

• Condiciones iniciales:

$$\dot{\alpha} = 2 \left(\frac{Mg l}{3I_1} \right)^{1/2} \quad \beta = 60^\circ$$

$$\dot{\gamma} = (3I_1 - I_3) \left(\frac{Mg l}{3I_1 I_3^2} \right)^{1/2} \quad \dot{\beta} = 0$$

• Momentum generalizados:

$$P_\alpha = \frac{\partial L}{\partial \dot{\alpha}} = I_1 \dot{\alpha} \sin^2 \beta + I_3 (\dot{\alpha} \cos \beta + \dot{\gamma}) \cos \beta \quad (\star)$$

$\underbrace{\hspace{10em}}_{P_\gamma}$

$$P_\gamma = \frac{\partial L}{\partial \dot{\gamma}} = I_3 (\dot{\alpha} \cos \beta + \dot{\gamma})$$

$$\left(\text{pues } L = \frac{1}{2} I_1 (\dot{\alpha}^2 \sin^2 \beta + \dot{\beta}^2) + \frac{1}{2} I_3 (\dot{\alpha} \cos \beta + \dot{\gamma})^2 - Mg l \cos \beta \right)$$

Como $\frac{\partial L}{\partial \alpha} = \frac{\partial L}{\partial \gamma} = 0$, los momentum P_α y P_γ son constantes en el tiempo; evalúo en $t=0$:

$$P_\gamma = I_3 \left(2 \left(\frac{Mg l}{3I_1} \right)^{1/2} \cdot \cos 60^\circ + (3I_1 - I_3) \left(\frac{Mg l}{3I_1 I_3^2} \right)^{1/2} \right)$$

$$= (3I_1 Mg l)^{1/2}$$

$$P_\alpha = I_1 \cdot 2 \left(\frac{Mg l}{3I_1} \right)^{1/2} \cdot \sin^2 60^\circ + (3I_1 Mg l)^{1/2} \cdot \cos 60^\circ$$

$$= (3I_1 Mg l)^{1/2}$$

Ahora para calcular el potencial efectivo, escribo la energía cinética;

$$T = \frac{1}{2} I_1 (\dot{\alpha}^2 \sin^2 \beta + \dot{\beta}^2) + \frac{1}{2} I_3 (\dot{\alpha} \cos \beta + \dot{\gamma})^2 +$$

y la potencial;

$$V = Mg l \cos \beta$$

Por último, puedo escribir $\dot{\alpha} = \frac{P_\alpha - I_3 P_\gamma \cos \beta}{I_1 \sin^2 \beta}$ (de \star)

Con esto, la energía ($E = T + V$) nos queda:

$$E = \frac{1}{2} I_1 (\dot{\alpha}^2 \sin^2 \beta + \dot{\beta}^2) + \frac{1}{2} I_3 (\dot{\alpha} \cos \beta + \dot{\gamma})^2 + Mg l \cos \beta$$

$$= \frac{1}{2} I_1 \dot{\beta}^2 + \frac{1}{2} I_1 \left(\frac{P_\alpha - I_3 P_\gamma \cos \beta}{I_1 \sin^2 \beta} \right)^2 \sin^2 \beta + \frac{1}{2} \frac{P_\gamma^2}{I_3} + Mg l \cos \beta$$