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Jeffrey E. Zabel

Journal of Labor Economics, Volume 11, Issue 2 (Apr., 1993), 387-416.

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Jeffrey E. Zabel, *Tufts University*

This article analyzes the relationship between hours of work and labor force participation (LFP) in Heckman's model, Cogan's fixed-cost model, Moffitt's minimum hours constraint model, and a generalized version of Heckman's model. First, the parameter restrictions between the LFP and reduced-form hours-of-work equations are compared. The models are then estimated, and the results support the weakening of the link between the LFP and hours-of-work decisions. One implication of the analysis is that Heckman's model overstates the standard labor supply elasticities because it confounds the direct effect on labor supply with the participation effect.

I. Introduction

The statistical model of labor supply behavior that is most prevalent in the literature was first analyzed by Heckman (1974). This model is derived from the comparison of the reservation and market wages and, hence, has a strong theoretical foundation. One important contribution of Heckman's work is the characterization of the simultaneous labor force participation and supply of hours decisions. In Heckman's model, these two decisions are intrinsically linked. One implication of the structure of this model is that the parameters in the reduced-form hours-of-work equation are pro-

I wish to thank Linda Datcher Loury, Thomas Lemieux, Amy Ellen Schwartz, Saul Schwartz, and participants at workshops at Tufts University and the Massachusetts Institute of Technology for their useful comments.

[*Journal of Labor Economics,* 1993, vol. 11, no. 2]

0734-306X/93/1102-0005$01.50

portional to the parameters in the (implicit) labor force participation equation.

At first thought, this strong link between labor force participation and the supply of hours is credible since these decisions appear to be closely related. But there are reasons to believe that this strong association implied by Heckman's model is too restrictive. Individuals are usually constrained, possibly by choice, in the number of hours that they can choose and seem to have more control over the decision to work rather than how much they work. For example, Card (1990) finds evidence of minimum hours constraints for a sample of men from the Survey of Income and Program Participation.

Cogan (1980, 1981) shows that there are a minimum (positive) number of hours an individual will choose to work due to the fixed costs of work. The fixed-cost model relaxes the restriction on the participation and hours decisions implied by Heckman's model, though some proportionality constraints between the parameters in the participation equation and the reduced-form hours-of-work equation remain. It is still possible to derive the fixed-cost model from a comparison of the reservation and market wages, but now the equivalence between the reservation wage and the value of home time at zero hours of work that holds in Heckman's model is not valid in Cogan's model.

Another reason for the weakening of the link between the participation and supply-of-hours decisions is that firms impose minimum hours constraints on their employees, giving them less choice over the number of hours they can work. Moffitt (1982) proposes a model that accounts for minimum hours constraints imposed by firms. An implication of this model is that there are fewer constraints between the parameters in the participation equation and the reduced-form hours-of-work equation than is the case for Heckman's model.

A general model will also be developed. This model imposes no constraints between the parameters in the participation equation and those in the reduced-form hours-of-work equation. Thus, it is possible that certain variables affect the participation decision, while others influence the hours decision. Zabel (1990) shows that this model nests the other three models and gives restrictions that must be imposed to obtain each of the more specific models.

In this article, the relationship between labor force participation and hours of work will be compared for these four models. This will be accomplished by first analyzing the theoretical relationships between the labor force participation and hours-of-work equations in terms of the parameter restrictions across these two equations that each model imposes. The models will then be estimated using a sample of married women from the Panel Study of Income Dynamics and an empirical comparison will follow.

It has been found in earlier studies that Heckman's model tends to overstate the magnitude of the standard elasticities of labor supply (Cogan 1980, 1981; Mroz 1987). Here, it is shown that this occurs because Heckman's model confounds the direct effect of wages on hours supplied, measured for workers only, with the participation effect, measured for both workers and nonworkers.

Overall, the results indicate that the relationship between the labor force participation and hours-of-work decisions implied by Heckman's model is too restrictive. The fixed-cost and minimum hours constraints models produce similar results, and it is not possible to make a strong distinction between them. Thus, one cannot determine whether fixed costs or minimum hours constraints offer a better explanation for the weakening of the association between the participation and supply-of-hours decisions. This may not be too surprising since both models impose lower bounds on the number of hours supplied. Thus, the generalized model is a useful and important model since it includes both the fixed-cost and minimum hours constraints models as special cases.

The four models are presented in Section II. A discussion of the data and the empirical analysis are in Section III. Section IV concludes.

II. Four Models of Labor Supply Behavior

In this section, the four models of labor supply behavior are presented. Each of the four models contains underlying hours-of-work and wage equations

$$H_i^* = \beta_1 \ln W_i^* + \beta_2 \mathrm{NI}_i + X_i\beta_3 + u_{1i} \tag{1}$$

and

$$\ln W_i^* = Y_i\alpha + u_{2i}, \tag{2}$$

with

H_i^* = latent value for desired hours of work,
$\ln W_i^*$ = latent value for the natural log of hourly wage,
NI_i = nonlabor income, and
X_i, Y_i = vectors of individual characteristics.

The linear and log-linear functional forms for the hours-of-work and wage equations are consistent with Heckman's original specification. When individuals work, their supply of hours and wages are observed. If they do not work, hours of work are zero and wages are not observed. The differences in the four models arise from their characterization of the labor force participation (LFP) decision and what this implies about the relationship between this decision and the supply-of-hours decision.

A. Heckman's Model

In Heckman's (1974) model, there are two equations that characterize labor supply behavior—the hours-of-work and wage equations—and there are two labor force states—working and not working. This model is derived from the comparison of reservation and market wages; individuals will work if their market wages are greater than their reservation wages at zero hours of work, and they will not work otherwise. In this case, the LFP decision is inextricably linked to the supply-of-hours decision. A version of Heckman's labor supply model that is used by Wales and Woodland (1980) explicitly models the LFP decision as one based on whether desired hours of work are greater or less than zero:

$$\left.\begin{array}{ll} \text{work} & \text{if } H_i^* > 0, \\ \text{do not work} & \text{if } H_i^* \leq 0 \rightarrow \dfrac{u_{1i} + \beta_1 u_{2i}}{\sigma} \leq -t_i, \end{array}\right\} \tag{3}$$

where

$$t_i = \frac{\beta_1 Y_i \alpha + \beta_2 \mathrm{NI}_i + X_i \beta_3}{\sigma} \tag{4}$$

and

$$\sigma = \text{std dev}(u_{1i} + \beta_1 u_{2i}) = (\sigma_1^2 + \beta_1^2 \sigma_2^2 + 2\beta_1 \sigma_{12})^{1/2}, \tag{5}$$

and where u_{1i} and u_{2i} are jointly normal with zero means and covariance matrix

$$\mathbf{\Sigma}_{12} = \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix}. \tag{6}$$

This model is a version of the Tobit model and will be referred to as a Tobit-type model. The corresponding log-likelihood function for this model is

$$\log L = \sum_0 \log[\Phi(-t_i)] + \sum_1 \log[b(u_{1i}, u_{2i}; \mathbf{\Sigma}_{12})], \tag{7}$$

where Φ is the standard normal cumulative density function (cdf), $b(v, w; \Omega)$ is the bivariate normal probability density function (pdf), and Σ_0 and Σ_1 indicate summation over nonworkers and workers, respectively. The first term in the log-likelihood function corresponds to the probability of not working, and the parameters in t_i are proportional to the parameters in the reduced-form hours-of-work equation with constant of proportionality equal to $1/\sigma$. The second term corresponds to the hours decision for workers.

B. The Fixed-Cost Model

The fixed-cost model, developed by Cogan (1980, 1981), differs from Heckman's model because it assumes a weaker association between the LFP and hours decisions due to the costs associated with labor market entry. Cogan shows that these fixed costs of work imply that there is a minimum number of hours that an individual will work so that these costs can be recovered. Cogan refers to the minimum number of hours as "reservation hours." Thus, if desired hours are less than reservation hours, the individual will get more utility from not working.

In addition to equations (1) and (2), a generalized version of the fixed-cost model contains a third equation, the reservation hours equation:[1]

$$H_i^r = \gamma_1 \mathrm{NI}_i + F_i \gamma_2 + u_{3fi}, \tag{8}$$

where F_i is a vector of individual characteristics. It is assumed that u_{1i}, u_{2i}, and u_{3fi} have a joint normal distribution with zero means and covariance matrix

$$\mathbf{\Sigma}_{123f} = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13f} \\ \sigma_{21} & \sigma_2^2 & \sigma_{23f} \\ \sigma_{13f} & \sigma_{23f} & \sigma_{3f}^2 \end{pmatrix}. \tag{9}$$

The LFP decision for the generalized fixed-cost model is

$$\left.\begin{array}{rl} \text{work} & \text{if } H_i^* > H_i^r, \\ \text{do not work} & \text{if } H_i^* \le H_i^r \rightarrow \dfrac{u_{1i} + \beta_1 u_{2i} - u_{3fi}}{\sigma_f} \le -f_i, \end{array}\right\} \tag{10}$$

[1] This model is a slight generalization of Cogan's model since there are separate sets of taste variables included in the hours-of-work and reservation hours equations. This generalization is made because there does not appear to be any a priori reason why these taste vectors should be the same. Also, Cogan derives the effect of a marginal change in nonlabor income on reservation hours and finds the sign to be ambiguous. Nonlabor income is explicitly included in the generalized reservation hours equation so that the sign of this effect can be determined empirically.

where

$$f_i = \frac{\beta_1 Y_i \alpha + (\beta_2 - \gamma_1)\mathrm{NI}_i + X_i\beta_3 - F_i\gamma_2}{\sigma_f}, \tag{11}$$

and

$$\begin{aligned}\sigma_f &= \text{std dev}(u_{1i} + \beta_1 u_{2i} - u_{3fi}) \\ &= (\sigma_1^2 + \beta_1^2\sigma_2^2 + \sigma_{3f}^2 + 2\beta_1\sigma_{12} - 2\sigma_{13f} - 2\beta_1\sigma_{23f})^{1/2}.\end{aligned} \tag{12}$$

The corresponding log-likelihood function is

$$\begin{aligned}\log L = \sum_0 \log[\Phi(-f_i)] \\ + \sum_1 \Bigg\{ \log\left[\Phi\left(\frac{H_i - \gamma_1\mathrm{NI}_i - F_i\gamma_2}{\sigma_{3f}}\bigg| u_{1i}, u_{2i}\right)\right] \\ + \log[b(u_{1i}, u_{2i}; \mathbf{\Sigma}_{12})]\Bigg\}.\end{aligned} \tag{13}$$

As in the Tobit-type model, the first term in the log-likelihood function is the probability of not working. The difference between the Tobit-type model and the generalized fixed-cost model can be seen by looking at t_i and f_i, the components of the LFP decisions for these two models. Notice that f_i includes regressors from the reservation hours equation. This leads to fewer constraints between the parameters in f_i and those in the reduced-form hours-of-work equation.[2] There will be at least one constraint if there is one regressor in Y_i or X_i that is not in F_i. The second term in the log-likelihood function corresponds to the decision to work, conditional on u_{1i} and u_{2i}, and the third term corresponds to the hours decision.

C. The Minimum Hours Constraint Model

Another explanation for the divergence of the LFP and hours decisions stems from the fact that the amount one works is limited by the available job opportunities, and these are influenced by the institutional structure of the workplace. Most jobs are full-time, so the option to work, say, 30 hours a week may not exist. There are good reasons why a firm will impose

[2] Consider a variable that is in both X_i and F_i, say, XF_i, with corresponding parameters in the hours-of-work and reservation hours equations of β_{xf} and γ_{xf}. Then in the Tobit-type model, the parameter associated with XF_i in t_i is β_{xf}/σ, which is proportional to β_{xf}. But in the fixed-cost model, the parameter associated with XF_i in f_i is $(\beta_{xf} - \gamma_{xf})/\sigma_f$, which is not proportional to β_{xf}.

minimum hours constraints (see Zabel 1990), hence, it is important to consider demand-side factors when analyzing the supply of labor. A model of labor supply that incorporates a minimum hours constraint imposed by firms is developed by Moffitt (1982). Wages are not included in this model, so it is modified to allow for the simultaneous determination of wages. Note that this makes it possible to calculate the labor supply elasticity with respect to wages. The minimum hours constraint imposed by firms, $H_i^{\min}$, is modeled as

$$H_i^{\min} = V_i\eta + u_{3di}, \tag{14}$$

where V_i is a vector of variables that affect $H_i^{\min}$.[3] In Moffitt's model it is assumed that u_{3di} is normally distributed with variance σ_{3d}^2 and is independent of u_{1i}. In the version of this model used here, no restrictions will be imposed on the three disturbance terms. Thus, it is assumed that u_{1i}, u_{2i}, and u_{3di} have a joint normal distribution with zero means and covariance matrix

$$\mathbf{\Sigma}_{123d} = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13d} \\ \sigma_{21} & \sigma_2^2 & \sigma_{23d} \\ \sigma_{13d} & \sigma_{23d} & \sigma_{3d}^2 \end{pmatrix} \quad \text{with } \mathbf{\Sigma}_{23d} = \begin{pmatrix} \sigma_2^2 & \sigma_{23d} \\ \sigma_{23d} & \sigma_{3d}^2 \end{pmatrix}. \tag{15}$$

If an individual's desired hours of work are greater than $H_i^{\min}$, the minimum hours constraint is not binding. But if desired hours are less than $H_i^{\min}$, the individual must decide between working $H_i^{\min}$ hours and not working. Consider the individual who is indifferent between working the minimum number of hours and not working. Let D be the difference between $H_i^{\min}$ and desired hours for this person (desired hours will be less than $H_i^{\min}$ for this individual). Moffitt (1982) shows that individuals whose desired hours are less than $H_i^{\min} - D$ will work zero hours and those whose desired hours are greater than $H_i^{\min} - D$ will work at least the minimum required hours. The LFP decision for the minimum hours constraint model is

[3] One question that arises is, What are the elements of V? Moffitt used age, education, and a nonwhite indicator. The V's are factors that affect the minimum hours required, so some firm- or job-specific variables should be included. For example, occupation or industry dummies could be included. But one problem with this is that the occupation of nonworkers is often unknown, so this type of variable cannot be included in the model. In fact, any firm-specific variables may not be included for the same reason. Hence, the minimum hours constraint function is a reduced-form equation where the elements of V reflect, among other things, the individual characteristics that influence the type of occupation the individual will choose. Other variables that will be included are local labor market conditions variables that are likely to affect the demand for labor.

$$\left.\begin{aligned}&\text{work desired hours} \quad \text{if } H_i^* > H_i^{\min}, \\ &\text{work minimum hours} \quad \text{if } H_i^{\min} - D < H_i^* \le H_i^{\min}, \\ &\text{and} \\ &\text{do not work} \quad \text{if } H_i^* \le H_i^{\min} - D \rightarrow \frac{u_{1i} + \beta_1 u_{2i} - u_{3di}}{\sigma_d} \le -m_i,\end{aligned}\right\} \quad (16)$$

where

$$m_i = \frac{\beta_1 Y_i \alpha + \beta_2 \mathrm{NI}_i + X_i \beta_3 - V_i \eta + D}{\sigma_d} \quad (17)$$

and

$$\begin{aligned}\sigma_d &= \text{std dev}(u_{1i} + \beta_1 u_{2i} - u_{3di}) \\ &= (\sigma_1^2 + \beta_1^2 \sigma_2^2 + \sigma_{3d}^2 + 2\beta_1 \sigma_{12} - 2\sigma_{13d} - 2\beta_1 \sigma_{23d})^{1/2}.\end{aligned} \quad (18)$$

The log-likelihood function is

$$\begin{aligned}\log L = {} & \sum_0 \log[\Phi(-d_i)] \\ & + \sum_1 \log\Bigg\{ \Phi\left(\frac{H_i - V_i\eta}{\sigma_{3d}} \middle| u_{1i}, u_{2i}\right) \cdot b(u_{1i}, u_{2i}; \Sigma_{12}) \\ & \qquad + \Bigg[\Phi\left(\frac{H_i - a_i}{\sigma_1} \middle| u_{2i}, u_{3di}\right) \\ & \qquad\qquad - \Phi\left(\frac{H_i - a_i - D}{\sigma_1} \middle| u_{2i}, u_{3di}\right)\Bigg] \\ & \qquad \times b(u_{2i}, u_{3di}; \Sigma_{23d})\Bigg\},\end{aligned} \quad (19)$$

where

$$a_i = \beta_1 \ln W_i + \beta_2 \mathrm{NI}_i + X_i \beta_3. \quad (20)$$

There are three states of work in this model: not working, working the constrained number of hours, and working desired hours.[4] The first term

[4] Note that only whether one works or does not work is observed.

in the log-likelihood function corresponds to the probability of not working. The remaining two terms correspond to working the minimum number of hours and working the desired number of hours. Again, one can see the difference between the Tobit-type model and the minimum hours constraint model by looking at t_i and m_i, the components of the LFP decision. Here, m_i includes regressors from the minimum hours equation as well as the parameter D. Their presence leads to fewer proportionality restrictions between the parameters in the reduced-form hours-of-work equation and m_i (see n. 2 above). Again, there will be at least one restriction if there is a variable in the hours-of-work or wage equation that is not in the minimum hours constraint equation.

D. A Generalized Labor Supply Model

A generalization of the Tobit-type model allows for a separate LFP equation.[5] This generalizes the Tobit-type model in two ways: first, the parameters in the LFP equation are not restricted to be proportional to the parameters in the reduced-form hours-of-work equation, and, second, the regressors included in the LFP equation are not necessarily identical to those in the reduced-form hours-of-work equation. While one may be inclined to believe that these two sets of variables should be the same, the presence of fixed costs of work or minimum hours constraints can lead to different regressors affecting the LFP and hours decisions. In the generalized Tobit-type model, these decisions are still related through the disturbance terms in these equations. The separate equation for labor force participation in the generalized Tobit-type model is

$$\mathrm{LFP}_i^* = Z_i\pi + u_{3gi}, \tag{21}$$

where LFP_i^* is a latent measure of the tendency to work and Z_i is a vector of variables. The three disturbance terms, u_{1i}, u_{2i}, and u_{3gi}, are assumed to have a joint normal distribution with zero means and covariance matrix[6]

$$\boldsymbol{\Sigma}_{123g} = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13g} \\ \sigma_{21} & \sigma_2^2 & \sigma_{23g} \\ \sigma_{13g} & \sigma_{23g} & 1 \end{pmatrix}. \tag{22}$$

The labor force participation decision is

$$\left.\begin{array}{ll} \text{work} & \text{if } \mathrm{LFP}_i^* > 0 \\ \text{do not work} & \text{if } \mathrm{LFP}_i^* \le 0 \rightarrow u_{3gi} \le -Z_i\pi. \end{array}\right\} \tag{23}$$

[5] This fits in the framework developed by Heckman (1978) for dummy endogenous variables in a simultaneous equation system.

[6] In order to identify the parameters in the LFP equation, the variance of the disturbance term in the LFP equation is set equal to one.

The log-likelihood function is

$$\log L = \sum_{0} \log[\Phi(-Z_i\pi)] + \sum_{1} \{\log[\Phi(Z_i\pi \mid u_{1i}, u_{2i})] + \log[b(u_{1i}, u_{2i}; \Sigma_{12})]\}. \tag{24}$$

For this model, there are three equations that characterize labor supply behavior: the LFP, hours-of-work, and wage equations. Again, two labor force states are observed: the individual either does or does not work. The differences between the log-likelihood functions for the Tobit-type and generalized Tobit-type models are related to the participation decision. The first term in the log-likelihood function is the probability of not working; recall that for the Tobit-type model this depends explicitly on the regressors and parameters in the reduced-form hours-of-work equation. These restrictions do not hold for the generalized Tobit-type model. Thus, the Tobit-type model includes the most restrictions between the parameters in the labor force participation and reduced-form hours-of-work equations. Fewer restrictions hold for the fixed-cost and minimum hours constraint model, while none exist for the generalized Tobit-type model.

The second term in the log-likelihood function is the probability of working conditional on u_{1i} and u_{2i}. This term would be zero under the restrictions implied by the Tobit-type model. Finally, the third term in the log-likelihood function corresponds to the hours decision. This term also appears in the log-likelihood functions for the other three models.

III. Empirical Results

The empirical analysis will focus on the labor supply behavior of married women. This group's labor supply behavior is very responsive to variables such as the wage rate, spouse's income, and the number of children. Hence, it was felt that limiting the analysis to married women would provide the best opportunity to see how the four models differ.

A. The Data Set and Initial Model Specifications

The data set that will be used for this analysis is a sample from the Panel Study of Income Dynamics (PSID). Data from the interview year 1987, which includes data for 1986, were chosen. The analysis is limited to wives aged 30–60 whose husbands earned labor income in 1986. The resulting sample data set contains observations on 830 married women, 640 (77%) of whom did some work for money in 1986. A list of the conditions for inclusion in the data set can be found in Appendix A.

The log of wages, nonlabor income, age, education, and variables related to the number of children and health are included in the hours-of-work equation, and age and education up to cubic terms, health, a variable indicating residence in a standard metropolitan statistical area (SMSA), variables related to parents' education, regional dummies, and three local labor market conditions variables are included in the wage equations in all four models. The union of the variables in the hours-of-work and wage equations are included in the LFP equation in the generalized Tobit-type model. The reservation hours equation in the generalized fixed-cost model contains the age and education variables up to cubic terms, nonlabor income, SMSA, and variables related to the number of children and health. The variables in the minimum hours equation in the minimum hours constraint model include age and education variables up to cubic terms, nonlabor income, the local labor market conditions variables, variables related to the education of the parents, SMSA, and regional variables. Definitions of these variables are given in Appendix B. Means and standard deviations for the variables by labor force status are provided in table 1.

Table 1
Means for Variables by Labor Force Status

Variable	Total	Workers	Nonworkers	Min	Max
Hours of work	1,158.22	1,501.45	...	4.0	4,742.0
		(697.85)			
LNWAGE	...	1.962	...	.106	3.831
		(.616)			
NONLINC	36.154	34.606	41.368	.20	106.295
	(18.911)	(17.810)	(21.467)		
AGE	40.080	39.661	41.489	30.0	60.0
	(8.134)	(7.779)	(9.113)		
EDUC	13.263	13.413	12.758	5.0	18.0
	(2.240)	(2.175)	(2.385)		
NKIDLT6	.325	.281	.474	.0	3.0
	(.646)	(.599)	(.768)		
NKID6T17	1.023	1.030	1.000	.0	5.0
	(1.076)	(1.074)	(1.089)		
UNPL	6.383	6.289	6.700	1.0	23.0
	(2.703)	(2.651)	(2.856)		
LNUNSKLL	1.498	1.506	1.473	1.209	2.282
	(.259)	(.261)	(.252)		
MATJ	.648	.641	.674	.0	1.0
LKAW	.122	.108	.168	.0	1.0
NORTHEST	.186	.175	.221	.0	1.0
NORTHCEN	.349	.348	.353	.0	1.0
SOUTH	.269	.260	.268	.0	1.0
WEST	.196	.208	.158	.0	1.0
MEDUCHS	.527	.542	.474	.0	1.0
MEDUCCOL	.092	.094	.084	.0	1.0
FEDUCHS	.405	.420	.353	.0	1.0
FEDUCCOL	.125	.138	.084	.0	1.0
SMSA	.494	.489	.511	.0	1.0

NOTE.—Standard deviations are in parentheses. N = 830. Number (percent) of nonworkers = 190 (22.89%). Number (percent) of workers = 640 (77.11%).

The measure of hours of work that is used in this study is the product of the number of hours usually worked per week and the number of weeks worked in 1986, giving an annual measure. The wage variable is obtained by dividing total labor income in 1986 by annual hours of work in 1986.[7]

The hours-of-work equations in all four models are identified since the wage equation includes a number of variables that are excluded from the hours equation. Note that the variables included in the LFP equation are the same as those that are included in the wage and hours equations. Thus, the LFP equation is not identified by exclusion restrictions, though the empirical results do indicate that different variables belong in the LFP equation and the wage and hours equations. Still, the LFP equation is identified by the nonlinear structure of the generalized Tobit-type model. Both the reservation and minimum hours equations are identified because the parameters in the reduced-form hours-of-work equations in the generalized fixed-cost and minimum hours constraint models do not appear in the part of the likelihood function corresponding to the probability of working.

Given that both the reservation and minimum hours equations are minimum hours constraint equations, it is important to include variables that will differentiate the two equations. A set of local labor market demand variables, regional variables, and variables related to parents' education are included in the minimum hours equation and excluded from the reservation hours equation. Variables relating to individual health and the number of children present at home are included in the reservation hours equation and not in the minimum hours equation.

[7] Given that there is measurement error in the observed values of annual hours of work, this induces a negative correlation between the dependent variable, annual hours of work, and a regressor, log(wages), that could lead to a negative bias in the estimate of the coefficient for log(wages). Call this wage measure LNWAGE. One other wage measure that does not suffer from this bias is the current wage or salary in 1986. Call this measure CURWAGE. The problem with CURWAGE is that it is only observed for women working during the week of the survey. This induces a form of selection bias. To see if a bias arises by using LNWAGE, the labor supply models are first estimated using CURWAGE. Women who worked during 1986 but not during the survey week are excluded. These estimates are compared to those based on LNWAGE for the restricted sample. If there is measurement error in annual hours of work, the estimated parameter for LNWAGE in the hours-of-work equation should be less than the one when CURWAGE is used. These two sets of estimates are very similar, and the estimated coefficient for LNWAGE is actually larger than the one for CURWAGE. Thus, this gives little indication of bias caused by the use of LNWAGE. One reason the bias appears to be minimal may be that LNWAGE is treated as endogenous. Thus, an instrument for LNWAGE is used in the hours-of-work equation, and this is a means for correcting for measurement error. See Borjas (1980) for a thorough analysis of this bias problem.

B. Estimation Results

The estimation results for the four models are given in table 2.[8] Parameter estimates and t-statistics are presented for variables in the hours equations for all four models, the reservations hours equation for the generalized fixed-cost model, the minimum hours equation for the minimum hours constraint model, and the LFP equation for the generalized Tobit-type model. The parameter estimates for the age and education variables are not listed for the latter three equations.

The goal of the empirical analysis is to compare the relationships between the LFP and hours-of-work decisions for the four models. This will be carried out in five ways. First, the wage and nonlabor income elasticities will be compared. Second, the different effects of the regressors on the LFP decisions for the four models are analyzed. Third, the derivatives of the unconditional expectation of labor supply with respect to nonlabor income and the number of young and older children present are calculated. This is a useful measure since it takes both workers and nonworkers into account and it provides a means for comparing the differential effects that variables have on the LFP and hours-of-work decisions. Fourth, the predictive accuracy of the four models with respect to labor force participation will be calculated. Finally, the four models will be directly compared using the likelihood ratio (LR) test.

C. Labor Supply Elasticities

The labor supply elasticities with respect to wages and nonlabor income are given in table 3. Note that these elasticities are calculated differently for the minimum hours constraint model. See Appendix C for details. The uncompensated wage elasticities for the fixed-cost, minimum hours constraint, and generalized Tobit-type models are similar, while the value for the Tobit-type model is much higher. This large value for the Tobit-type model has also been found by Mroz (1987) and Cogan (1980, 1981) and is an indication that the implicit restriction in the Tobit-type model that does not allow for separate hours-of-work and LFP decisions does not hold.

[8] The exogeneity of variables relating to nonlabor income (NONLINC), the number of children (NKIDLT6, NKID6T17), and health (LKAW) has been questioned in different studies of labor supply behavior, so it is important to determine the validity of the assumption that these variables are exogenous. The exogeneity of NONLINC, NKIDLT6, and NKID6T17 was tested using a version of the Hausman test (Smith and Blundell 1986). The null hypothesis of exogeneity was not rejected for these three variables. Zabel (1991) finds little evidence to reject the exogeneity assumption for LKAW for prime-aged married women, so this assumption is maintained for this study. The exogeneity of previous experience was also tested and not rejected. There is little change in the results when it is added to the models, so previous experience is excluded from the analysis.

Table 2
Parameter Estimates for Models
A. Labor Supply Equation

Variable	Tobit Type	Generalized Fixed Cost	Minimum Hours Constraint	Generalized Tobit Type
CONSTANT	1,507.2 (4.01)	2,121.2 (7.68)	1,452.8 (2.87)	2,100.5 (7.22)
LNWAGE	831.49 (2.11)	275.58 (2.20)	305.10 (1.04)	295.36 (1.55)
NONLINC	−13.96 (6.72)	−5.07 (3.22)	−14.34 (4.90)	−5.01 (3.18)
NKIDLT6	−570.38 (8.31)	−222.13 (4.06)	−762.92 (5.69)	−221.09 (4.04)
NKID6T17	−138.47 (3.62)	−57.77 (2.02)	−194.13 (3.47)	−57.49 (2.02)
EDUC	−7.77 (.16)	−45.85 (2.69)	59.23 (1.55)	−47.65 (2.32)
AGE	−27.22 (4.66)	−3.08 (.70)	−33.02 (3.56)	−3.08 (.66)
LKAW	−175.30 (1.28)	−83.19 (.96)	−372.28 (2.40)	−78.20 (.90)
		Reservation Hours Equation	Minimum Hours Equation	LFP Equation
CONSTANT		6,974.4 (1.12)	31,501 (2.11)	−21.522 (1.34)
NONLINC		−3.71 (1.70)	1.97 (.85)	−.009 (3.73)
NKIDLT6		−154.30 (1.69)	...	−.414 (4.88)
NKID6T17		−34.61 (.87)	...	−.144 (3.01)
LKAW		−64.08 (.59)	...	−.209 (1.46)
SMSA		51.07 (1.40)	111.78 (1.42)	−.069 (.70)
UNPL		...	20.44 (1.31)	−.031 (1.63)
MATJ		...	−128.95 (1.35)	−.001 (.01)
LNUNSKLL		...	103.58 (.63)	.319 (1.67)
NORTHEST		...	206.24 (1.64)	−.367 (2.41)
NORTHCEN		...	109.72 (.99)	−.108 (.80)
SOUTH		...	330.70 (2.83)	−.170 (1.12)
MEDUCHS		...	121.56 (1.29)	.034 (.30)
MEDUCCOL		...	−35.15 (.23)	−.139 (.68)
FEDUCHS		...	−145.36 (1.60)	.127 (1.10)
FEDUCCOL		...	−250.01 (1.77)	.273 (1.41)

B. Covariance Terms

Variable	Tobit Type	Generalized Fixed Cost	Minimum Hours Constraint	Generalized Tobit Type
σ_1	994.8 (16.7)	681.7 (27.5)	1,020.3 (8.48)	680.5 (25.8)
σ_2	.571 (32.6)	.662 (26.6)	.583 (27.2)	.656 (27.3)
σ_3	...	846.5 (6.30)	778.1 (9.02)	1
ρ_{12}	−.217 (0.95)	.120 (.95)	−.393 (2.27)	.102 (.54)
ρ_{13}	...	.926 (14.6)	−.165 (.84)	−.478 (2.82)
ρ_{23}	...	.482 (4.70)	.545 (8.47)	−.871 (28.2)
D			1,919.0 (4.03)	

NOTE.—Absolute values of t-statistics are in parentheses.

The labor supply elasticities with respect to nonlabor income are negative and significant for all four models. Again, the nonlabor income elasticity for the Tobit-type model is much larger in magnitude than the values for the other three models.

Table 3
Estimated Wage and Nonlabor Income Elasticities

Model	Wage Coefficient	Elasticity	Nonlabor Income Coefficient	Elasticity
Tobit type	831.49	.554	−13.96	−.322
Generalized fixed cost	275.58	.184	−5.07	−.117
Minimum hours*	305.10	.078	−14.34	−.127
Generalized tobit type	295.36	.197	−5.01	−.115
Two-stage	334.60	.223	−4.69	−.108

* The wage and nonlabor income elasticities for the minimum hours constraint model are not computed in the same way as they are for the other three models. This is because the interpretation of the coefficients in the labor supply equation is different than for the other three models. See App. C for details.

The last row in table 3 gives the estimated value of the uncompensated wage and nonlabor income elasticities using the two-step estimation procedure developed by Heckman (1978). These values are very close to those for the generalized Tobit-type model. One can think of the estimation procedure applied to the generalized Tobit-type model as the maximum likelihood version of the two-step estimation procedure with a stronger assumption of the joint normality of the disturbance terms in the LFP, hours, and wage equations. Thus the fact that these two procedures give similar estimates provides support for the joint normality assumption for the disturbance terms.

D. A Comparison of the Effects of Regressors on LFP

What distinguishes the four models is the structure of the LFP decision. This decision is dependent on the variables in the LFP equation for the generalized Tobit-type model, in the reservation hours equation for the generalized fixed-cost model, and in the minimum hours equation for the minimum hours constraint model. For the Tobit-type model, the LFP decision is based on the reduced-form hours-of-work equation.

Looking at the variables in the LFP equation for the generalized Tobit-type model, it appears that nonlabor income and the presence of young children (NKIDLT6) have a major effect on the LFP decision. Note that NKIDLT6 is highly significant in the hours-of-work equation in the Tobit-type model and the estimated coefficient is much larger than the corresponding value in the generalized Tobit-type model. The results from the Tobit-type model do not allow one to directly differentiate the effects of NKIDLT6 on the decisions to work and supply hours.

If the Tobit-type model is correctly specified, the ratio between the parameters for each variable in the LFP and reduced-form hours-of-work equations in the generalized Tobit-type model should be constant. These restrictions can be tested using the LR test. The results strongly reject the null hypothesis that the parameters in the LFP and reduced-form hours equation are proportional. Thus, it does not appear that the restrictions imposed by the Tobit-type model are valid for this sample.[9]

The three variables relating to local labor market conditions that are included in the minimum hours constraint model are not significant. There does appear to be some regional variation in minimum hours constraints imposed by firms. Parents' education variables are included to account for the type of job a person might have. It appears that higher levels of the father's education are correlated with types of jobs for the daughter that have lower minimum hours constraints.

In the generalized fixed-cost model, the presence of young children has a marginally negative effect on reservation hours. This result is consistent with Cogan (1980). He mentions that this implies that the presence of children increases the time costs of work, and younger children increase time costs more than older children. Note that nonlabor income also has a marginally negative effect on reservation hours.

It was mentioned in the introduction that both the generalized fixed-cost model and the minimum hours constraint model restrict the choice set of hours supplied. Some idea of the restrictions implied by these models is given by the average reservation and minimum hours for the generalized fixed-cost model and the minimum hours constraint model, respectively. The discontinuity in the hours-of-work equation implied by the generalized fixed-cost model is 1,275 hours (approximately 25 hours per week for someone who works 52 weeks a year). Cogan (1980, 1981) estimates the discontinuity to be 1,151 and 1,257 hours, which are comparable to the value found here.

The average minimum hours constraint is 1,435 hours (approximately 28 hours per week). The estimated value of D is 1,919.[10] This estimated

[9] Let X_{1i}, Y_{1i}, and XY_i denote the vectors of variables that are included in the hours-of-work equation, the wage equation, and both equations. Thus, $X_i = (X_{1i}, XY_i)$ and $Y_i = (Y_{1i}, XY_i)$. The parameter vectors $\beta_3 = (\beta_{31}, \beta_{32})$ and $\alpha = (\alpha_1, \alpha_2)$ are similarly partitioned. Z_i, the vector of variables in the LFP equation, includes the variables in the reduced-form hours equation. Thus, it can be decomposed as $Z_i = (\mathrm{NI}_i, X_{1i}, XY_i, Y_{1i})$. The corresponding parameter vector π can be expressed as $\pi = (\pi_{\mathrm{NI}}, \pi_x, \pi_{xy}, \pi_y)$. The restrictions that are imposed on the LFP parameters are

$$\pi_{\mathrm{NI}} = \frac{\beta_2}{\sigma}, \qquad \pi_x = \frac{\beta_{31}}{\sigma}, \qquad \pi_y = \frac{\beta_1\alpha_1}{\sigma}, \qquad \text{and} \qquad \pi_{xy} = \frac{\beta_{32} + \beta_1\alpha_2}{\sigma}.$$

Note that this restricted model has only two more parameters than the Tobit-type model, σ_{13g} and σ_{23g}. See Sec. IIIG for a further discussion of the test results.

[10] Recall that D is the difference between desired hours and the minimum required hours for the individual who is indifferent between working the minimum number of hours and not working.

value for D is very similar to the value that Moffitt found (37 hours a week). The estimated value for the correlation between the disturbance terms in the wage and minimum hours equations is 0.545, with a t-value of 8.47. This positive value might arise because both equations are demand-side equations and the dependent variables are likely to be affected by the same set of unobservables.

One other important result to consider is the percentage of workers that are constrained to work the minimum hours demanded by firms. The results for the minimum hours constraint model indicate that 66% of the workers are constrained, while 34% work more than the minimum hours demanded. Moffitt's (1982) results show that 83% of the workers in his sample of men are constrained. This might reflect the ability of women to work part time while men usually work full time and often overtime.

E. Elasticities of the Unconditional Expectation of Labor Supply

Another means for comparing the four models is to consider the elasticities of the *unconditional expectation* of labor supply since this takes into account all individuals in the sample. The unconditional expectation of labor supply for the generalized Tobit-type model is

$$\mathrm{E}[H_i] = \mathrm{E}[H_i \mid \mathrm{LFP}_i = 1] \cdot \mathrm{prob}(\mathrm{LFP}_i = 1). \tag{25}$$

Thus, the change in the unconditional expectation of labor supply due to a marginal change in a given variable, X_i, that appears in the hours-of-work, wage, and LFP equations is

$$\frac{\partial \mathrm{E}[H_i]}{\partial X_i} = \frac{\partial \mathrm{E}[H_i \mid \mathrm{LFP}_i = 1]}{\partial X_i} \cdot \Phi(Z_i\pi) + \mathrm{E}[H_i \mid \mathrm{LFP}_i = 1] \cdot \frac{\partial \Phi(Z_i\pi)}{\partial X_i}. \tag{26}$$

The derivatives for the four labor supply models are computed in Appendix D. The effect of a marginal change in X_i on the unconditional expectation of labor supply has two components: (1) the change in hours of work for those who continue to work and (2) the participation effect. The elasticity of the unconditional expectation of labor supply will be considered with respect to nonlabor income (NONLINC) and the presence of young children (NKIDLT6) and older children (NKID6T17). The elasticities are listed in table 4.

The relative values of these elasticities are quite different than the relative estimates of the standard elasticities for labor supply. This is seen by looking at the results for NONLINC. Note that, for the generalized Tobit-type model, the direct effect, −0.162, is similar to the standard elasticity of labor supply with respect to NONLINC, −0.117. However, the total effect for

Table 4
Elasticities of the Unconditional Expectation of Labor Supply

Variable	Tobit Type	Generalized Fixed Cost	Minimum Hours Constraint	Generalized Tobit Type
NONLINC:				
Total effect	−.347	−.264	−.144	−.274
	(.042)	(.025)	(.040)	(.029)
Direct effect	−.239	−.162	.006	−.170
	(.064)	(.019)	(.046)	(.006)
Participation effect	−.109	−.102	−.150	−.104
	(.027)	(.026)	(.040)	(.004)
NKIDLT6:				
Total effect	−.128	−.112	−.069	−.115
	(.015)	(.011)	(.019)	(.013)
Direct effect	−.088	−.066	.003	−.069
	(.024)	(.007)	(.022)	(.008)
Participation effect	−.040	−.046	−.072	−.045
	(.010)	(.012)	(.019)	(.013)
NKID6T17:				
Total effect	−.097	−.116	−.055	−.112
	(.012)	(.020)	(.015)	(.014)
Direct effect	−.067	−.067	.002	−.062
	(.018)	(.010)	(.018)	(.006)
Participation effect	−.031	−.049	−.057	−.050
	(.007)	(.012)	(.015)	(.014)
Average expected labor supply	1,242.9	1,157.6	1,150.7	1,156.5

NOTE.—These elasticities are obtained by first taking the average of the appropriate derivatives calculated for each individual in the sample and then multiplying by the average of the particular variable divided by the average expected labor supply. The latter step is taken rather than taking the average of the elasticities evaluated over individuals because individual elasticities with respect to NKIDLT6 and NKID6T17 would be zero when these variables are zero. Elasticities for NKIDLT6 and NKID6T17 are also calculated as a change in the unconditional expectation of labor supply with respect to an increase of one additional child. The results are similar. Standard errors are in parentheses.

the Tobit-type model, −0.347, is very close to the elasticity for observed labor supply, −0.322. Thus, one reason earlier studies, and this one, have found such large values for the standard elasticities for Heckman's model is that they are actually a combination of the direct effect on hours of work and the indirect participation effect.[11] When the overall effect is considered, there is not such a large difference between the elasticities for the generalized Tobit-type model and the Tobit-type model.

The unconditional elasticity of labor supply with respect to NONLINC for the generalized fixed-cost model is very close to the value for the generalized Tobit-type model. The elasticity for the minimum hours constraint model is much lower (in magnitude) than for the other three models.

[11] This is something that Cogan (1980, p. 352) notes: "The large elasticities reported in earlier studies . . . result primarily from variations in hours worked among women entering the labor force and not from variations in hours worked among working women."

The elasticities with respect to the presence of children are significant and very close in magnitude for all models but the minimum hours constraint model, which, again, is lower in magnitude. The smaller (in magnitude) elasticities for this latter model probably arise because a large proportion of the workers are at their minimum hours constraint and hence cannot reduce their hours when nonlabor income increases or when another child is present.

F. The Predictive Accuracy of Labor Force Participation

Another means for comparing the participation decisions of the four labor supply models is to look at their labor force participation prediction accuracy. First, the probability of participation for each individual is calculated. If this value is greater than or equal to 0.5, it is predicted that individuals will work. Otherwise, it is predicted they will not work. These predicted values can be compared with the actual labor force status of each individual to calculate a rate of successful prediction. The generalized Tobit-type model has the highest success rate, correctly predicting the labor force status of 651 of the individuals out of the full sample of 830. This gives a success rate of 78.4%. The fixed-cost and minimum hours constraint models accurately predict the labor force participation of the women in the sample in 78.1% of the cases, while the Tobit-type model lags behind with a 77.5% success rate. Thus, the generalized Tobit-type model, the generalized fixed-cost model, and the minimum hours constraint model appear to be marginally better than the Tobit-type model at predicting labor force status. Cogan (1981) makes the same comparison between the fixed-cost model and the Tobit-type model but finds that the latter has a slightly higher prediction rate.

G. A Direct Comparison of the Four Labor Supply Models

One way of analyzing the participation and hours-of-work decisions is by directly comparing the four labor supply models using their log-likelihood values. These values and the number of parameters in each model are given in table 5. The generalized Tobit-type model has the highest

Table 5
Estimated Log-Likelihood Values

Model	Log-Likelihood Value	Number of Parameters
Generalized tobit type	−3,004.06	60
Generalized fixed cost	−3,011.30	51
Minimum hours constraint	−3,013.52	59
Tobit type	−3,095.47	33

estimated log-likelihood value, the generalized fixed-cost model and the minimum hours constraint model have very similar values, while the Tobit-type model has an estimated log-likelihood value that is well below the other three.

Zabel (1990) shows that the Tobit-type model is nested in the other three models when they contain a common set of regressors. Thus, one can use the LR test to compare the Tobit-type model with the other three models. But a problem arises because, by imposing the constraints on the generalized Tobit-type, fixed-cost, and minimum hours constraint models, their covariance matrices become nonsingular, implying that the restricted parameter vector is on the boundary of the parameter space. Thus, the standard distribution theory for the LR test does not hold.[12] Still, if one considers the values of the three statistics ($2 \cdot \mathrm{LR} = 182.82$, 168.34, and 163.9), it is clear that there is a significant difference between the Tobit-type model and the other three models.

It was mentioned in Section IIID that, if the Tobit-type model is correctly specified, the parameters in the reduced-form hours-of-work equation are proportional to the parameters in the LFP equation. When these restrictions are imposed on the generalized Tobit-type model, the resulting model has two more parameters than the Tobit-type model: σ_{13g} and σ_{23g}. This implies that there are 25 restrictions being imposed. When the restricted model is estimated, the estimation procedure comes close but does not converge because the parameter vector is close to the one for the Tobit-type model, which implies a nonsingular covariance matrix. The estimated log-likelihood value is very close to the value for the Tobit-type model. Thus, the hypothesis that the restrictions hold is clearly rejected. This implies that the generalized Tobit-type model is significantly better than the Tobit-type model.

Zabel (1990) also shows that the generalized fixed-cost model and the minimum hours constraint model are nested in the generalized Tobit-type model. But they are no longer nested in the generalized Tobit-type model when the models are estimated since the variance of the disturbance term in the LFP equation in the generalized Tobit-type model is set to one. Thus, it is not possible to use the standard likelihood ratio (LR) test for nested models to compare these three models.

Recently, Vuong (1989) has developed a model comparison procedure based on the LR test that can be used for nonnested models. The test results based on this version of the LR test indicate that the null hypothesis that any pair of the three models are equivalent cannot be rejected at the 5% significance level even when the difference in the number of parameters is taken into account.

[12] This point is made by Heckman (1980, p. 231, n. 15).

Thus, it appears that the generalized Tobit-type, fixed-cost, and minimum hours constraint models are significantly better than the Tobit-type model. But it is not possible to distinguish among these three models using the LR test. Thus, it is not clear if fixed costs of work or minimum hours constraints are the cause for the weakening of the link between the LFP and hours-of-work equations.

IV. Summary of Results and Conclusions

In this article, the relationship between the participation and supply-of-hours decisions was analyzed for four models of labor supply behavior. This was first done by looking at the models and the structure of the labor force participation equation. The models displayed varying numbers of restrictions between the parameters in the labor force participation and hours equations. The Tobit-type model includes the most restrictions, while there are no such restrictions in the generalized Tobit-type model.

The models were then estimated and compared using a sample data set of married women taken from the Panel Study of Income Dynamics. Initial analysis of the labor supply elasticities with respect to wages and nonlabor income indicated that the generalized model, the generalized fixed-cost model, and the minimum hours constraint model are very similar. These elasticities were found to be much larger for the Tobit-type model, which is a result found in a number of other studies.

Elasticities of the unconditional expectation of labor supply were also considered since these contain a direct effect and a participation effect. Elasticities were taken with respect to variables relating to nonlabor income and the presence of children. The values for the Tobit-type model were much closer to the corresponding values for the other three models than was the case for the standard labor supply elasticities. It appears that the standard labor supply elasticities for the Tobit-type model compound the direct effect and the participation effect, and this is why they tend to be so large in magnitude.

One way of characterizing the differences among the four models is with respect to the implied participation decision. An analysis of the predictive accuracy of the four models showed that the Tobit-type model was slightly less effective in predicting the labor force status of the sample individuals.

Finally, the LR test was used to compare the models directly. The Tobit-type model was found to be significantly worse than the other three models. Thus, it appears that the labor force participation and hours decisions are not as strongly tied as the Tobit-type model specifies. The fixed costs of work and minimum hours constraints imposed by firms are explanations for the weakening of this link. But the fixed-costs and minimum hours constraints models are quite similar, and hence it is not possible to determine which one is the primary cause of the severing of the strong relationship between labor force participation and hours of work. Thus, the generalized Tobit-type model, which includes these two models as special cases, is a useful model to estimate.

Appendix A

Construction of the Sample Data Set

The following are the selection rules used to create the sample of women from the PSID data set:

1. not from low-income sample;
2. married, white, aged 30–60, not in school or retired;
3. spouse present, aged 30–60;
4. no change in household status for head and wife from 1986 to 1987; and
5. spouse reported positive earnings for 1986.

Appendix B

Definitions of Variables

H = (number of hours usually worked per weak at 1986 job) × (number of weeks worked at 1986 job)
LNWAGE = log of (total labor income in 1986 divided by H)
NONLINC = total taxable earnings of household minus wife's total labor income in thousands of dollars
NKIDLT6 = number of kids less than 6 years old living at home
NKID6T17 = number of kids aged 6–17 living at home
EDUC = number of years of education
AGE = age in years
LKAW = 1 if respondent is limited in the kind or amount of work that she can perform, 0 otherwise
UNPL = unemployment rate in county of residence—September 1986
MATJ = 1 if more applicants than jobs in county of residence, 0 otherwise
LNUNSKLL = the log of the typical wage that an unskilled worker might receive in county of residence
NORTHEST = 1 if living in the Northeast, 0 otherwise
NORTHCEN = 1 if living in the north-central part of the country, 0 otherwise
SOUTH = 1 if living in the South, 0 otherwise
WEST = 1 if living in the West, 0 otherwise
SMSA = 1 if living in an SMSA, 0 otherwise
MEDUCHS = 1 if mother's highest degree is high school degree, 0 otherwise
MEDUCCOL = 1 if mother has college degree, 0 otherwise
FEDUCHS = 1 if father's highest degree is high school degree, 0 otherwise
FEDUCCOL = 1 if father has college degree, 0 otherwise

Appendix C

A Derivation of the Wage and Nonlabor Income Elasticities for the Minimum Hours Constraint Model

It is shown in Zabel (1990) that, for the minimum hours constraint model, the observed labor supply function is a weighted sum of the desired-hours-of-work function and the minimum hours supply function where the weights are the probability of being at the desired labor supply or minimum hours level given participation. The probability of participation, as predicted by the minimum hours constraint model, is 0.770. Of this 77%, 26.9% of the sample are predicted to be working their desired hours, while 50.1% are predicted to be working the minimum hours imposed by the firms. Thus conditional on working, these are 34.9% and 65.1%, respectively. A marginal change in either wages or nonlabor income will not only affect those who are working their desired hours but will also cause those workers on the margin to switch from working their desired hours to the minimum hours or vice versa. Thus, the derivative of observed labor supply with respect to the log of wages ($\ln W$) is

$$\frac{\partial \mathrm{LS}_i}{\partial \ln W_i} = p_1\beta_1 + [(\beta_1 \ln W_i + \beta_2 \mathrm{NI}_i + X_i\beta_3) - V_i\eta]\cdot\frac{\partial p_1}{\partial \ln W_i}, \quad \text{(C1)}$$

where

$$\begin{aligned}\frac{\partial p_1}{\partial \ln W_i} &= \frac{\partial P(\mathrm{LS}_i^* > \mathrm{LS}_i^{\min})/\partial \ln W_i}{P(\mathrm{LS}_i^* > \mathrm{LS}_i^{\min} - D)} \\ &= \frac{\partial \Phi(b_i)/\partial \ln W_i}{P(\mathrm{LS}_i^* > \mathrm{LS}_i^{\min} - D)} \\ &= \frac{\phi(b_i)}{P(\mathrm{LS}_i^* > \mathrm{LS}_i^{\min} - D)}\cdot\frac{\beta_1}{(\sigma_1^2 + \sigma_{3d}^2)^{1/2}}\end{aligned} \quad \text{(C2)}$$

and

$$b_i = \frac{\beta_1 \ln W_i + \beta_2 \mathrm{NI}_i + X_i\beta_3 - V_i\eta}{(\sigma_1^2 + \sigma_{3d}^2)^{1/2}}. \quad \text{(C3)}$$

Note that, since the wage and nonlabor income elasticities correspond to observed hours of work (and not expected hours of work, which is considered in Section IIIE), the probability of working, $P(H_i^* > H_i^{\min} - D)$, is held constant. Thus, the derivative in equation (C2) is the marginal change in the probability of working desired hours versus working the minimum required hours.

Appendix D

The Unconditional Expectation of Labor Supply and Derivatives for the Four Labor Supply Models

I. The Tobit-Type Model

The following term will be needed to calculate the unconditional expectation of labor supply for the Tobit-type model:

$$\begin{aligned} \mathrm{E}[\ln W_i | H_i > 0] &= Y_i\alpha + \mathrm{E}[u_{2i} | H_i > 0] \\ &= Y_i\alpha + \mathrm{E}\left[u_{2i} \middle| \frac{u_{1i} + \beta_1 u_{2i}}{\sigma} > -t_i \right] \\ &= Y_i\alpha + \frac{\sigma_{12} + \beta_1\sigma_2^2}{\sigma}\cdot\lambda(t_i), \end{aligned} \tag{D1}$$

where, in general,

$$\lambda(x_i) = \frac{\phi(x_i)}{\Phi(x_i)}, \tag{D2}$$

ϕ and Φ are the standard normal pdf and cdf, and t_i and σ are given in equations (4) and (5). Thus,

$$\begin{aligned} \mathrm{E}[H_i] &= \mathrm{E}[H_i | H_i > 0]\cdot P(H_i > 0) \\ &= \Bigg(\beta_1 \mathrm{E}[\ln W_i | H_i > 0] + \beta_2 \mathrm{NI}_i + X_i\beta_3 \\ &\quad + \mathrm{E}\left[u_{1i} \middle| \frac{u_{1i} + \beta_1 u_{2i}}{\sigma} > -t_i \right] \Bigg)\cdot \Phi(t_i) \\ &= [\beta_1 Y_i\alpha + \beta_2\mathrm{NI}_i + X_i\beta_3 + \sigma\cdot\lambda(t_i)]\cdot\Phi(t_i). \end{aligned} \tag{D3}$$

The change in the unconditional expectation of labor supply due to a marginal change in a given variable X_i that appears in the hours-of-work and wage equations is

$$\begin{aligned} \frac{\partial \mathrm{E}[H_i]}{\partial X_i} &= \frac{\partial \mathrm{E}[H_i | H_i > 0]}{\partial X_i}\cdot\Phi(t_i) + \mathrm{E}[H_i | H_i > 0]\cdot\frac{\partial\Phi(t_i)}{\partial X_i} \\ &= \{(\beta_x + \beta_1\alpha_x)\cdot[1 - \lambda(t_i)^2 - \lambda(t_i)\cdot t_i]\}\cdot\Phi(t_i) \\ &\quad + \mathrm{E}[H_i | H_i > 0]\cdot\phi(t_i)\cdot\frac{\beta_x + \beta_1\alpha_x}{\sigma} \\ &= (\beta_x + \beta_1\alpha_x)\cdot\Phi(t_i), \end{aligned} \tag{D4}$$

where β_x and α_x are the coefficients for X_i in the hours-of-work and wage equations.

II. The Generalized Fixed-Cost Model

The following term will be needed to calculate the unconditional expectation of labor supply for the generalized fixed-cost model:

$$\begin{aligned} E[\ln W_i | H_i^* > H_i^r] &= Y_i\alpha + E[u_{2i} | H_i^* > H_i^r] \\ &= Y_i\alpha + E\left[u_{2i} \middle| \frac{u_{1i} + \beta_1 u_{2i} - u_{3fi}}{\sigma_f} > -f_i\right] \\ &= Y_i\alpha + \frac{\sigma_{12} + \beta_1\sigma_2^2 - \sigma_{23f}}{\sigma_f} \cdot \lambda(f_i), \end{aligned} \tag{D5}$$

where f_i and σ_f are given in equations (11) and (12). Thus,

$$\begin{aligned} E[H_i] &= E[H_i | H_i^* > H_i^r] \cdot P(H_i^* > H_i^r) \\ &= \Bigg(\beta_1 E[\ln W_i | H_i^* > H_i^r] + \beta_2 \mathrm{NI}_i + X_i\beta_3 \\ &\qquad + E\left[u_{1i} \middle| \frac{u_{1i} + \beta_1 u_{2i} - u_{3fi}}{\sigma_f} > -f_i\right]\Bigg) \cdot \Phi(f_i) \\ &= \left[\beta_1 Y_i\alpha + \beta_2 \mathrm{NI}_i + X_i\beta_3 + \frac{\sigma_{f1}^2}{\sigma_f} \cdot \lambda(f_i)\right] \cdot \Phi(f_i), \end{aligned} \tag{D6}$$

where

$$\sigma_{f1}^2 = \sigma_1^2 + 2\beta_1\sigma_{12} + \beta_1^2\sigma_2^2 - \sigma_{13f} - \beta_1\sigma_{23f}. \tag{D7}$$

The change in the unconditional expectation of labor supply due to a marginal change in a given variable X_i that appears in the hours-of-work, wage, and reservation hours equations is

$$\begin{aligned} \frac{\partial E[H_i]}{\partial X_i} &= \frac{\partial E[H_i | H_i^* > \mathrm{H}_i^r]}{\partial X_i} \cdot \Phi(f_i) \\ &\quad + E[H_i | H_i^* > H_i^r] \cdot \frac{\partial \Phi(f_i)}{\partial X_i} \\ &= \Bigg\{\beta_x + \beta_1\alpha_x - \frac{\sigma_{f1}^2}{\sigma_f} \\ &\qquad \times \left[\frac{\beta_x + \beta_1\alpha_x - \gamma_x}{\sigma_f}\right] \cdot [\lambda(f_i)^2 + \lambda(f_i) \cdot f_i]\Bigg\} \cdot \Phi(f_i) \\ &\quad + E[H_i | H_i^* > H_i^r] \cdot \phi(f_i) \cdot \frac{\beta_x + \beta_1\alpha_x - \gamma_x}{\sigma_f} \end{aligned} \tag{D8}$$

$$= (\beta_x + \beta_1\alpha_x)\cdot\Phi(f_i) + \frac{\beta_x + \beta_1\alpha_x - \gamma_x}{\sigma_f}\cdot\phi(f_i) \times \left[\beta_1 Y_i\alpha + \beta_2 \mathrm{NI}_i + X_i\beta_3 - f_i\cdot\frac{\sigma_{f1}^2}{\sigma_f}\right],$$

where β_x, α_x, and γ_x are the coefficients for X_i in the hours-of-work, wage, and reservation hours equations, respectively.

III. The Minimum Hours Constraint Model

The following term will be needed to calculate the unconditional expectation of labor supply for the minimum hours constraint model:

$$\begin{aligned} \mathrm{E}[\ln W_i | H_i^* > H_i^{\min}] &= Y_i\alpha + \mathrm{E}[u_{2i} | H_i^* > H_i^{\min}] \\ &= Y_i\alpha + E\left[u_{2i} \middle| \frac{u_{1i} + \beta_1 u_{2i} - u_{3di}}{\sigma_d} > -d_i\right] \qquad \text{(D9)} \\ &= Y_i\alpha + \frac{\beta_1\sigma_2^2 + \sigma_{12} - \sigma_{23d}}{\sigma_d}\cdot\lambda(d_i), \end{aligned}$$

where d_i and σ_d are given in equations (17) and (18). Thus,

$$\begin{aligned} \mathrm{E}[H_i] &= \mathrm{E}[H_i | H_i^* > H_i^{\min}]\cdot P(H_i^* > H_i^{\min}) \\ &\quad + \mathrm{E}[H_i | H_i^{\min} - D < H_i^* < H_i^{\min}] \\ &\quad \times P(H_i^{\min} - D < H_i^* < H_i^{\min}) \\ &= \Bigg(\beta_1 \mathrm{E}[\ln W_i | H_i^* > H_i^{\min}] + \beta_2 \mathrm{NI}_i + X_i\beta_3 \\ &\quad + \mathrm{E}\left[u_{1i} \middle| \frac{u_{1i} + \beta_1 u_{2i} - u_{3di}}{\sigma_d} > -d_i\right]\Bigg)\cdot\Phi(d_i) \qquad \text{(D10)} \\ &\quad + \left(V_i\eta + E\left[u_{3di} \middle| -d_i > \frac{u_{1i} + \beta_1 u_{2i} - u_{3di}}{\sigma_d} > -(d_i + D_1)\right]\right) \\ &\quad \times P\left[-d_i > \frac{u_{1i} + \beta_1 u_{2i} - u_{3di}}{\sigma_d} > -(d_i + D_1)\right] \\ &= \left[\beta_1 Y_i\alpha + \beta_2 \mathrm{NI}_i + X_i\beta_3 + \frac{\sigma_{d1}^2}{\sigma_d}\cdot\lambda(d_i)\right]\cdot\Phi(d_i) \\ &\quad + \left\{V_i\eta + \frac{\sigma_{d3}^2}{\sigma_d}\cdot\left[\frac{\phi(d_i + D_1) - \phi(d_i)}{\Phi(d_i + D_1) - \Phi(d_i)}\right]\right\} \\ &\quad \times [\Phi(d_i + D_1) - \Phi(d_i)] \end{aligned}$$

where

$$\left.\begin{aligned} D_1 &= \frac{D}{\sigma_d}, \\ \sigma_{d1}^2 &= \sigma_1^2 + 2\beta_1\sigma_{12} + \beta_1^2\sigma_2^2 - \sigma_{13d} - \beta_1\sigma_{23d}, \\ \text{and} \qquad \sigma_{d3}^2 &= \sigma_{13d} + \beta_1\sigma_{23d} - \sigma_{3d}^2. \end{aligned}\right\} \tag{D11}$$

The change in the unconditional expectation of labor supply due to a marginal change in a given variable X_i that appears in the hours-of-work, wage, and minimum hours equations is

$$\begin{aligned} \frac{\partial \mathrm{E}[H_i]}{\partial X_i} &= \frac{\partial \mathrm{E}[H_i|H_i^* > H_i^{\min}]}{\partial X_i} \cdot \Phi(d_i) \\ &\quad + \mathrm{E}[H_i|H_i^* > H_i^{\min}] \cdot \frac{\partial \Phi(d_i)}{\partial X_i} \\ &\quad + \frac{\partial \mathrm{E}[H_i|H_i^{\min} - D < H_i^* < H_i^{\min}]}{\partial X_i} \\ &\quad \times [\Phi(d_i + D_1) - \Phi(d_i)] \\ &\quad + \mathrm{E}[H_i|H_i^{\min} - D < H_i^* < H_i^{\min}] \\ &\quad \times \left[\frac{\partial \Phi(d_i + D)}{\partial X_i} - \frac{\partial \Phi(d_i)}{\partial X_i}\right] \\ &= \left(\beta_x + \beta_1\alpha_x - \frac{\sigma_{d1}^2}{\sigma_d} \cdot \left[\frac{\beta_x + \beta_1\alpha_x - \eta_x}{\sigma_d}\right]\right) \\ &\quad \times [\lambda(d_i)^2 + \lambda(d_i) \cdot d_i] \cdot \Phi(d_i) \\ &\quad + \mathrm{E}[H_i|H_i^* > H_i^{\min}] \cdot \phi(d_i) \cdot \frac{\beta_x + \beta_1\alpha_x - \eta_x}{\sigma_d} \\ &\quad + \left\{\eta_x - \frac{\sigma_{d3}^2}{\sigma_d} \cdot \frac{\beta_x + \beta_1\alpha_x - \eta_x}{\sigma_d}\right. \\ &\qquad \times \left(\left[\frac{\phi(d_i + D_1) - \phi(d_i)}{\Phi(d_i + D_1) - \Phi(d_i)}\right]^2\right. \\ &\qquad\qquad \left.\left. + \frac{(d_i + D_1) \cdot \phi(d_i + D_1) - d_i \cdot \phi(d_i)}{\Phi(d_i + D_1) - \Phi(d_i)}\right)\right\} \\ &\quad \times [\Phi(d_i + D_1) - \Phi(d_i)] \\ &\quad + \mathrm{E}[H_i|H_i^{\min} - D < H_i^* < H_i^{\min}] \end{aligned} \tag{D12}$$

$$
\begin{aligned}
&\times \frac{\beta_x + \beta_1\alpha_x - \eta_x}{\sigma_d} \cdot [\phi(d_i + D_1) - \phi(d_i)] \\
&= (\beta_x + \beta_1\alpha_x) \cdot \Phi(d_i) + \eta_x \cdot [\Phi(d_i + D_1) - \Phi(d_i)] \\
&+ \frac{\beta_x + \beta_1\alpha_x - \eta_x}{\sigma_d} \\
&\times \left\{ \phi(d_i) \cdot \left(\beta_1 Y_i\alpha + \beta_2 \mathrm{NI}_i + X_i\beta_3 - d_i \cdot \frac{\sigma_{d1}^2}{\sigma_d} \right) \right. \\
&\quad + [\phi(d_i + D_1) - \phi(d_i)] \cdot \left(V_i\eta - d_i \cdot \frac{\sigma_{d3}^2}{\sigma_d} \right) \\
&\quad \left. - D_1 \cdot \phi(d_i + D_1) \cdot \frac{\sigma_{d3}}{\sigma_d} \right\},
\end{aligned}
$$

where β_x, α_x, and η_x are the coefficients for X_i in the hours-of-work, wage, and minimum hours constraint equations, respectively.

IV. The Generalized Tobit-Type Model

The following term will be needed to calculate the unconditional expectation of labor supply for the generalized Tobit-type model:

$$
\begin{aligned}
\mathrm{E}[\ln W_i | \mathrm{LFP}_i^* > 0] &= Y_i\alpha + E[u_{2i} | \mathrm{LFP}_i^* > 0] \\
&= Y_i\alpha + E[u_{2i} / u_{3gi} > -Z_i\pi] \qquad \text{(D13)} \\
&= Y_i\alpha + \sigma_{23g} \cdot \lambda(Z_i\pi).
\end{aligned}
$$

Thus,

$$
\begin{aligned}
\mathrm{E}[H_i] &= \mathrm{E}[H_i | \mathrm{LFP}_i^* > 0] \cdot P(\mathrm{LFP}_i^* > 0) \\
&= (\beta_1 \mathrm{E}[\ln W_i | u_{3gi} > -Z_i\pi] + \beta_2 \mathrm{NI}_i + X_i\beta_3 \\
&\quad + \mathrm{E}[u_{1i} | u_{3gi} > -Z_i\pi]) \cdot \Phi(Z_i\pi) \\
&= \{\beta_1 \cdot [Y_i\alpha + \sigma_{23g} \cdot \lambda(Z_i\pi)] + \beta_2 \mathrm{NI}_i + X_i\beta_3 \qquad \text{(D14)} \\
&\quad + \sigma_{13g} \cdot \lambda(Z_i\pi)\} \cdot \Phi(Z_i\pi) \\
&= [\beta_1 Y_i\alpha + \beta_2 \mathrm{NI}_i + X_i\beta_3 + (\sigma_{13g} + \beta_1\sigma_{23g}) \cdot \lambda(Z_i\pi)] \\
&\quad \times \Phi(Z_i\pi).
\end{aligned}
$$

The change in the unconditional expectation of labor supply due to a marginal change in a given variable, X_i, that appears in the hours-of-work, wage, and LFP equations is

$$\begin{aligned}\frac{\partial \mathrm{E}[H_i]}{\partial X_i} &= \frac{\partial \mathrm{E}[H_i|\mathrm{LFP}_i = 1]}{\partial X_i} \cdot \Phi(Z_i\pi) + \mathrm{E}[H_i|\mathrm{LFP}_i = 1] \\ &\quad \times \frac{\partial \Phi(Z_i\pi)}{\partial X_i} \\ &= \{\beta_x + \beta_1\alpha_x - (\sigma_{13g} + \beta_1\sigma_{23g}) \cdot \pi_x \\ &\quad \times [\lambda(Z_i\pi)^2 + \lambda(Z_i\pi) \cdot Z_i\pi]\} \cdot \Phi(Z_i\pi) \\ &\quad + \mathrm{E}[H_i|\mathrm{LFP}_i = 1] \cdot \phi(Z_i\pi) \cdot \pi_x \\ &= (\beta_x + \beta_1\alpha_x) \cdot \Phi(Z_i\pi) + \pi_x \cdot \phi(Z_i\pi) \\ &\quad \times [\beta_1 Y_i\alpha + \beta_2\mathrm{NI}_i + X_i\beta_3 - (\sigma_{13g} + \beta_1\sigma_{23g}) \cdot Z_i\pi],\end{aligned} \tag{D15}$$

where β_x, α_x, and π_x are the coefficients for X_i in the hours-of-work, wage, and LFP equations, respectively.

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