_{max}, \quad \sigma_2 = \sigma_{int}, \quad \sigma_3 = \sigma_{min}} \tag{8.29}$$

that is, $\sigma_1 \geq \sigma_2 \geq \sigma_3$. To each principal stress σ_i there is a unit normal vector that defines the corresponding **principal direction**, that is, the normal to the plane on which that principal stress acts. If we draw the stress element whose faces are all principal planes, we get Fig. 8.19. The principal directions are labeled p_1, p_2, and p_3, with $\sigma_1 \geq \sigma_2 \geq \sigma_3$. Since all faces of this element are free of shear stress, this element is said to be in a state of **triaxial stress**.

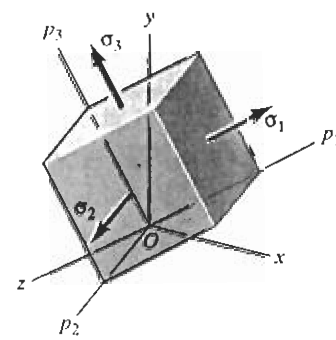

FIGURE 8.19 Principal stresses acting on a three-dimensional element.

Absolute Maximum Shear Stress—General Stress State. In Section 8.4 we examined *maximum in-plane shear stress* for the case of plane stress. For a general state of stress at a point, including the case of plane stress, we need to determine the **absolute maximum shear stress,** that is, the largest-magnitude shear stress acting in any direction on any plane passing through the point.[10] To do so, it is convenient to assume that we already know the principal directions and the principal stresses at the point. Figure 8.20*a* represents the element on which the principal stresses at the point act.

[9] For a detailed discussion of procedures for determining principal stresses and principal directions, see Sections 75–78 of *Theory of Elasticity*, Third Edition, by S. P. Timoshenko and J. N. Goodier, McGraw-Hill Book Company, New York, 1970, [Ref. 8-1].

[10] For a detailed derivation, see Section 79 of *Theory of Elasticity*, Third Edition, by S. P. Timoshenko and J. N. Goodier, McGraw-Hill Book Company, New York, 1970, [Ref. 8-1].

ME 46 A RESISTENCIA DE MATERIALES

28/11/02 **EXAMEN** **Prof.: M. Elgueta**

Problema 1

Una viga en voladizo de sección en T está cargada por una fuerza inclinada de *10 kN*, como se muestra en la figura 1. Obtenga los esfuerzos principales y el esfuerzo cortante máximo en los puntos A y B mostrados (recuerde que las acciones internas se calculan en el centro de gravedad de la sección)

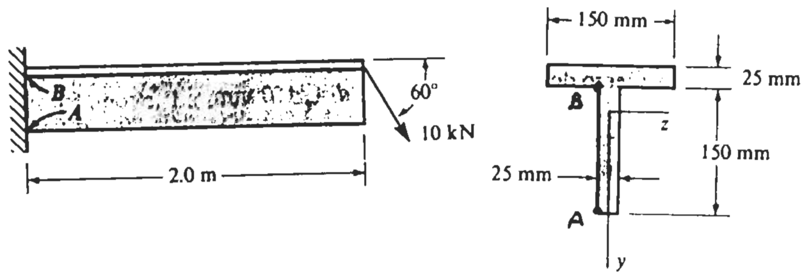

Figura 1

Problema 2.

La estructura de la figura 2 se encuentra simplemente apoyada en A y empotrada en B. Cuando se aplica la carga *P*, calcule la reacción en A utilizando el método de Castigliano. Considere que en la sección transversal hay fuerza interna normal y momento flector. Datos: *A* área de la sección transversal; *E* módulo de elasticidad; *I* momento de inercia.

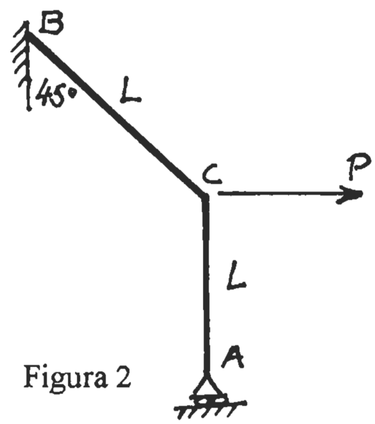

Figura 2

Problema 3.

Para la columna mostrada en la figura, se pide determinar la ecuación, en su expresión más sencilla, que permite calcular la carga crítica.

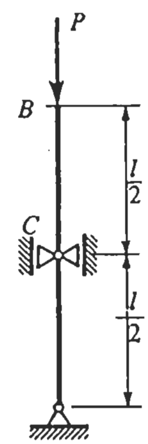

Figura 3.

11/07/2002

ME-46A RESISTENCIA DE MATERIALES
EXAMEN ADICIONAL

Prof.: M. Elgueta

PROBLEMA 1

Determine el momento en B para la viga mostrada en la figura 1. EI es constante Considere sólo flexión. Si va a calcular algún giro o deflexión, utilice Castigliano.

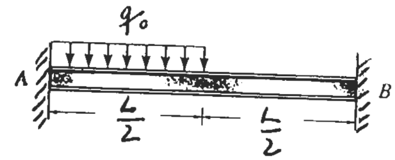

Figura 1.

PROBLEMA 2

El ancla de ½ pulgada de diámetro mostrada en la figura 1,está sometida a una carga F = 150 lb. Calcule los esfuerzos principales en lkos puntos A y B.

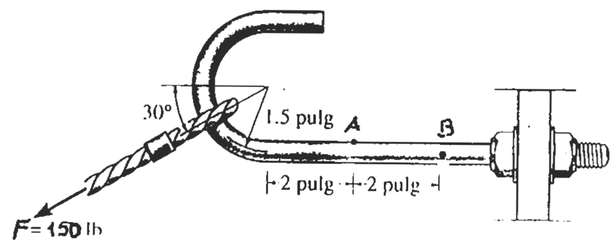

Figura 2.

con una fuerza ... planos rígidos que permanecen

... coeficiente de seguridad 2 y asuma:

$E = 30,0 \cdot 10^{6}$ psi

$\alpha = 9,0 \cdot 10^{-6}$ 1/°F

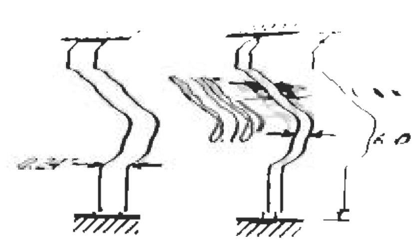

Figura 3

PROBLEMA 1

Determine los esfuerzos principales que actúan en los puntos A y B de la viga que se muestra en la figura 1.

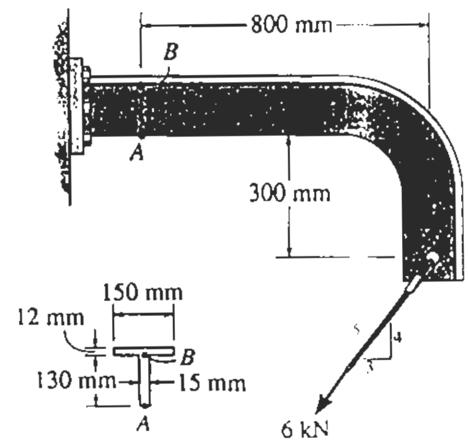

Figura 1

PROBLEMA 2

La viga ABC mostrada en la figura 2 tiene una rigidez a la flexión $EI = 4,0\ MN \cdot m^2$. Cuando se aplican las cargas, el apoyo B se asienta verticalmente una distancia de $3,0\ mm$. Calcular la reacción R_B. Nota: Utilice superposición. Para calcular las deflexiones utilice sólo el método de Castigliano y considere sólo flexión.

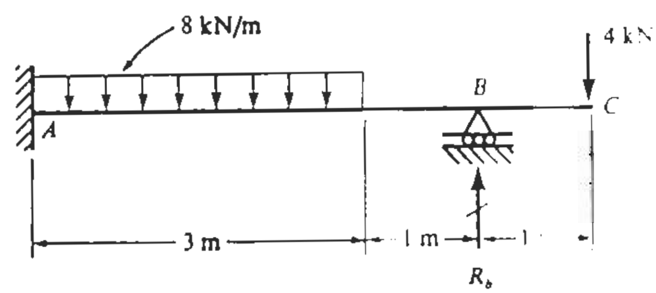

Figura 2

PROBLEMA 3

Una barra se precarga, a temperatura ambiente, con una fuerza de compresión de $20\ lb$ entre dos cuerpos planos rígidos que permanecen siempre a la distancia $6\ pulg$. La barra se encuentra articulada en el extremo superior y empotrada en el inferior. Determine cuánto puede aumentar la temperatura de modo que no se produzca falla por pandeo. Utilice un coeficiente de seguridad 2 y asuma:

$E = 30,0 \cdot 10^6\ psi$
$\alpha = 9,0 \cdot 10^{-6}\ 1/°F$

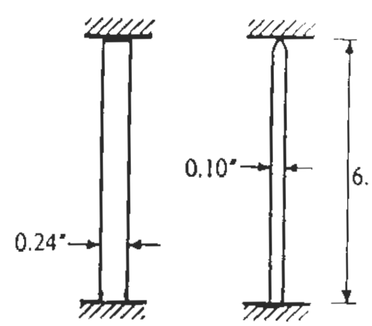

Figura 3

Prof: M. Elgueta

ME-46 A Resistencia de Materiales.

Examen Preg. 3. 28/11/02

1.° De las condiciones de equilibrio en la posición de equilibrio indiferente

$$\begin{cases} R_A - R_C = 0 \\ V_A - P = 0 \\ R_A \dfrac{l}{2} = Pf \end{cases}$$

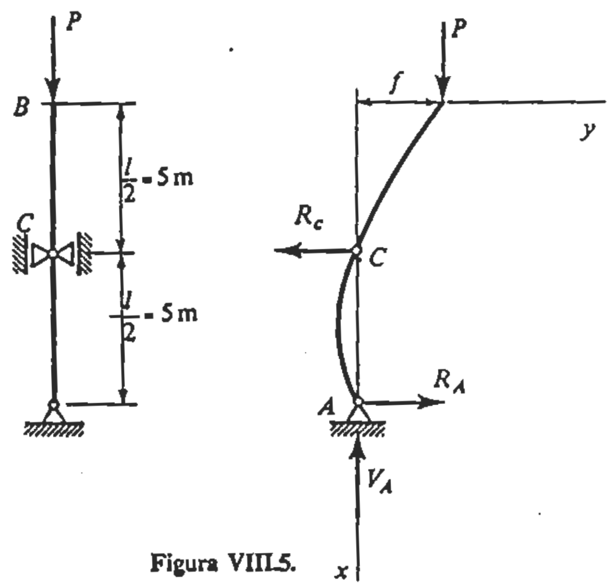

Figura VIII.5.

se deducen los valores de las reacciones en las ligaduras A y C

(1,0)

$$V_A = P \quad ; \quad R_A = R_C = \frac{2Pf}{l}$$

Las leyes de momentos flectores en el soporte sometido a carga son:

$$M = P(f - y_1) \qquad ; \quad 0 \leqslant x \leqslant \frac{l}{2}$$

$$M = P(f - y_2) - \frac{2Pf}{l}\left(x - \frac{l}{2}\right) \quad ; \quad \frac{l}{2} \leqslant x \leqslant l$$

por lo que las ecuaciones de la elástica serán:

$$EIy_1'' = P(f - y_1) \quad 0 \leqslant x \leqslant \frac{l}{2}$$

(1,5)

$$y_1'' + k^2 y_1 = k^2 f \quad \text{siendo} \quad k^2 = \frac{P}{EI}$$

$$y_1 = A \operatorname{sen} kx + B \cos kx + f$$

$$EIy_2'' = Pf - Py_2 - \frac{2Pf}{l}x + Pf \quad ; \quad \frac{l}{2} \leqslant x \leqslant l$$

$$y_2'' + k^2 y_2 = 2k^2 f - \frac{2k^2 f}{l}x$$

(1,5)

$$y_2 = C \operatorname{sen} kx + D \cos kx + 2f - \frac{2f}{l}x$$

Determinación de las constantes de integración

$$x = 0 \;\; ; \;\; y_1 = f \quad \Rightarrow \quad f = B + f \quad \Rightarrow \quad B = 0$$

$$x = \frac{l}{2} \;\; ; \;\; y_1 = 0 \quad \Rightarrow \quad A \operatorname{sen} \frac{kl}{2} + f = 0$$

(1,0)

$$x = \frac{l}{2} \;\; ; \;\; y_2 = 0 \quad \Rightarrow \quad C \operatorname{sen} \frac{kl}{2} + D \cos \frac{kl}{2} + 2f - f = 0$$

$$x = \frac{l}{2} \;\; ; \;\; y_1' = y_2' \quad \Rightarrow \quad Ak \cos \frac{kl}{2} = Ck \cos \frac{kl}{2} - Dk \operatorname{sen} \frac{kl}{2} - \frac{2f}{l}$$

$$x = l \;\; ; \;\; y_2 = 0 \quad \Rightarrow \quad C \operatorname{sen} kl + D \cos kl + 2f - 2f = 0$$

Estas condiciones de contorno constituyen un sistema homogéneo de cuatro ecuaciones con cuatro incógnitas: A, C, D y f. La condición para que este sistema tenga solución distinta de la trivial, que no interesa, es que el determinante de los coeficientes sea igual a cero

(0,5)

$$\begin{vmatrix} \operatorname{sen} \frac{kl}{2} & 0 & 0 & 1 \\ 0 & \operatorname{sen} \frac{kl}{2} & \cos \frac{kl}{2} & 1 \\ k \cos \frac{kl}{2} & -k \cos \frac{kl}{2} & k \operatorname{sen} \frac{kl}{2} & \frac{2}{l} \\ 0 & \operatorname{sen} kl & \cos kl & 0 \end{vmatrix} = 0$$

Desarrollándolo por los elementos de la primera fila, se tiene:

$$\operatorname{sen} \frac{kl}{2} \begin{vmatrix} \operatorname{sen} \frac{kl}{2} & \cos \frac{kl}{2} & 1 \\ -k \cos \frac{kl}{2} & k \operatorname{sen} \frac{kl}{2} & \frac{2}{l} \\ \operatorname{sen} kl & \cos kl & 0 \end{vmatrix} + k \cos \frac{kl}{2} \begin{vmatrix} \operatorname{sen} \frac{kl}{2} & \cos \frac{kl}{2} \\ \operatorname{sen} kl & \cos kl \end{vmatrix} = 0$$

(0,5)

$$\operatorname{sen} \frac{kl}{2} \left[\frac{2}{l} \operatorname{sen} kl \cos \frac{kl}{2} - k \cos \frac{kl}{2} \cos kl - k \operatorname{sen} \frac{kl}{2} \operatorname{sen} kl - \frac{2}{l} \cos kl \operatorname{sen} \frac{kl}{2} \right] +$$

$$+ k \cos \frac{kl}{2} \left(\operatorname{sen} \frac{kl}{2} \cos kl - \cos \frac{kl}{2} \operatorname{sen} kl \right) = 0$$

Simplificando:

$$\frac{2}{l} \left(\operatorname{sen} kl \cos \frac{kl}{2} - \cos kl \operatorname{sen} \frac{kl}{2} \right) - k \left(\cos \frac{kl}{2} \cos kl + \operatorname{sen} \frac{kl}{2} \operatorname{sen} kl \right) - k \cos \frac{kl}{2} = 0$$

$$\frac{2}{l} \operatorname{sen} \frac{kl}{2} - 2k \cos \frac{kl}{2} = 0 \quad \Rightarrow \quad \operatorname{tg} \frac{kl}{2} = lk$$