FI21B Control 1 primavera 2005

P3]

(1) $T = \frac{1}{2} M \vec{V}_{CM}^{\,2} + \frac{1}{2} \vec{\omega}^{t} I \vec{\omega}$ 0,5

caso ejes principales $\Rightarrow$ $T = \frac{1}{2} M \vec{V}_{CM}^{\,2} + \frac{1}{2} I_1 w_1^2 + \frac{1}{2} I_2 w_2^2 + \frac{1}{2} I_3 w_3^2$ 0,5

(2) b) $\vec{\omega} = \dot{\phi}\, \hat{a} = \begin{pmatrix} \dot{\phi} \cos\alpha \\ \dot{\phi} \cos\beta \\ \dot{\phi} \cos\gamma \end{pmatrix}$ 0,5 $\quad |\vec{v}_{CM}| = L \dot{\phi}$ 0,5

$T = \frac{1}{2} M L^2 \dot{\phi}^2 + \frac{1}{2} I_1 \dot{\phi}^2 \cos^2\alpha + \frac{1}{2} I_2 \dot{\phi}^2 \cos^2\beta + \frac{1}{2} I_3 \dot{\phi}^2 \cos^2\gamma$ 0,5

$V = -MgL\cos\phi$ 0,5

$\Rightarrow L = \frac{1}{2}\left(ML^2 + I_1\cos^2\alpha + I_2\cos^2\beta + I_3\cos^3\gamma\right)\dot{\phi}^2 + MgL\cos\phi$ 0,5

a) $I_1 = I_2 \equiv I$

$\gamma = \pi/2 \Rightarrow \alpha + \beta = \pi/2$ 0,5

$\Rightarrow L = \frac{1}{2}\left[ML^2 + I\,\underbrace{(\cos^2\alpha + \cos^2\beta)}_{1}\right]\dot{\phi}^2 + MgL\cos\phi$

$\Rightarrow L = \frac{1}{2}\left(ML^2 + I\right)\dot{\phi}^2 + MgL\cos\phi$ 0,5

(3) $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\phi}}\right) = \left(ML^2 + I_1\cos^2\alpha + I_2\cos^2\beta + I_3\cos^2\gamma\right)\ddot{\phi}$

1,0

$\frac{\partial L}{\partial \phi} = -MgL\sin\phi$

E-L $\Rightarrow$ $\left(ML^2 + I_1\cos^2\alpha + I_2\cos^2\beta + I_3\cos^2\gamma\right)\ddot{\phi} + MgL\,\underbrace{\sin\phi}_{\approx\phi} = 0$

$\Rightarrow \ddot{\phi} + \underbrace{\frac{MgL}{ML^2 + I_1\cos^2\alpha + I_2\cos^2\beta + I_3\cos^2\gamma}}_{= w^2}\phi = 0$ 1,0