

The settlement and bearing capacity of very large foundations on strong soils: 1996 R.M. Hardy keynote address

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Abstract: Strong soils are not typically problem soils, and hence their behaviour has not been extensively studied. Strong soils are best defined on the basis of their geologic history, but for this paper they can be roughly defined as cohesive soils with an N value of about 15 or over and cohesionless soils with N values over 30. Settlement of tall buildings on strong soils has always been of interest. The means of estimating settlement of the large foundations or pile foundations associated with these structures varies but is generally understood to be predominantly elastic. Although predictions of settlement based on laboratory tests or in situ tests may vary as much as an order of magnitude, there now exists a reasonable data base which suggests that large buildings will settle similar amounts regardless of the size or bearing pressure of the foundations or, for that matter, the type of foundations. No data base exists for quantifying the maximum bearing pressure that will be tolerated by large foundations without failure. The angle of internal friction is known to be critical and to decrease with increasing pressure. It is difficult to measure the undisturbed strength of strong soils, since undisturbed samples are very difficult to secure. Centrifuge model tests of large foundations of different shapes confirm that the bearing capacity factor N_γ decreases with increased size of footing, but the decrease of N_γ may not be accounted for entirely by the friction angle change with pressure. Selection of a friction angle to determine the peak capacity of very large foundations must be done very carefully and with a great deal of judgement, since it cannot be accurately measured.

Key words: settlement, bearing capacity, foundation behaviour.

Résumé : Les sols compétents ne posent en général pas de problèmes et leur comportement n'a donc pas été très étudié. La meilleure définition de ces sols se fait à partir de leur histoire géologique mais dans cet article on considère qu'un sol cohérent est compétent s'il a une valeur « N » d'environ 15 ou plus et qu'un sol pulvérulent est compétent pour des valeurs de N supérieures à 30. Le tassement d'immeubles de grande hauteur sur des sols compétents a toujours été un sujet d'intérêt. Les méthodes d'évaluation des déplacements des fondations de grande taille ou des fondations sur pieux associées à ces structures font généralement appel à l'élasticité. Bien que les prévisions de tassement basées sur des essais de laboratoire ou des essais en place puissent différer par un ordre de grandeur, il existe aujourd'hui un nombre raisonnable de données qui suggèrent que de grands immeubles tasseront du même montant quels que soient la taille de la fondation, la pression au sol ou encore le type de fondation. Il n'existe pas de données permettant de quantifier la pression portante maximale qui peut être tolérée sans rupture par de grandes fondations. On sait que l'angle de frottement interne est un facteur critique et que sa valeur décroît lorsque la pression augmente. Il est difficile de mesurer la résistance non remaniée des sols compétents car le prélèvement d'échantillons intacts est très malaisé. Des essais en centrifugeuse sur de grandes fondations de formes variées ont confirmé que le coefficient de capacité portante décroît lorsque la taille de la semelle augmente mais on ne peut pas complètement prendre en compte la diminution de N_γ seulement à partir du changement de l'angle de frottement avec la pression. Le choix d'une valeur de cet angle lors de la détermination de la capacité portante maximum des très grandes fondations doit être fait avec beaucoup de discernement car cet angle ne peut être mesuré avec précision.

Mots clés : tassement, capacité portante, comportement des fondations.

[Traduit par la rédaction]

Introduction

Conventional practice for the design of foundations for large structures is to select a design bearing capacity that will limit settlement to an amount that can be tolerated by the building without suffering structural distress. Terzaghi and Peck (1948) recommended a maximum allowable differential settlement of three-quarters of an inch (1 in = 25.4 mm) between adjacent

columns. MacDonald and Skempton (1955) observed that cracking of outside wall panels would occur when differential settlement exceeded a slope of about 1/300, and damage to frame and floors would be experienced at differential slopes exceeding 1/150. A more refined criteria emerged about 35 years ago when allowable rotational settlement according to the type of structure was adopted (Bjerrum 1963). Displacement criteria in common use were presented in the first draft of the Canadian manual on foundation engineering in 1975 (National Research Council of Canada 1975), the forerunner of the *Canadian foundation engineering manual* (Canadian Geotechnical Society 1992). This reflected a trend to a limit-state design by foundation engineers which is now widely

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adopted in civil engineering practice. Estimation of rotational settlement requires a settlement analysis. A settlement analysis for foundations on soft soils will typically be based on consolidation theory. Conventional settlement analyses take into account consolidation and elastic deformation, depending upon the properties of the soil.

For strong soils, elastic settlement will be predominant and consolidation tests are often carried out to determine a compression index. Elastic parameters may also be determined by calibrated field tests or in laboratory triaxial tests, and a settlement analysis will be carried out. For both soft soils and strong soils, if the analysis indicates settlements that could exceed the limiting values accepted for design, then the foundation is changed. Individual footings may be increased in size or combined into a raft or often the loads will be distributed to a greater depth by piles.

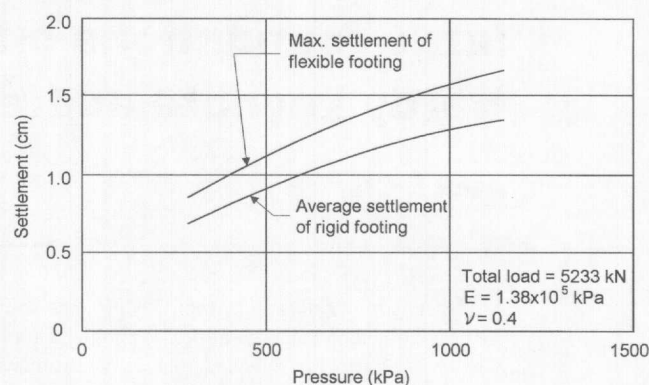
The practice is well founded for soft soils or loose soils but less so for strong soils. The same theories are used to calculate settlements of foundations on strong soils as are used for soft soils. Only the numbers change. The numbers typically come from laboratory tests or penetration tests in the field. Both laboratory tests and field tests are difficult to carry out for strong soils.

Foundations on strong soils have been neglected in soil mechanics and foundation engineering research. Thousands of articles and hundreds of textbooks have been written on the behaviour of clays and sands which are not overconsolidated, but very little attention has been given to strong soils. Strong soils are typically those which have been glaciated. Soft rock also behaves as a strong soil. Dense sands and gravels are also considered strong soils. For the cohesive soils to be considered as a strong soil they must have been deposited or overridden by a glacier and would typically have a standard penetration resistance N of at least 15 blows for 0.3 m of penetration or an undrained strength of about 100 kPa. These soil types cover much of Canada and indeed much of the world. In many places they occur to a great depth or may overlie soft rock which is not much better or may even be worse in terms of foundation support. Cohesionless soils require an N value of at least 30 to be in the strong soil category.

The determination of ultimate bearing capacity is most often based on conventional bearing capacity theory. The bearing capacity factors are derived from assumed friction angles or from tests made on "relatively undisturbed" samples in the laboratory. Deformation analyses usually assume that settlements will be predominantly elastic and will occur mostly during construction. Sometimes the settlement analysis is based on conventional laboratory consolidation tests. Where there are historical performance data, parameters are derived from back analysis and the settlement analysis of large buildings can be accurately made. Finite element programs can be created to account for increasing modulus with depth, various structural configurations, and overburden.

Clark et al. (1980) showed that for a constant elastic modulus in an elastic half-space elastic settlement was relatively insensitive to the bearing pressure for a given load. This analysis is reproduced as Fig. 1 for flexible and rigid foundations. It can be seen that if the bearing pressure for a given load is increased by a factor of 4, the settlement increases by a factor of 2. The analysis is conservative, since it assumes a constant elastic modulus for the full range of loads considered. In

Fig. 1. Settlement vs. pressure for square footings (modified after Clark et al. 1980).



reality, each increment of load will result in an increase in elastic modulus, which will be greater for a small footing with a higher bearing pressure and less for a larger footing carrying the same total load. Hence, the difference in settlement will be much less than shown for larger loads and the difference between flexible and rigid footings will disappear at large loads.

The increase in elastic modulus with each increment of load has been shown for a large ring foundation on a soil that is predominantly clay shale. Increments of load were measured with great accuracy (Clark and Robinson 1972). The load-settlement relationship shows the "elastic" modulus increasing by a factor of almost 10 from the load increment at 0.45×10^4 kN to the load increment at 3.2×10^4 kN (Fig. 2). An increasing modulus with depth was also shown very clearly by seismic tests at the same site (Fig. 3). These tests also indicated the anisotropy of the soil.

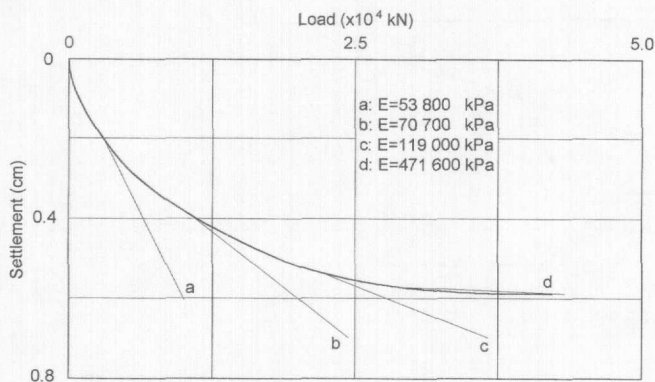
The seismic data and the observational data indicate that for each doubling of the load the elastic modulus increases by a factor of about 1.5 to 2 times. The effect of this feature can be demonstrated by a simple analysis of two footings, one four times larger in area than the other but each having the same total load. Figure 4 shows the results. For convenience, one footing is assumed to be 1.54 m square and the second 3.1 m square. A value of 42 MPa is taken for the initial elastic modulus, E_0 . For the first increment of load considered, from 0 to 0.11 MN, the small footing settles about twice that of the large footing, as one would expect. But as the load increases the amount of the settlement converges because of the greater increase of the deformation modulus E beneath the small footing. Above a load of 0.56 MN the large footing will settle more than the small footing. This is entirely consistent with field observations, as will be seen later.

Figure 4 shows the relationship of E versus P and q versus P used for this analysis, where E is the deformation modulus, q is the bearing pressure, s is the settlement, and P is the total load. The simple settlement equation (Harr 1966) is used, where B is the footing width, ν is Poisson's ratio, and α is a shape factor.

For this simple comparison the depth of soil below the footing is assumed to be infinite, the shape factor is equal to 1, and the elastic modulus increases with increasing stress as shown in Fig. 4.

Although tolerable settlement usually dictates design, as structures have become larger and larger and subjected to

Fig. 2. Load-settlement record (modified after Clark and Robinson 1972).



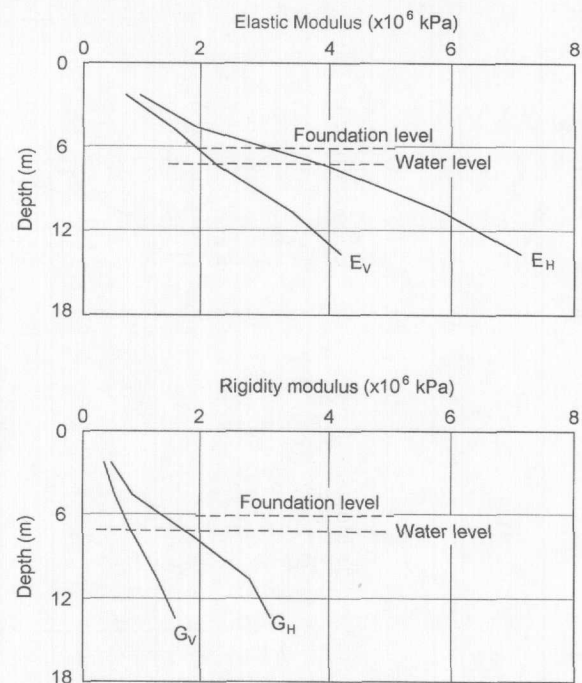
higher and higher loads, particularly offshore structures, the foundation bearing capacity of such structures, even though on very strong soils, has become an issue that requires detailed attention. Very large inclined environmental loads must be accommodated, particularly where the structures must resist ice loads. For example, such is the case for the Hibernia structure and the structures supporting the Confederation Bridge. The design of many of the gravity-based platforms in the North Sea, even though they are on very strong soils, has had to consider bearing capacity as a very important factor. In the case of the Confederation Bridge, which connects Prince Edward Island to the mainland, very wide spans were designed to produce what may be the most economical and elegant bridge of its length and type ever to have been built. The bearing capacity of the pier foundations was a major consideration, even though founded on very strong strata, in terms of soil consistency.

There are no case histories of bearing capacity failures of large structures on strong soils, so it is not possible to derive strength parameters from back analysis. The soil properties in place become very important but are extremely difficult to measure. Tests that have been made for bearing capacity usually are relatively small scale at 1g and hence may not replicate full-scale behaviour. To a large extent this problem can be overcome by centrifuge modelling. Small-scale models of large prototype foundations can be constructed and tested in the correct gravitational field to the bearing capacity failure load to give an indication of the validity of the theoretical and analytical methods used for design of these foundations. The bearing capacity factors selected for design are very sensitive to the angle of internal friction, which is usually derived from laboratory tests or estimated. The factors should be the same for centrifuge models as for full-scale prototype design.

Sampling and testing of strong soils

It is impossible to secure an undisturbed sample of a strong soil. This is obvious for gravels and dense sands but it is also the case for strong cohesive soils and soft rocks. The concept that a tube can be pushed into the soil at the bottom of a hole, withdrawn to the surface, examined, sealed and transported back to the laboratory, extruded from the tube with a very strong thrust, typically from a hydraulic ram, trimmed and enclosed in a rubber membrane in a triaxial test sample or a

Fig. 3. Elastic moduli from seismic data (modified after Clark and Robinson 1972). Subscripts: H, horizontal; V, Vertical.

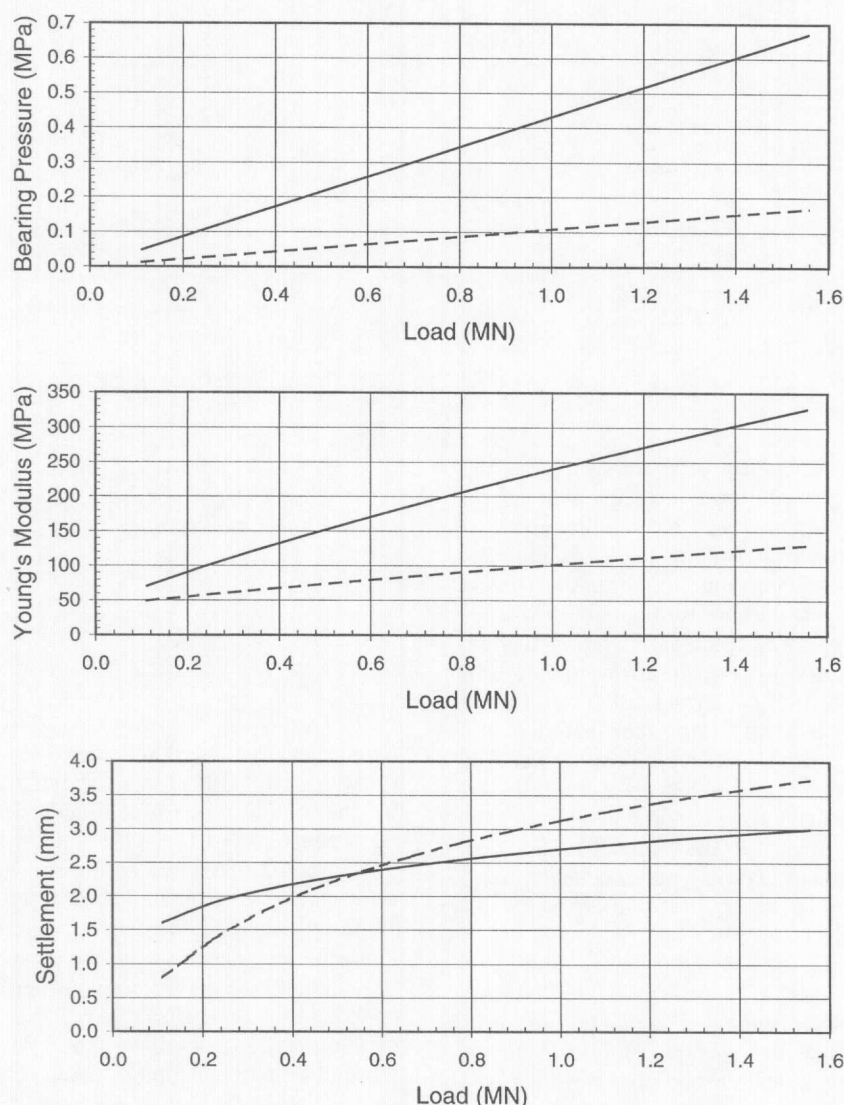


simple shear device, and then subjected to a consolidation pressure that will restore the integrity of the sample to the in situ condition is a pipe dream. Samples of soils that have been consolidated typically under a load of several hundreds to thousands of metres of ice and held under that stress for thousands of years cannot be restored to their in situ condition.

Sample disturbance cannot be prevented. In situ tests, particularly the piezocone, are the most reliable means of assessing strength and deformation properties. Even these tests may be exceptionally difficult in heterogeneous soils. The drilling of a hole will usually disturb the soil beneath the bottom of the boring before the sample is taken. The disturbed zone may be deeper than the length of the tube sample that is subsequently pushed into it.

The effect of soil disturbance below a bore hole was demonstrated at an offshore site where a layer of very strong soil was encountered with remarkably uniform cone resistance. Piezometric cone tests were made ahead of the boring as the hole was advanced. The disturbed soil was then drilled out and a tube sample was taken below the boring, a common procedure. The process was repeated to obtain cone penetration resistance and secure relatively undisturbed samples for the full depth of the layer. Figure 5 shows a section of the test hole log and some of the cone test results in this relatively uniform deposit with constant cone resistance. It can be seen that as the cone push starts at the bottom of the hole, the resistance gradually builds up for a distance of about 0.3 m and then becomes relatively constant. All cone tests showed essentially the same configuration and resistance. An example of the test results is enlarged in Fig. 5. The curved portion of the top of the cone tip resistance, or what can be referred to as the cobra effect, would not be there to that depth of about 10 cone diameters if the soil had not been disturbed by the drilling process. When a tube sampler is pushed through that 0.3 m depth below the

Fig. 4. Comparison of the elastic settlement of a small footing (1.5×1.5 m; solid lines) with a large footing (3.0×3.0 m broken lines), with each carrying the same load.



boring, the sample that is secured is already disturbed before the tube is pushed.

Retrieving the tube to the surface will also obviously affect the sample. There is a stress release when the sample is removed, constrained somewhat by the sampling tube. If a wet drilling procedure is used or if the sample is being taken from an offshore test boring, it will pick up water on the trip to the surface because of the stress release and resulting negative pore pressure. This is a factor that is often neglected and hence moisture-content tests that are taken at the surface, even if they are taken immediately after the sample is retrieved, are often not representative and may be much higher than the in situ condition. This can be demonstrated by moisture-content tests that were carried out on another offshore layer of material which was not uniform and was extremely difficult to sample or to test with cones. The samples that were secured were mostly short but nevertheless moisture contents were run on portions taken from the sample tubes. Although it is not known exactly where the moisture-content samples were taken, one

can assume that the engineers or technicians would take it from as close to the middle of each tube as possible to avoid picking up the disturbed material or contaminated material at the ends of the tube. But with very short samples there may be no choice and all of it may be required for the moisture-content test. Figure 6 shows an unusual relationship between moisture content and sample length for this particular layer. There is no doubt that there is a relationship between water content and sample recovery length.

On another project where exceptionally good core samples and very detailed core logs were recorded in a soft mudstone, the exact location of the moisture-content sample from each core was carefully recorded. The trend of increased moisture content with increasing distance from the bottom of the core is opposite to that of tube samples, for a very good reason. The greater the length of the core, the longer the core at the top is exposed to the drilling fluid or water. In a tube sample only the ends are exposed to water. Figure 7 shows the trend of increased moisture with increasing distance from the bottom

Fig. 5. Results of cone penetration tests in a soil of uniform cone resistance.

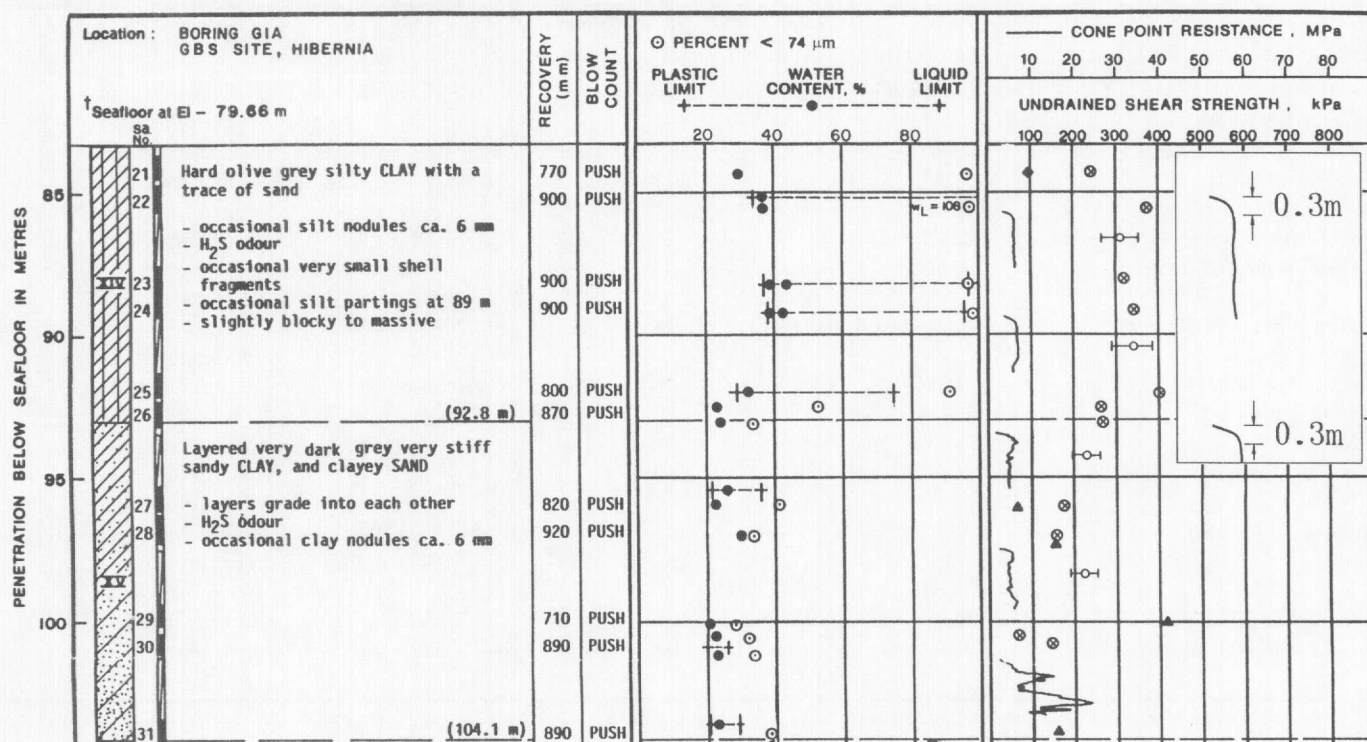
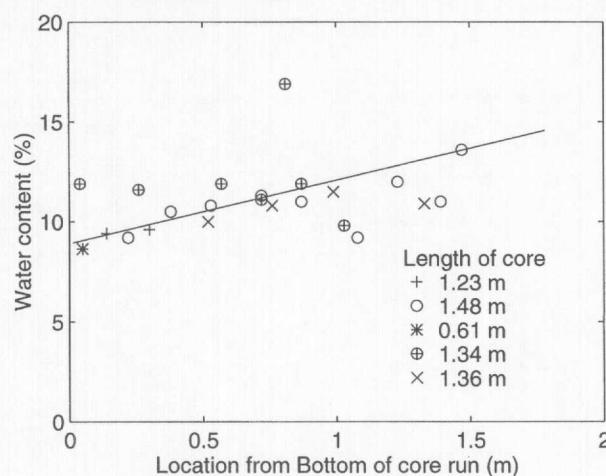


Fig. 6. Water content vs. sample length for a strong silty clay.

Fig. 7. Water content vs. location bottom of core run for a mudstone, Confederation bridge.



of the core. Moreover, for core samples, the moisture content was highest at the outer skin of the core. Figure 8 shows the result of moisture-content tests taken at different distances from the bottom of a core sample as well as the moisture variation from the outside of the core to the middle. These relationships indicate the level of disturbance and contamination of cores of very strong soils as they are brought to the surface.

The interpretation of laboratory tests on samples for strength and consolidation properties requires a great deal of judgement no matter how carefully the tests are carried out. Sample disturbances, such as an increase in moisture content, will result in the strength being underestimated and potential consolidation and settlement being overestimated. The judge-

ment required to select the appropriate design parameters is best conditioned by review of relevant case histories.

Settlement behaviour of foundations on strong soils

During the 1970s and 1980s a number of buildings in Canada that were founded on strong soils were instrumented to measure settlement. This was often at the cost of the consultant who did the instrumentation to check predictions from settlement analyses.

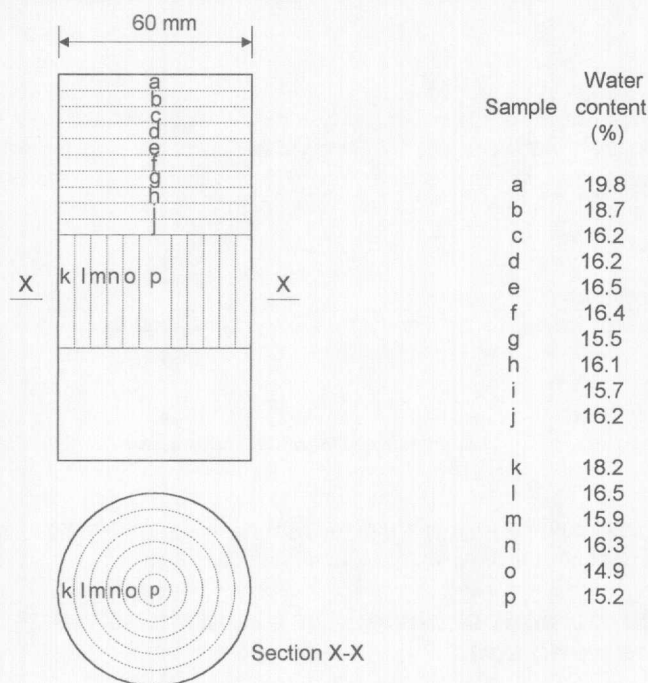
The instrumentation work carried out proved to be very useful and provided insight into the settlement behaviour of

Table 1. Settlement of buildings.

Name and location of building	Foundation type	Below grade (m)	No. of storeys	Settlement (mm)	Soil type	Reference
Social Science, University of Calgary, Calgary	Cast-in-place pile	2	13	33	Clay till	AGRA Earth and Environmental ^b
Science 3B, University of Calgary, Calgary	Pile	2	3	11	Clay till	Agra Earth and Environmental ^b
CN Tower, Edmonton	Raft	6.7–7.9	26	30	Clay till	DeJong and Harris 1971
Avord Arms, Edmonton	Expanded pile	—	27	32	Clay till	DeJong and Harris 1971
A.G.T. Tower, Edmonton	Raft	13.7	34	48	Sand–clay till	DeJong and Morgenstern 1973
Oxford Building, Edmonton	Raft	7.3–13.7	28	28	Sand–clay till	DeJong and Morgenstern 1973
Omni International Hotel, Miami	Raft	—	26	30	Overconsolidated soils	Leary and Langan 1982
Sun Oil Building, Calgary	Spread footing – mat	6.1	32	25	Gravel, silt, and till	Toovey 1983
Scotia Centre, Calgary	Spread footing – mat	7.3	40	49	Silt till	Toovey 1983
First Canadian Centre, West Tower, Calgary	Raft	—	42	53	Till	AGRA Earth and Environmental ^b
Building A, Toronto	Raft	4.9	43	48	Clay–silt till	Trow and Bradstock 1972
Bankers Hall, Calgary	Raft	18	47	55	Clay–silt till	Agra Earth and Environmental ^b
Calgary Tower, Calgary ^a	Ring	7	8	8	Shale	Clark and Robinson 1972
Canada Malting Building, Calgary ^a	Raft	6	44	33	Till–shale	AGRA Earth and Environmental ^b

^a Corrected to equivalent storeys from load on foundation.

^b Data from files of AGRA Earth and Environmental (formerly Hardy Associates).

Fig. 8. Variation of moisture content from bottom of a core run and from outside skin to the middle.

very large buildings. In a few cases the observations were carried on well beyond the end of construction.

Table 1 presents a summary of settlement observations at the end of the construction period for 14 buildings on strong soils. Most of these are in Alberta, but other settlement records

are also available from Toronto, Miami, Houston, and New York. Most of the buildings are on mat foundations or rafts, but some also are on drilled pile foundations or dynamically cast-in-place concrete piles. Most of the case histories in Table 1 are office buildings, but two other structures for which very good total load and bearing pressure data are available have been converted for comparison by computing the number of storeys that would correspond to the foundation load. These were the Calgary Tower and the Canada Malting Building (Toovey 1983).

By far the most comprehensive program of settlement monitoring ever carried out and reported in the literature is that by Mueser Rutledge on behalf of the New York City Housing Authority (NYCHA) (Gould and Parsons 1975; Parsons 1976). It is reported that the consultants felt that expensive pile foundations were being used where they might not be needed. Over 70 buildings were instrumented, of which 43 are reported in the literature. The foundation systems included tapered piles, mat foundations, and spread footings. The building heights ranged from about 7 to 25 storeys. The soil profile typically consists of a medium dense sand overlying an overconsolidated varved clay. The thickness of the sand layer and the varved clay varies but there was no apparent correlation with the settlement behaviour of the buildings. Identical buildings with the same foundation showed similar settlement patterns, but occasional anomalies were recorded. These have been included in calculating the average settlement. The paper by Gould and Parsons (1975) gives an excellent analysis of differential settlement and long-term settlement.

It can be seen from the table that settlements are not large and are quite similar for buildings of the same approximate height. The data from the NYCHA settlement measurement

have been plotted on a semi-logarithm plot in Fig. 9 along with the data from Table 1. The average settlement for each type of foundation for the same building height has been calculated and the number of buildings monitored is also shown in Fig. 9. The averages shown are for the end of construction only to be consistent with the other buildings. Typically the end of construction settlement was about 80% of the final total settlement recorded which is about the same as for those buildings in Table 1 where longer term records were available. The paper by Parsons (1976) presents detailed information on the soil profile and soil properties and settlement calculations compared with actual settlements.

Two references of settlement were found by the author in the collection of Terzaghi records on piles that are kept in the Terzaghi Library of the Norwegian Geotechnical Institute. Both were for buildings in Houston, Texas. One of 26 stories settled "just over an inch" which is taken as 28 mm, and the second, a 30 storey office building, settled 37 mm. Terzaghi referenced these buildings as satisfactory examples where footings or a mat was used instead of the common practice of using piles.

The data in Fig. 9 reflect the maximum settlement recorded at the end of construction for each building. It can be seen that if one were to estimate 1 mm per storey of construction plus 10%, the prediction for all of the buildings would be relatively close. Only a few cases shown in Table 1 include settlement records after construction, and they indicate fairly consistently that about 80% of settlement occurs during construction; after about 2 years, settlement is almost negligible. Hence a 50 mm settlement at the end of construction would indicate an eventual total settlement of about 65 mm. The rigidity of the building would obviously have an influence on the amount of differential settlement, but the maximum settlement typically occurs at the centre of the building.

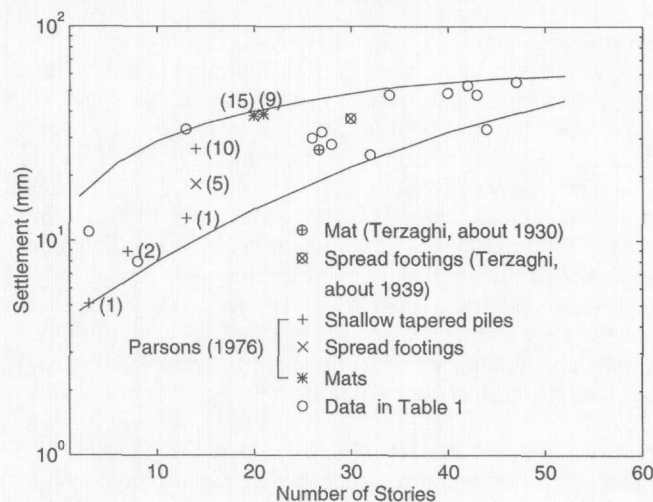
These data indicate that for strong soils, where the settlement is predominantly elastic, it is also relatively insensitive to foundation type. The settlements are about the same, whether the building is on footings, a raft, drilled piles, or dynamically cast-in-place concrete piles. This suggests that as loads increase, the elastic response for different foundations and soils converges. Lightly loaded foundations show variations in settlement, but as the load increases (i.e., as the number of storeys increase) the differences between foundation types are much less. This is not to suggest that detailed settlement analysis should not be carried out, but if the results are very different from the curves shown, the input parameters should be checked. Figure 9 suggests that analyses for settlement of foundations on strong soils require only the ability to count and multiply by small numbers.

Bearing capacity of large foundations

As pointed out previously, the bearing capacity of large foundations has not usually been a major consideration in the design. The design has usually focused on potential deformation. Even in soft soils or loose sands where piles are often used instead of spread footings or mat foundations they are intended to limit settlement rather than to increase the ultimate bearing capacity.

It has been known for many years that an increase in footing size will result in a decrease in unit bearing capacity and will

Fig. 9. Settlement of buildings at end of construction. The values in parentheses indicate the number of buildings averaged (Terzaghi case histories are unpublished notes from Terzaghi Library, Norwegian Geotechnical Institute, Norway).



change the failure mode of the soil (De Beer 1965a). The increase in size substantially increases the total load carrying capacity, but not in direct proportion to the width or diameter of the foundation as traditional bearing capacity theory suggests. Unlike settlement, there are no full-scale case records of ultimate bearing capacity for large foundations on strong soils to test theoretical analyses against actual performance.

Centrifuge modelling has been used to investigate the bearing capacity of large prototype square and circular foundations. The centrifuge tests for this study were carried out on both dry sand and saturated sand. Model footings up to 7 m in prototype diameter were tested on the dry sand and up to 10 m diameter on the saturated sand. The C-CORE centrifuge used for the tests is an Acutronics model 682 with a 5.5 m arm and a payload capacity of up to 1 t. It can be operated to a maximum of 200g.

Over the past two decades extensive use has been made of geotechnical centrifuge modelling to investigate the effect of footing size on bearing capacity, but the tests have usually been limited to prototype footings within the 3–4 m size. The bearing capacity of very large foundations, particularly those on strong soils, is still not well understood. This centrifuge program was designed to investigate the effect of footing size on the bearing capacity and failure mode for large foundations. The phenomenon of progressive failure and the decrease of soil friction angle with increasing stress level as well as the effects of shape and repeated loading were investigated.

Soil properties of the test bed

Dense silica sand was used for the tests, and the properties were investigated by means of a series of drained triaxial compression tests. The density index for the soils tested was 88%, the specific gravity 2.65, the mean grain size (d_{50}) 0.20 mm, and the uniformity coefficient (C_u) 1.69. The void ratio varied from a maximum value of 1.11 to a minimum value of 0.68.

Typical results of stress-strain and volume-strain from these tests are shown in Fig. 10 for the dense sand. Using the

curve for a confining pressure σ_3 of 25 kPa, point M corresponds to the peak while point C corresponds to the critical shear strength. The peak friction ϕ_{\max} and the critical state friction angle ϕ_{crs} can be derived from the stress parameters at M and C, respectively. Figure 10 shows the decrease in the deviator stress ratio $(\sigma_1 - \sigma_3)/\sigma_1$ with the increase in cell pressure, where σ_1 is the vertical stress. The strain required to reach the peak strength as well as the critical strength increases with stress level. It can also be seen that dilation of the sand during shear decreases significantly with confining pressure.

It has been widely recognized that the peak friction angle of soils decreases with stress level (Meyerhof 1950; De Beer 1965a; Bolton 1986). The Mohr failure envelope is a curve rather than a straight line. In contrast, the stress dependency of the critical state friction angle has been less widely recognized and there are only a few reports of this fact (Chu 1995). The triaxial tests conducted show that both the peak friction angle ϕ_{\max} and the critical state friction angle ϕ_{crs} decrease with the stress level shown in Fig. 11. When the confining pressure increases from 25 to 2500 kPa, ϕ_{\max} decreases from approximately 49 to 39° while ϕ_{crs} changes from approximately 40 to 34°. The difference between ϕ_{\max} and ϕ_{crs} ranges from about 5 to 9°, decreasing with stress.

Centrifuge modelling of footings

Since it is very difficult and costly to conduct in situ tests of foundations, small-scale footing tests have played an important part in studies of bearing capacity. It has been found, however, that the bearing capacity factor N_γ decreases with footing size, mainly as a result of the decrease of soil friction angle with stress level and the phenomenon of progressive failure. In recent years, centrifuge modelling has been widely adopted for the study of bearing capacity (Kimura et al. 1985; Pu and Ko 1988; Bakir and Garnier 1994). In centrifuge tests, reduced-scale footings are tested at high acceleration levels to simulate the behaviour of large-scale prototype foundations.

For this work centrifuge tests up to 160g were carried out on the silica sand described above using circular, square, and ring footings. Sand samples were prepared using a raining technique (Zhu et al. 1996) and had a density index (I_D) of 88%. The samples were dry or saturated with water.

Failure modes of soil

There are three failure modes of soil supporting foundations: general shear failure, local shear failure, and punching shear failure (Vesic 1973). In the case of general shear failure, there usually exists a continuous failure surface from one edge of the footing to the ground surface. The ultimate bearing capacity (q_u) is the peak load applied. In contrast, punching shear failure is characterized by a failure pattern that is not obvious. The foundation penetrates due to the compression of the soil immediately beneath it. The penetration increases as the loading is increased and there is no peak load. Local shear failure is a transitional mode between general failure and punching failure. There is a visible bulging of the soil adjacent to the foundation. The failure surfaces usually end in the soil.

The failure mode of a foundation depends upon the soil compressibility, foundation size, and depth. A strip footing on the surface of a very dense sand may fail in general shear. In

Fig. 10. Results of triaxial tests on sand used for centrifuge tests.

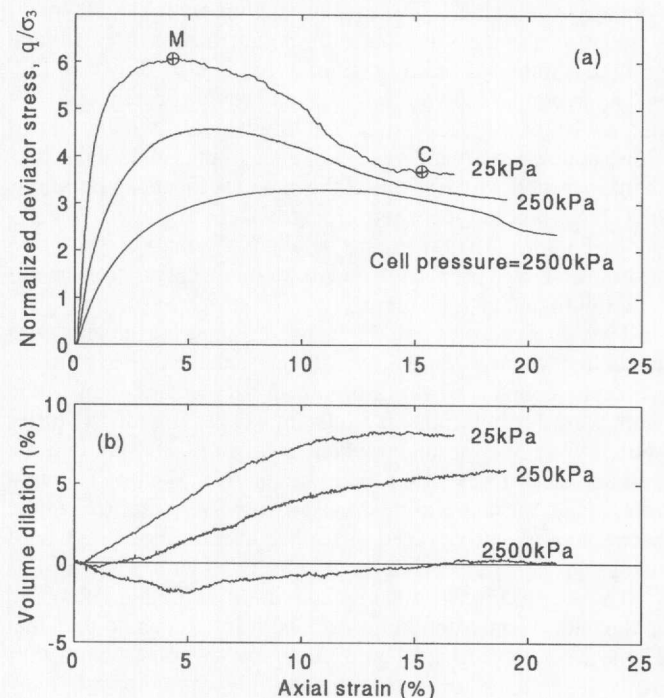
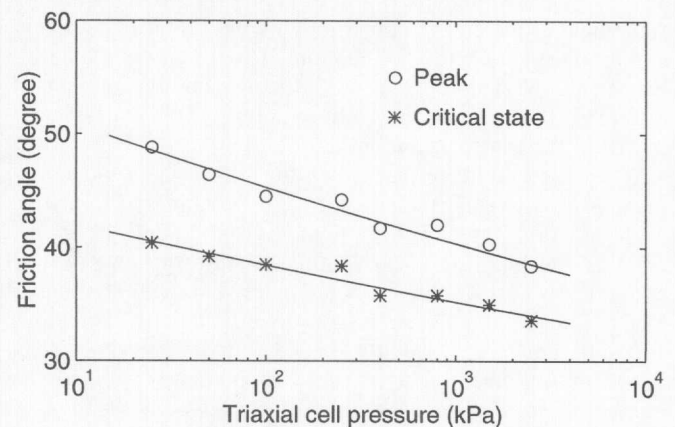


Fig. 11. Decrease of peak and critical friction angles with increasing cell pressure.



contrast, this footing on the surface of a loose sand will fail in punching shear. By increasing the dimension or the embedment depth of a footing, the failure mode tends to move from general shear failure to punching shear failure.

Figure 12 presents the relationship between load and settlement of circular footings on dry dense sand tested at different acceleration levels. The model footing 43.7 mm in diameter was tested at accelerations of 1, 10, 40, 100, and 160g. The corresponding diameters of the prototypes were 0.044, 0.437, 1.75, 4.37, and 6.99 m. In Fig. 12, s represents the settlement of the footings, and the loading ratio R_p is given by

$$[1] \quad R_p = \frac{p}{\gamma D}$$

in which p is the unit load applied, γ is the unit weight of soil, and D is the diameter of the circular footing.

It is obvious that the failure mode of the footing at 1g is general shear; the maximum load occurs at a relative settlement of about 6%. It was observed that the failure surface was extended to the surface of the soil. For the 10g test, there was not an obvious failure surface at the top of the soil. After reaching the peak value, the load decreased slightly with displacement and then increased with continuing settlement. The failure mode in this case is still general shear but it is approaching the local shear mode. For the tests at 40 and 160g, there are no peak loads; the soil heaved around the footings during loading and there were no failure lines observed on the surface of the soil samples. The failure mode was local shear failure.

Comparison of the footing test results in Fig. 12 and the data of triaxial tests of the same sand in Fig. 10 provides a further understanding of the failure mode. In the triaxial tests, the sand dilated significantly during shearing at a low stress level. At a high confining pressure of 2500 kPa, the dilation phenomenon disappeared. Both the relative compressibility of the sand and the strain at which the load reached the maximum increase with stress level. For small footings, since the stress level in the soil is low, the movement and heave of soil around the footing are caused by both the dilation of soil during shearing and the expulsion of soil due to the penetration of the footing.

Because of the high dilatancy and the low strain at failure of the dense sand, the strength mobilization at points A, B, and C in Fig. 13 along the failure surface of a small footing is more uniform with a general failure mode. With the increase of footing size, the stress level in the soil increases. The dilation of the soil becomes smaller and the strain at failure becomes higher. To mobilize the strength along the failure surface in the soil, larger displacement of the footing is required and the failure mode becomes more localized. It should be emphasized that the volume change of the soil under the footings is also an important factor affecting the failure mode.

Basic equations for bearing capacity

The well-known Terzaghi equation, accepted widely as a basic formula for the bearing capacity of strip foundations, is given by

$$[2] \quad q_u = cN_c + qN_q + 0.5\gamma BN_\gamma$$

where q_u is the ultimate bearing capacity; c is the soil cohesion; q is the overburden pressure; γ is the soil unit weight; B is the width of foundations; and N_c , N_q , and N_γ are the bearing capacity factors. In the literature there is a variety of bearing capacity theories. While the bearing capacity factors N_c and N_q proposed by Prandtl (1921) and Reissner (1924) are widely accepted, the variation in N_γ is substantial (Terzaghi 1943; Caquot and Kerisel 1953; Meyerhof 1963; Brinch-Hansen 1970). For strip foundations resting on the surface of cohesionless soil, [2] becomes

$$[3] \quad q_u = 0.5\gamma BN_\gamma$$

and for a circular foundation the bearing capacity is expressed by (Terzaghi 1943)

$$[4] \quad q_u = 0.3\gamma DN_\gamma$$

where D is the diameter of a circular foundation.

In using the above equations it is usually assumed that the

Fig. 12. Load settlement relation for circular footings of different diameters.

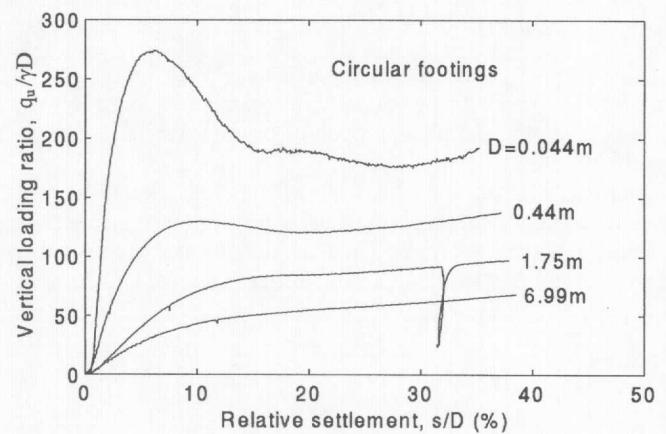
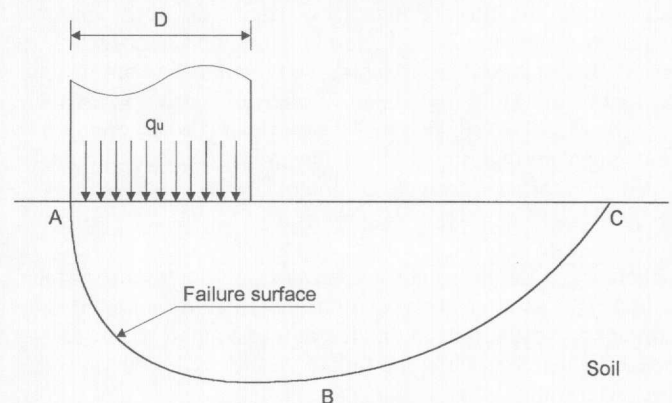


Fig. 13. General failure mode for small footings.



value of N_γ is constant and independent of footing size. For a constant N_γ , the bearing capacity expressed by [2]–[4] increases linearly with foundation dimension. Experimental data collected by De Beer (1965a), however, show that the bearing capacity factor N_γ decreases with foundation size. Centrifuge test results (Zhu et al. 1996; Taylor 1995) indicate that the bearing capacity increases proportional with foundation size when plotted on a log–log scale diagram. This suggests that N_γ decreases linearly with footing size on a double-log scale diagram. Studies of the effect of footing size by De Beer (1965a) and Vesic (1965) suggest that the average shear strength mobilization along the failure surface of soil supporting a shallow foundation decreases with footing size. The decrease of the mobilized strength is a result of the curvature of Mohr's strength envelope (Meyerhof 1950; De Beer 1965a) and the progressive rupture along the failure surface (De Beer 1965b; Muhs 1965). The relative compressibility of soils increases with footing size.

To illustrate the effect of footing size on the bearing capacity, centrifuge test results of a model circular footing 43.7 mm in diameter on the surface of a dry dense sand described above ($\gamma = 15.04 \text{ kN/m}^3$) are shown in Fig. 14, where D is the prototype diameter of the footings. In Fig. 14a, the relationship between the bearing capacity and the footing diameter is given by

$$[5] \quad q_u = 1470D^{0.68}$$

in which q_u is in kilopascals and D is in metres. Using [4] and [5], N_γ in Fig. 14b can be written as

$$[6] \quad N_\gamma = 325D^{-0.32}$$

The value of N_γ decreases significantly with increasing D , mainly as a result of the decrease of soil friction angle (ϕ) with stress level and the phenomenon of progressive failure of the soil.

In Fig. 11, both the peak friction angle (ϕ_{\max}) and the critical state friction angle (ϕ_{crs}) of the sand from triaxial shearing decrease with stress level. Their relationships with the confining pressure σ_3 can be approximately expressed as

$$[7] \quad \phi_{\max} = 57\sigma_3 - 0.05$$

and

$$[8] \quad \phi_{\text{crs}} = 46\sigma_3 - 0.04$$

where ϕ_{\max} and ϕ_{crs} are in degrees, and σ_3 is in kilopascals.

With the increase of footing size, the stress level in the soil supporting the footing increases. The stress level increase leads to a decrease of soil friction angle, and the bearing capacity factor N_γ will be reduced. To take into account the curvature of the Mohr envelope of failure, Meyerhof (1950) and De Beer (1965a) suggest that the value of ϕ corresponding to the mean normal stress along the failure surface should be used. For surface footings, De Beer (1965a) suggests

$$[9] \quad \sigma_m = 0.25q_u(1 - \sin \phi)$$

where σ_m is the mean normal stress along the failure surface, and ϕ is the friction angle of soil. Using [9] and the relationship among σ_m , σ_1 , and σ_3 in the diagram of Mohr's envelope, the mean value of σ_3 along the failure surface of foundations can be given roughly by

$$[10] \quad \sigma_3 = 0.25q_u \tan^2(45^\circ - \phi/2)$$

A rough quantitative evaluation of the effect of footing size on bearing capacity is shown in Table 2. When the diameter of a footing is given, the bearing capacity q_u can be calculated by [5]; using [7] and [8], together with [10] for estimating the mean stress level along the failure surface, the average ϕ_{\max} and ϕ_{crs} corresponding to the mean stress level can be obtained, as listed in Table 2. If N_γ is estimated by the formula of Brinch-Hansen (1970), which is recommended in the *Canadian foundation engineering manual* (Canadian Geotechnical Society 1992), the values of $N_{\gamma\max}$ and $N_{\gamma\text{crs}}$ corresponding to ϕ_{\max} and ϕ_{crs} can be derived as shown in Table 2. Therefore, if the peak values of ϕ are used, when the footing diameter increases from 0.1 to 10.0 m, the value of ϕ decreases from 50.9 to 42.6° due to the increase of stress level in the soil. As a result, the bearing capacity factor $N_{\gamma\max}$ of Brinch-Hansen (1970) decreases from 697 to 127. If the critical state values of ϕ are adopted, the value of $N_{\gamma\text{crs}}$ decreases from 99 to 40. Table 2 also shows the bearing capacity factor (actual N_γ) back calculated from [6] and the corresponding value of ϕ (actual ϕ). The variation of calculated theoretical $N_{\gamma\max}$ and $N_{\gamma\text{crs}}$ and the back-calculated actual N_γ is shown in Table 2. It is very clear that the selection of the appropriate value of ϕ according to footing size (or stress level) is critical to evaluate the bearing capacity. The relationship between N_γ calculated from the critical state friction angle ϕ_{crs} and the back-calculated N_γ from the tests is shown in Fig. 15.

Table 2. Effect of the size of circular footings on sand.

Diameter D (m)	q_u (kPa)	ϕ_{\max} (°)	ϕ_{crs} (°)	$N_{\gamma\max}^a$	$N_{\gamma\text{crs}}^a$	Actual N_γ^b	Actual ϕ (°) ^a
0.1	309	50.9	41.2	697	99	679	50.8
0.5	918	47.8	39.3	354	70	406	48.5
1.0	1466	46.5	38.5	271	61	325	47.4
5.0	4351	43.8	36.8	159	46	194	44.8
10.0	6952	42.6	36.0	127	40	156	43.7

^a Estimated by the formula of Brinch-Hansen (1970).

^b Estimated by [6].

In the classic bearing capacity theories (Terzaghi 1943; Meyerhof 1950; Sokolovskii 1960), in which soils are assumed to be rigid – perfectly plastic, the mobilization of shear strength along the failure surface is uniform. The soil elements at points A, B, and C (Fig. 12) fail simultaneously when the foundation collapses. In reality, because soil is elastoplastic, the failure of soil along the failure surface is a progressive process (Muhs 1965; De Beer 1965b). The failure surface begins at point A and develops gradually to point C. The mobilization of shear strength is not uniform. The influence of progressive failure on the bearing capacity depends on the deformation before failure and will be more prominent when the settlement is large.

It should be mentioned here that the values of N_γ , ϕ_{\max} , and ϕ_{crs} in Table 2 and Fig. 15 are only rough estimations. The purpose of the quantitative analysis above is intended to provide a clearer understanding of the influence of footing size and stress level on bearing capacity. In engineering practice, it is very important to carefully select the soil strength parameter ϕ . The error caused by improper selection of ϕ in estimating the bearing capacity may be very large. The peak friction angle should be used for determining the ultimate bearing capacity, but N_γ should take into account the effect of footing size on ϕ_{\max} .

As seen in Fig. 14, the load-carrying capacity increases with foundation size, even though the unit bearing capacity decreases. Therefore, conducting bearing capacity tests for very large foundations is difficult and very costly. If the relationship between the bearing capacity and the footing diameter is extrapolated as shown by the broken line in Fig. 14, the bearing capacities will be 7.0, 11.1, 14.6, and 20.6 MPa when the footing diameters are 10, 20, 30, and 50 m, respectively. Conducting bearing capacity tests of a foundation with a dimension of over, say, 10 m is impossible at 1g and it is difficult to apply the huge load to cause the foundation to fail, even in a centrifuge at 160g. Because of the high pressure in the soil under the foundation, soil particles will be crushed (Hardin 1985; Lade and Yamamuro 1996) and will behave differently.

As the failure mode tends to move from general shear failure to punching shear failure with the increase of foundation size, the relative settlement of foundations at failure increases as shown in Fig. 12.

Square, circular, and ring footings

Footing shape is one of the factors affecting the bearing capacity. To examine the effect of footing shape, four footings on the surface of dry sand have been tested in the centrifuge at 100g. The area of model footings was 15 cm². The first two footings

Fig. 14. The effect of footing size on bearing capacity.

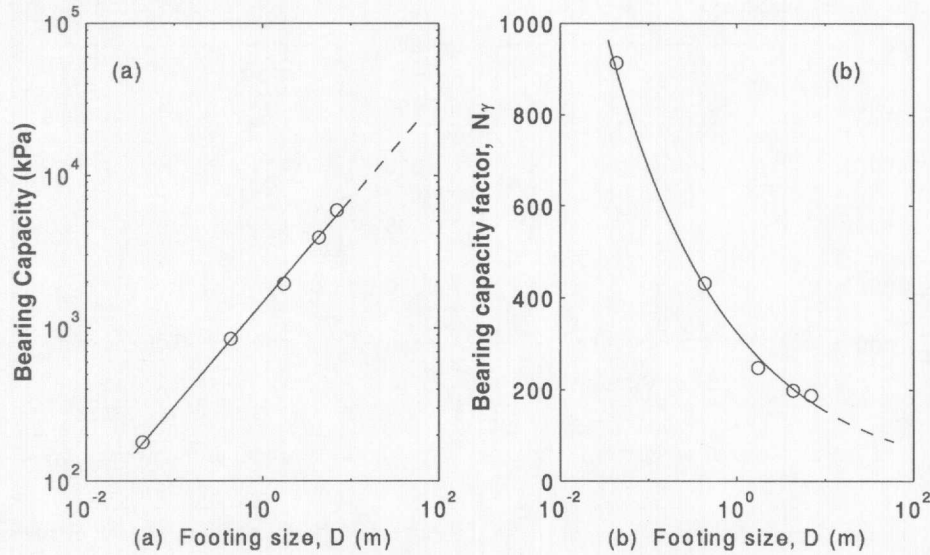
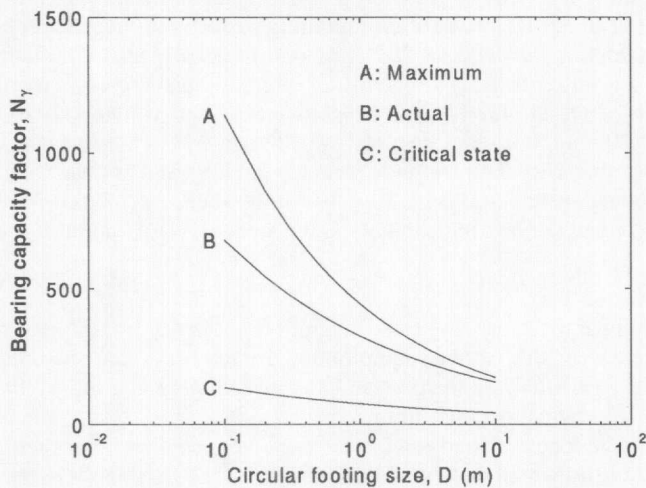


Fig. 15. Variation in bearing capacity factor with footing size.



were a full circular footing and a full square footing. The other two were a circular ring footing and a square ring footing with a ring radii ratio (n) of 0.7. The ring radii ratio is defined as the ratio of the inside diameter to the outside diameter for the circular ring footing, or the ratio of the inside width to the outside width for the square ring footing. The relationships between the loads and settlements of the footings are shown in Fig. 16. Figure 17 shows the corresponding prototype footings.

Two important conclusions can be drawn from these Figs. 16 and 17. One relates to the fact that to simplify analysis, a circular ring foundation is often assumed to be a square ring with the same contact area. These results support that assumption. The much greater resistance of lateral loads provided by the same contact area for circular and square ring foundations is achieved at no reduction in vertical load-carrying capacity.

It can be seen from Fig. 16a that the load-settlement responses of the circular and square footings are virtually identical. There are no peak loads and the failure modes are local

shear failures. The bearing capacity of the two footings is approximately 3900 kPa. In Fig. 16b, the responses of settlements to the loads of the two ring footings are also similar. The failure modes are obviously general shear failures. The bearing capacities are about 2900 kPa for the square ring and 2700 kPa for the circular ring. Compared with square and circular footings, the bearing capacities and failure modes of the ring footings are different. The bearing capacity of ring footings is dependent on the ring radii ratio. Ring foundations having high capacity to resist lateral loading have been used in Canada for large structures (Clark and Robinson 1972; Kosar et al. 1994).

To further illustrate the effect of the shape on the behaviour of foundations, five model square rings of 40 mm outside width were tested on dry dense sand in the centrifuge at 100g. The outside width (B) of the prototype ring footings corresponding to 100g was 4.0 m. The ring radii ratio (n), as in Fig. 18, is defined as

$$[11] \quad n = \frac{b}{B}$$

where b is the inside width, and B is the outside width. With a constant B , the values of n of the five square ring footings were 0, 0.3, 0.45, 0.6, and 0.8 for values of b of 0, 12, 18, 24, and 32, respectively.

The relationships between load and settlement of the footings are presented in Fig. 18. The bearing capacities of the footings are 3800, 4150, 3730, 2750, and 1690 kPa when the corresponding n values are 0, 0.3, 0.45, 0.6, and 0.8, respectively. Figure 19 shows the variation of the bearing capacity with n . When n changed from 0 to 0.3, the bearing capacity increased about 10% from 3800 to 4150 kPa, due to the arching effect of the soil under the centre of the ring footing. With the further increase of n , the bearing capacity decreases because the width of the ring footings, $t = (1 - n)B/2$ decreases with n . When B is constant, t is a factor representing the overall size of the ring footings. It can also be seen from Fig. 18 that the failure mode of the footings is related to the value of n . When n increases, the failure mode tends to move from local shear failure to general shear failure.

Fig. 16. (a) Load settlement comparison for square and circular foundations of the same area. (b) Load settlement comparison for square ring and circular ring foundations of the same area.

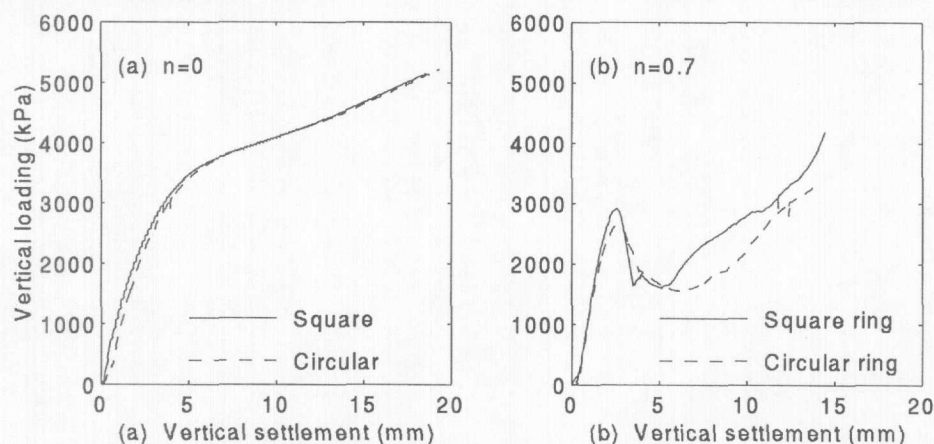
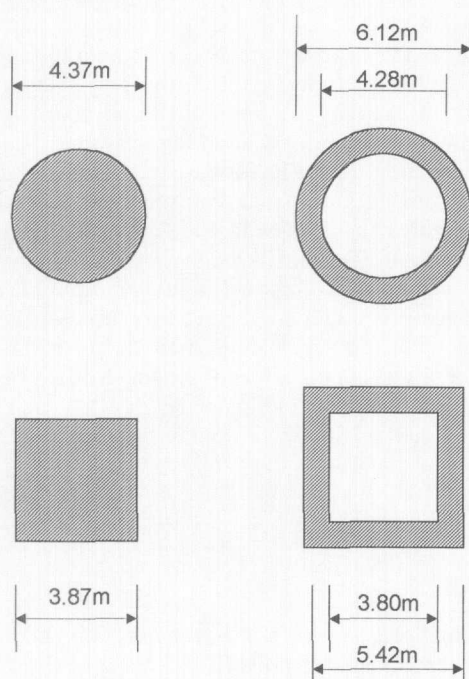


Fig. 17. Prototype size of solid and ring footings of the same area tested in centrifuge.



Foundations under repeated loading

Many foundations, such as those for offshore structures, are often subjected to high-magnitude repeated loads. The displacement of the foundations under repeated loading is an important criterion for the operation of the structures and can be evaluated using the deformation modulus of the soil supporting the foundations. For a rigid circular foundation, the deformation modulus E_d may be given by (Winterkorn and Fang 1975)

$$[12] \quad E_d = \frac{0.79(1 - \nu^2)Dp}{s}$$

where p is the loading applied to the foundation, D is the di-

ameter of the foundation, s is the displacement, and ν is Poisson's ratio, assumed to be 0.3.

Repeated loading tests of footings on a saturated, dense sand ($I_D = 88\%$) in the centrifuge were conducted. A model footing 62.5 mm in diameter was tested at centrifuge accelerations of 32, 100, and 160g, simulating corresponding prototype diameters of 2.00, 6.25, and 10.00 m, respectively. The load-settlement relationships of the footings are shown in Fig. 20a, and the bearing capacities are approximately 1070, 2400, and 3500 kPa, respectively. The bearing capacity data, together with data from the tests for the dry sand as described above, are given in Fig. 20b. As can be seen, the relationship between the bearing capacity and a stress-level parameter

$$[13] \quad \Sigma = 0.3\gamma D$$

is linear in the log-log scale diagram for both dry and saturated sand. In the above equation, γ is the dry unit weight (15.04 kN/m³) for the dry sand and the submerged unit weight (5.23 kN/m³) for the saturated sand.

The repeated loads on the footings were applied by pushing and lifting dead weights. The ratios of the repeated loads to the ultimate bearing capacities were 24, 35, and 31% for the footings when the accelerations were 32, 100, and 160g, respectively. The values of the deformation modulus (E_d) of the soil calculated by [12] during repeated loads are shown in Fig. 21. For each footing, E_d increases with footing dimension and the number of load cycles. For the footings with prototype diameters D of 2.00, 6.25, and 10.00 m, the values of E_d during the first cycle of loading are 9.4, 19.4, and 32.4 MPa, respectively. The increases of E_d from the first loading to the second loading are significant. The ratios of E_d in the second loading to E_d in the first loading are 11.3, 9.8, and 7.6, respectively. After the second loads, the increases of E_d are much less.

In this work, centrifuge tests of footings on a dense sand were conducted to investigate the effects of footing size and shape on the bearing capacity of large foundations. For ring footings with a constant outside diameter, both the bearing capacity and the failure mode are influenced by the ring radii ratio. The effect of repeated loading on the deformation modulus of soil supporting foundations, which increases with the number of loading cycles, has also been presented. In the triaxial tests of the sand, both the peak friction angle (ϕ_{max}) and

Fig. 18. Load-settlement relationship for footings of the same width with variable n .

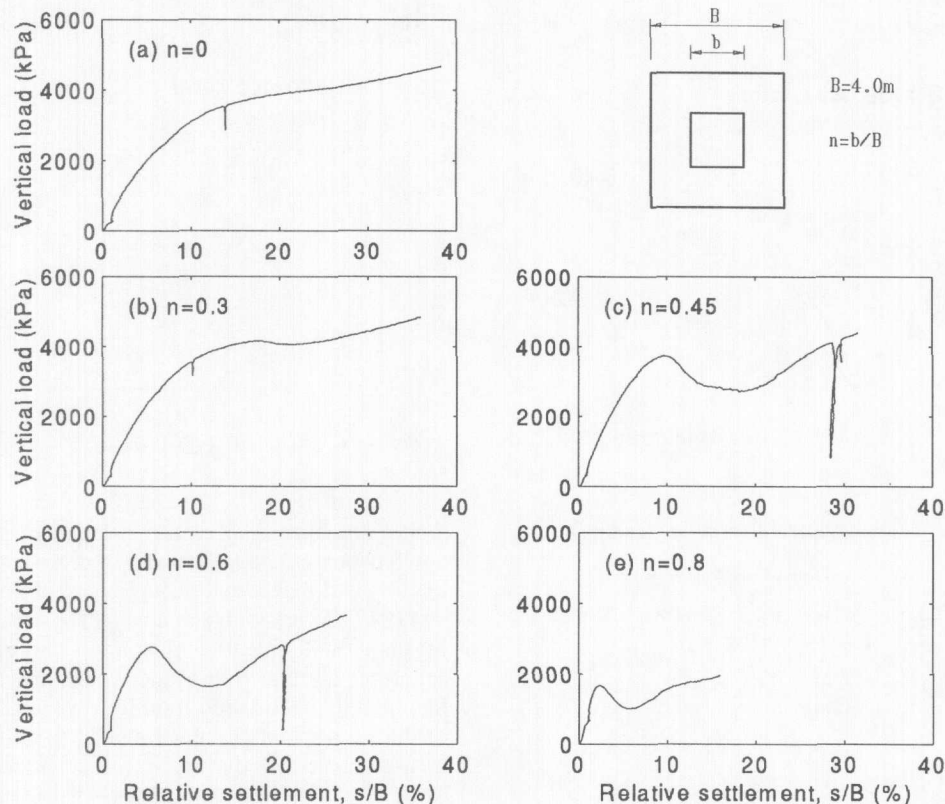
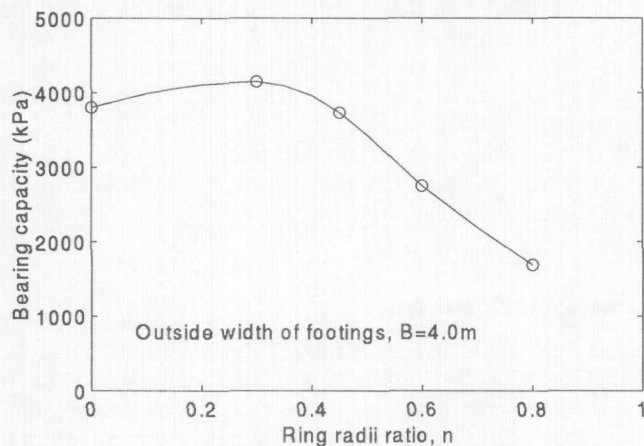


Fig. 19. Bearing capacity of square ring foundations with variable n .



the critical state friction angle (ϕ_{crs}) decrease with confining pressure. Using the triaxial test data and the centrifuge test results, a quantitative estimation of the effect of footing size on the bearing capacity factor N_γ has been presented. The decrease of N_γ with footing size is due to the fact that soil friction angles ϕ_{max} and ϕ_{crs} decrease with stress level and progressive failure of soil, which reduces soil friction angle to a value between ϕ_{max} and ϕ_{crs} .

Conclusions

- (1) The settlement of foundations of large buildings on

strong soils is very similar for buildings of the same height. The data suggest that settlement would be similar whether or not the buildings are on spread foundations, rafts, mats, driven piles, drilled piles, or dynamically cast-in-place piles. The reason for this is that as incremental loads are applied during construction the modulus of deformation increases in proportion to the load on the foundation and tends toward similar values for a wide range of strong soils and soft rocks and foundation types. In the early stages of loading of building foundations, settlement may vary according to soil type and foundation type, but the significance of this variance with respect to total settlement is obliterated as the height of the building increases. This holds true only for soils and loads where the deformation is predominately elastic.

- (2) Each load increment produces an increase in elastic modulus which reduces settlement under the next load increment.

- (3) A reasonable rule of thumb for settlement at the end of construction of large buildings on strong soils is approximately 1 mm per storey plus 10%.

- (4) It is impossible to obtain undisturbed samples of very strong soils. The drilling of the bore hole in itself disturbs the soil below the boring for most of the length of the sampling tube that is pushed into what is believed to be undisturbed soil below the boring. The trip to the surface results in pickup of water and hence increase in moisture content in the sample. Typically, the shorter the tube sample, the higher the moisture content, whereas with cores the longer the core, the higher the moisture content.

- (5) Traditional bearing capacity analyses are based on

Fig. 20. (a) Load-settlement relationship for repeated loads. (b) Bearing capacity relationship for dry and submerged sand.

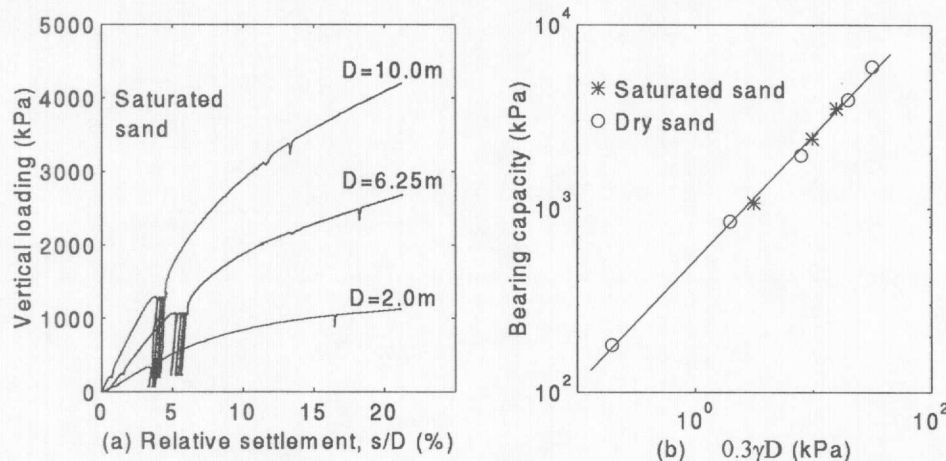
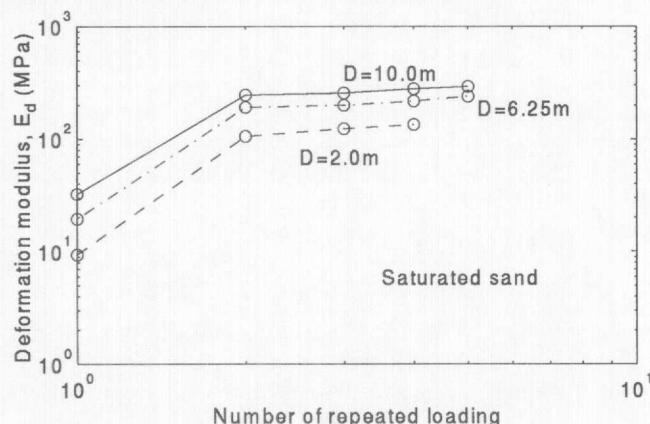


Fig. 21. Change in deformation moduli with repeated loading.



friction angles and soil properties which are exceptionally difficult if not impossible to measure for very strong soils. Estimates or tests usually yield values much lower than reality, resulting in very significant understatement of the factors of safety. The bearing capacity factor increases rapidly with increasing friction angle. The friction angle of the soil decreases with increasing stress level. Engineering judgement conditioned by experience is required to arrive at realistic design values.

(6) Foundations for very large buildings can be sized for construction convenience unless the bearing load starts to approach the ultimate bearing capacity. Ultimate bearing capacity is usually underestimated by a factor of several times.

(7) A straight-line relationship of bearing capacity and footing size is obtained on a log-log scale as opposed to a simple arithmetical scale as conventional bearing capacity theory would indicate.

(8) The bearing capacity factor N_γ decreases dramatically with increased footing size. This is partly due to the change in friction angle but is also due to the transition from a general failure to progressive failure for small footings to larger footings. As a general rule of thumb, the bearing capacity factor N_γ decreases by about 50% for each log cycle of footing diameter.

(9) The elastic deformation for a constant load repeatedly applied decreases significantly from first loading to second loading and then slowly decreases with repeated loads.

(10) The vertical load capacity for square and circular footings is virtually identical over a wide range of penetration. For square or circular ring foundations vertical loading is almost identical to that of solid footings for the early part of the curve of the square and circular foundation but then drops with increased settlement due again to the change in the failure mode. The removal of bearing area from the interior part of the footing causes no reduction in vertical load capacity until the ratio of the interior diameter to the exterior diameter exceeds about one third.

(11) For the most economical design of foundations of large structures on strong soils, bearing capacity rather than settlement should govern. A design bearing capacity should be used to provide the required degree of safety of the foundation. The settlement of foundations designed for that bearing capacity will not be significantly greater than that for much larger footings at lower bearing pressures carrying the same load or for piled foundations distributing loads over a greater depth within the same soil type.

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