PAUTA P3 CONTROL 2 FI21b

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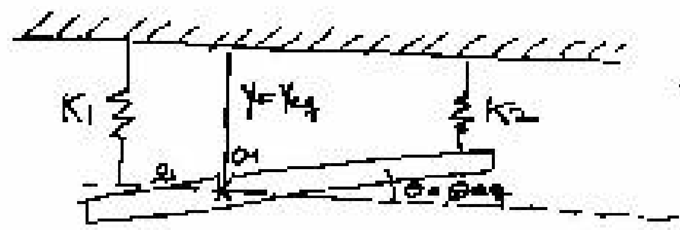
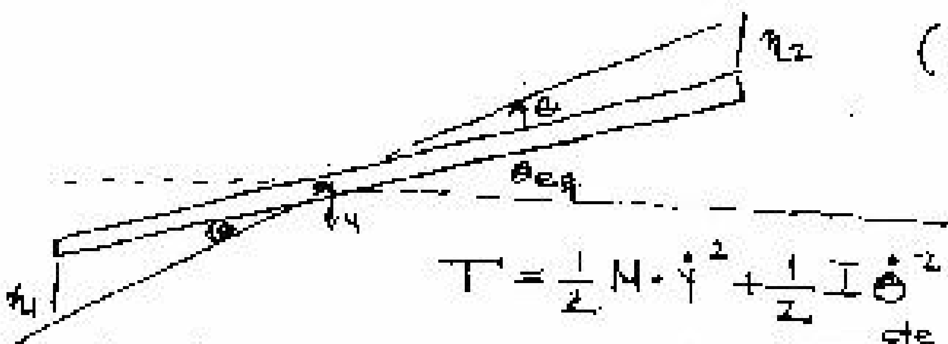


$$V = -mgy + k_1 (y + \ell_1 \operatorname{sen}\theta)^2 \cdot \frac{1}{2} + k_2 (y - \ell_2 \operatorname{sen}\theta)^2 \cdot \frac{1}{2}$$

(Supongo largos naturales iguales a cero)

Para pequeñas oscilaciones en torno a $y = y_{eq}$ y $\theta = \theta_{eq}$, redefino mis variables y y θ

(Ahora y y θ están medidos c/r a la posición de eq.)



$$T = \frac{1}{2} M \cdot \dot{y}^2 + \frac{1}{2} I \dot{\theta}^2$$

Serie de Taylor $\rightarrow$ $V(y_{eq}+y, \theta_{eq}+\theta) = \overbrace{V(y_{eq}, \theta_{eq})}^{cte} + \nabla V(y_{eq}, \theta_{eq}) \begin{pmatrix} y \\ \theta \end{pmatrix}$

$$+ \frac{1}{2} \begin{pmatrix} y & \theta \end{pmatrix} \underbrace{\begin{pmatrix} \frac{\partial^2 V}{\partial y^2} & \frac{\partial^2 V}{\partial \theta \partial y} \\ \frac{\partial^2 V}{\partial y \partial \theta} & \frac{\partial^2 V}{\partial \theta^2} \end{pmatrix}}_{H(V)}\Bigg|_{\substack{\theta = \theta_{eq} \\ y = y_{eq}}} \begin{pmatrix} y \\ \theta \end{pmatrix}$$

Cálculo de las derivadas parciales:

$$\frac{\partial^2 V}{\partial y^2} = \frac{\partial}{\partial y}\left(-mg + k_1(y + \ell_1 \operatorname{sen}\theta) + k_2 (y - \ell_2 \operatorname{sen}\theta)\right)$$

$$= k_1 + k_2$$

$$\frac{\partial^2 V}{\partial \theta \partial y} = \frac{\partial^2 V}{\partial y \partial \theta} = (k_1 \ell_1 - k_2 \ell_2) \cdot \cos\theta$$

$$\frac{\partial^2 V}{\partial \theta^2} = \frac{\partial}{\partial \theta}\left(k_1 (y + \ell_1 \operatorname{sen}\theta) \cdot \ell_1 \cos\theta + k_2 (y - \ell_2 \operatorname{sen}\theta)(-\ell_2) \cdot \cos\theta\right)$$

$$= \frac{\partial}{\partial \theta}\left((k_1 \ell_1 \cdot y - k_2 \ell_2 \cdot y)\cos\theta + (k_1 \ell_1^2 + k_2 \ell_2^2)\operatorname{sen}\theta \cdot \cos\theta\right)$$

$$= -(k_1 \ell_1 \cdot y - k_2 \ell_2 y)\operatorname{sen}\theta + (k_1 \ell_1^2 + k_2 \ell_2^2) \cdot \cos^2\theta$$