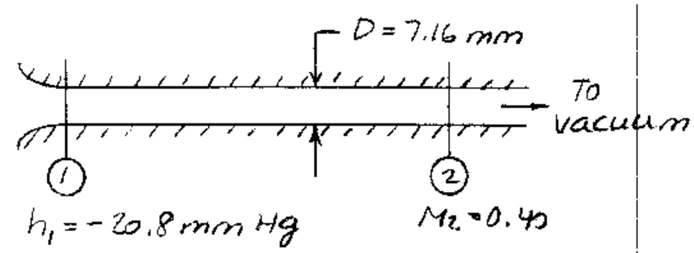
Given: Fanno line flow apparatus in laboratory, smooth brass tube fed by converging nozzle.

$T = 23°C$

$h_{barometer} = 755.1$ mm Hg



Find: (a) M_1

(b) Mass flow rate in tube

(c) p_2

Solution: Apply equations for steady, 1-D compressible flow:

Computing equations: $T_0 = T\left(1+\frac{k-1}{2}M^2\right)$ {Entire flow adiabatic}

$p_0 = p\left(1+\frac{k-1}{2}M^2\right)^{\frac{k}{k-1}}$ {Isentropic in nozzle}

Assume: (1) Stagnation conditions in laboratory

(2) Ideal gas

Then $\frac{p_0}{p_1} = \left(1+\frac{k-1}{2}M_1^2\right)^{\frac{k}{k-1}} = \frac{755.1 \text{ mm Hg}}{(755.1-20.8) \text{ mm Hg}} = 1.03$

$M_1 = \left\{\frac{2}{k-1}\left[\left(\frac{p_0}{p_1}\right)^{\frac{k-1}{k}}-1\right]\right\}^{1/2} = \left\{5\left[\left(\frac{755.1}{734.3}\right)^{0.286}-1\right]\right\}^{1/2} = 0.200$ M_1

From continuity, $\dot{m} = \rho_1 V_1 A_1$; $\rho_1 = \frac{p_1}{RT_1}$

$p_1 = \rho_{Hg} g h_1 = (13.5)1000 \frac{kg}{m^3} \times 9.81 \frac{m}{s^2} \times 0.734 \, m \times \frac{N \cdot s^2}{kg \cdot m} = 97.2$ kPa (abs)

$T_1 = \frac{T_0}{1+\frac{k-1}{2}M_1^2} = \frac{(273+23)K}{1+0.2(0.200)^2} = 294$ K ; $c_1 = \sqrt{kRT_1} = 344$ m/s

$\rho_1 = 97.2\times10^3 \frac{N}{m^2} \times \frac{kg \cdot K}{287 \, N \cdot m} \times \frac{1}{294 \, K} = 1.15 \; kg/m^3$

$V_1 = M_1 c_1 = 0.200 \times 344$ m/s $= 68.8$ m/s

$A_1 = \frac{\pi D^2}{4} = \frac{\pi}{4}(0.00716)^2 m^2 = 4.03\times10^{-5} m^2$

$\dot{m} = 1.15 \frac{kg}{m^3} \times 68.8 \frac{m}{s} \times 4.03\times10^{-5} m^2 = 3.19\times10^{-3}$ kg/s $\dot{m}$

Since T_0 = constant, $T_2 = T_0 / \left(1+\frac{k-1}{2}M_2^2\right) = 287$ K ; $c_2 = 340$ m/s ; $V_2 = M_2 c_2 = 136$ m/s

$\rho_2 = \rho_1 \frac{V_1}{V_2} = 1.15 \frac{kg}{m^3} \times \frac{68.8}{136} = 0.582 \; kg/m^3$

Then $p_2 = \rho_2 R T_2$

$p_2 = 0.582 \frac{kg}{m^3} \times 287 \frac{N \cdot m}{kg \cdot K} \times 287 \, K = 47.9$ kPa (abs) p_2