

$$R = \frac{3mg}{k}$$

$$l_0 = \frac{R}{2}$$

$$L = 2R \cos \theta$$

$$\Delta l = 2R \cos \theta - \frac{R}{2}$$

$$\therefore \Delta l = \frac{R}{2}(4 \cos \theta - 1) \quad (1)$$

$$\begin{aligned} \Delta l &= 0 \text{ para } \\ 4 \cos \theta - 1 &= 0 \\ \cos \theta &= \frac{1}{4} \end{aligned}$$

$$\therefore V = \frac{1}{2} k (\Delta l)^2 - mg 2R \cos \theta \cdot \cos \theta$$

$$\therefore V = \frac{1}{2} k \frac{R^2}{4} (4 \cos \theta - 1)^2 - 2mgR \cos^2 \theta \quad (2)$$

$$V' = \frac{kR^2}{2 \cdot 4} \cdot 2(4 \cos \theta - 1)(-4 \sin \theta) + 4mgR \sin \theta \cos \theta$$

$$\therefore V' = \frac{kR^2}{4} [-16 \sin \theta \cos \theta + 4 \sin \theta] + 4mgR \sin \theta \cos \theta$$

$$= -4kR^2 \sin \theta \cos \theta + kR^2 \sin \theta + 4mgR \sin \theta \cos \theta$$

$$= -12mgR \sin \theta \cos \theta + 3mgR \sin \theta + 4mgR \sin \theta \cos \theta$$

$$\therefore V' = -8mgR \sin \theta \cos \theta + 3mgR \sin \theta$$

$$V' = -3mgR \sin \theta \left( \frac{8}{3} \cos \theta - 1 \right)$$

$$V' = 0 \Rightarrow \sin \theta = 0 \Rightarrow \theta_1 = 0 \quad \checkmark$$

$$\frac{8}{3} \cos \theta = 1 \Rightarrow \cos \theta_2 = \frac{3}{8} \Rightarrow \theta_2 = \arccos \frac{3}{8} = 68^\circ$$

C17C

$$V'' = -3mgR \cos \theta \left( \frac{8}{3} \cos \theta - 1 \right) - 3mgR \sin \theta \left( -\frac{8}{3} \sin \theta \right)$$

$$V'' = -3mgR \cos \theta \left( \frac{8}{3} \cos \theta - 1 \right) + 8mgR \sin^2 \theta$$

$$\theta_1 = 0 \Rightarrow V''(0) = -3mgR \left( \frac{8}{3} - 1 \right) = -5mgR < 0$$

$\Rightarrow \theta_1 = 0$  is unstable

$$\cos \theta_2 = \frac{3}{8} \Rightarrow \begin{array}{c} 8 \\ \text{ } \backslash \\ \theta_2 \text{ } \triangle \\ \text{ } / \\ 3 \end{array} \quad x = \sqrt{8^2 - 3^2} = \sqrt{55}$$

$$\Rightarrow \sin \theta_2 = \frac{\sqrt{55}}{8} \Rightarrow \sin^2 \theta_2 = \frac{55}{64}$$

$$\cos \theta_2 = \frac{3}{8} \Rightarrow V''(\theta_2) = -3mgR \cdot \frac{3}{8} \left( \frac{8}{3} \cdot \frac{3}{8} - 1 \right) + 8mgR \cdot \frac{55}{64}$$

$$\Rightarrow V''(\theta_2) = \frac{55}{8} mgR > 0 \Rightarrow \theta_2 \text{ is eq. stable}$$

Para  $\omega^2$  ; se necesita  $v^2$  :

$$\text{en coord. polares } (r, \phi) \Rightarrow v^2 = \dot{r}^2 + r^2 \dot{\phi}^2 \quad / r=R \Rightarrow \dot{r}=0$$

$$\Rightarrow v^2 = R^2 \dot{\phi}^2$$

$$\text{Pero } \phi = \pi - 2\theta \Rightarrow \dot{\phi} = -2\dot{\theta} \Rightarrow \dot{\phi}^2 = 4\dot{\theta}^2$$

$$\Rightarrow K = \frac{1}{2} m v^2 = \frac{1}{2} m R^2 \cdot 4\dot{\theta}^2 \Rightarrow \alpha = 4mR^2$$

$$\Rightarrow \omega^2 = \frac{V''(\theta_2)}{\alpha} = \frac{55g}{32R} \Rightarrow \boxed{T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{32R}{55g}}}$$

$$2. \quad a) \quad \vec{\omega} = \vec{\omega}_1 + \vec{\omega}_3 \\ = -4\hat{i} + 3\hat{j}$$

$$\therefore \boxed{\vec{\omega} = -4\hat{i} + 3\hat{j}}$$

← Sistema móvil

acompaña rotación de la barra

DCA y a la rotación de la plataforma

fornu

$$b) \quad \vec{v}_A = \vec{\omega}_3 \times \vec{r}_{DA} \\ = -4\hat{i} \times (0,9\hat{i} + 0,6\hat{j})$$

$$\therefore \boxed{\vec{v}_A = -2,4\hat{k}}$$

$$c) \quad \vec{v}_B = \vec{v}_A + \vec{v}_{rel} + \vec{\omega} \times \vec{r}_{AB}$$

$$\text{Pero, } \vec{v}_{rel} = \vec{\omega}_2 \times \vec{r}_{AB}$$

$$= 2\hat{k} \times (1,5\cos\theta\hat{i} + 1,5\sin\theta\hat{j})$$

$$\begin{array}{c} 5 \\ \triangle \\ 4 \end{array} : \quad \sin\theta = \frac{3}{5} \quad y \quad \cos\theta = \frac{4}{5}$$

$$\therefore \vec{v}_{rel} = 2\hat{k} \times (1,2\hat{i} + 0,9\hat{j})$$

$$\therefore \boxed{\vec{v}_{rel} = -1,8\hat{i} + 2,4\hat{j}}$$

$$\text{Además: } \vec{\omega} \times \vec{r}_{AB} = (-4\hat{i} + 3\hat{j}) \times (1,2\hat{i} + 0,9\hat{j}) \\ = -3,6\hat{k} - 3,6\hat{k}$$

$$\therefore \boxed{\vec{\omega} \times \vec{r}_{AB} = -7,2\hat{k}}$$

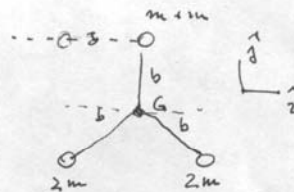
$$\text{Finalmente, : } \vec{v}_A = -2,4\hat{k} - 1,8\hat{i} + 2,4\hat{j} - 7,2\hat{k}$$

$$\therefore \boxed{\vec{v}_A = -1,8\hat{i} + 2,4\hat{j} - 9,6\hat{k}}$$

Velocidad del C. de M.

Antes del choque plástico:  $6m \vec{V}_G = m v_0 \hat{i}$

$$\therefore \boxed{\vec{V}_G = \frac{v_0}{6} \hat{i}} \quad (1) \quad 1.0$$

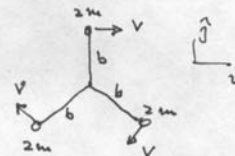


Las fuerzas que se desarrollan durante el impacto son fzras. internas. del sistema. No hay componentes de fzras. externas en el plano del movimiento. Por lo tanto,  $\vec{V}_G$  se mantiene inalterado y el C. de M. se mueve en movimiento rectilíneo uniforme.

Rotación del sistema:  $\vec{\tau}_G = \vec{0} \Rightarrow \frac{d\vec{L}_G}{dt} = \vec{0} \Rightarrow \vec{L}_G = \text{cte.} \quad 1.0$

$$\therefore L_G(t=0^-) = L_G(t=0^+)$$

$$\begin{aligned} \therefore m v_0 \cdot b &= 3 \cdot [(2m) V \cdot b] && \text{y como } V = b \omega \\ &= 3 \cdot [(2m) b \omega \cdot b] \end{aligned}$$



$$v_0 \cdot b = 6 b^2 \omega \Rightarrow \boxed{\omega = \frac{v_0}{6b}} \quad 2.0 \quad \vec{\omega} = \frac{v_0}{6b} (-\hat{k})$$

$\omega$  y cte.

$$\text{luego, de } \theta = \omega t \rightarrow \frac{2\pi}{3} = \frac{v_0}{6b} t^* \rightarrow t^* = \frac{4\pi b}{v_0} \quad 1.0$$

luego, la distancia  $d$  recorrida por G es

$$d = V_G t^* \rightarrow \boxed{d = \frac{2}{3} \pi b} \quad // \quad 1.0$$