# Mass Conservation Applied to Environmental Engineering

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# Mass Conservation Equation

Rate of Accumulation = Net Fluxes across Interfaces

- For chemical species:

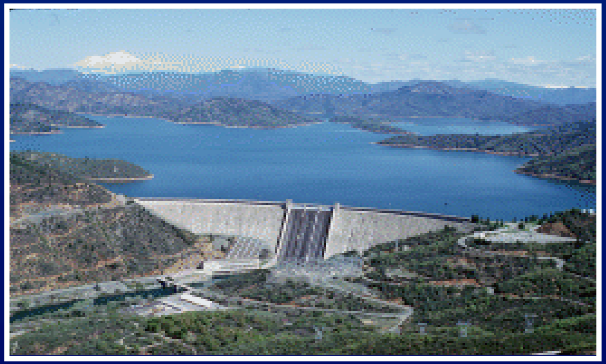
Rate of Accumulation = Net Fluxes across Interfaces + Rate of (Sources – Sinks)

- Ignore Sources - Sinks

LAKES

WASTEWATER
TREATMENT

# Applications of Aeration Systems



RESERVOIRS

RIVERS

# Types of Aerators

Surface Fountain

Surface Aerator

Oxygen Injector

# Submerged Diffusers (most common)

~ Release air or pure oxygen bubbles at depth

~ Free turbulent bubble plume rises to the water surface through buoyant forces

~ Provide both oxygen transfer and mixing

- OXYGEN TRANSFER occurs through the water surface and bubble interfaces as they rise to the surface.
- MIXING occurs as water is entrained by the rising bubble plume, causing circulation.

# Objectives

- Explain standard method for determining oxygen transfer in aeration tests.

    - Show results from standard aeration tests up to 10 m deep.

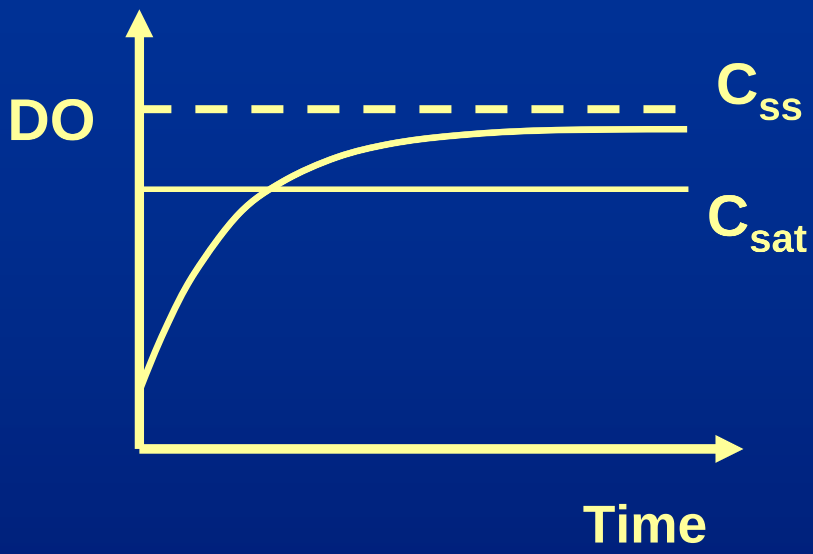
**Standard Aeration Tank Test**

- Explain challenges with standard method and better describe oxygen transfer process through mass conservation.

    - Show results from mass transfer model that give insight into oxygen transfer in deep water systems.

# Standard Aeration Tank Test

Assume that the water in the tank is well mixed




**ASCE *Standard for Measurement of Oxygen Transfer in Clean Water* (1992)**

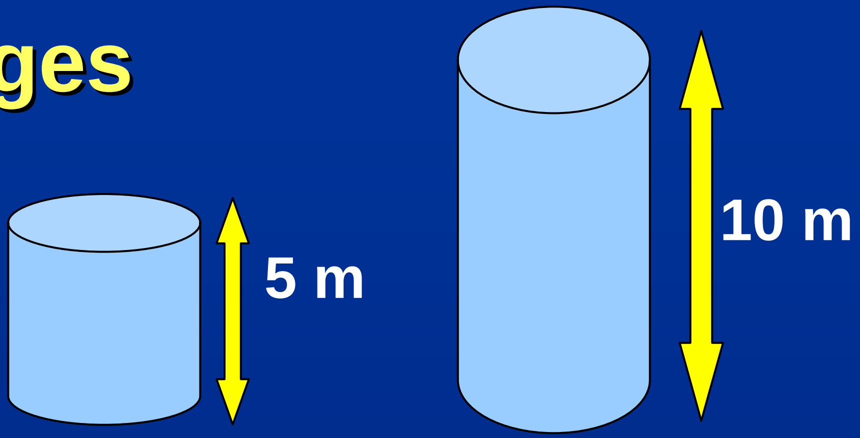
$$\left(V\frac{dC}{dt}\right) = k_L A\left(C_{ss} - C\right)$$

**~ $k_L A$ used to calculate performance parameters ~**

# Problems with the Standard Aeration Tank Test

- Fit $K_La$ and $C_{ss}$ to experiments
- Mass Conservation is violated
  - Mass comes from bubbles, but does not decrease in bubbles
  - $K_La$ and $C_{ss}$ are difficult to interpret
- Cannot apply resulting $K_La$ to deeper water bodies
  - Typical tank is 3 – 5 m in depth
- Understanding of factors that affect $K_La$ is limited

# Application Challenges




## 1.) DIFFUSER DEPTH

- ~ Manufacturer's aeration tests are typically conducted in tanks at depths of 3-5 meters, replicating activated sludge tanks
- ~ Lakes and reservoirs are typically 10 meters or greater in depth

**How can we apply test results at depths of 5 meters to deep lakes and reservoirs?**

# Aeration Experiments

Large cylindrical tank used for tests with water depths of 3.4 m, 6.7 m, and 9.6 m.

10.3 m

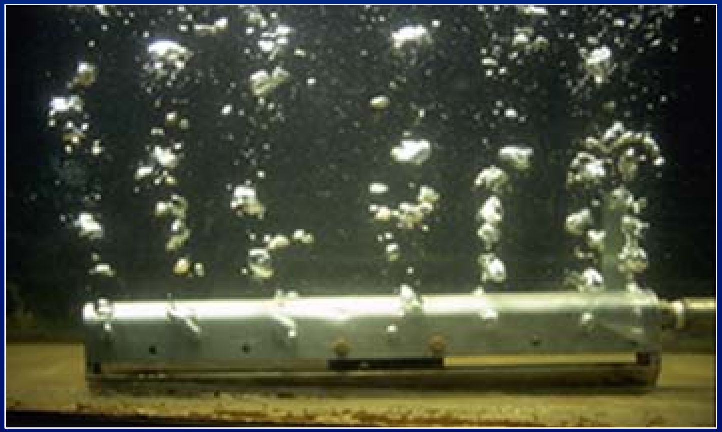
7.6 m

40 Membrane Diffuser tests
24 Coarse Bubble Diffuser tests
10 Soaker Hose tests

# Diffuser Types


**Soaker Hose**
**2 – 3 mm fine bubbles**

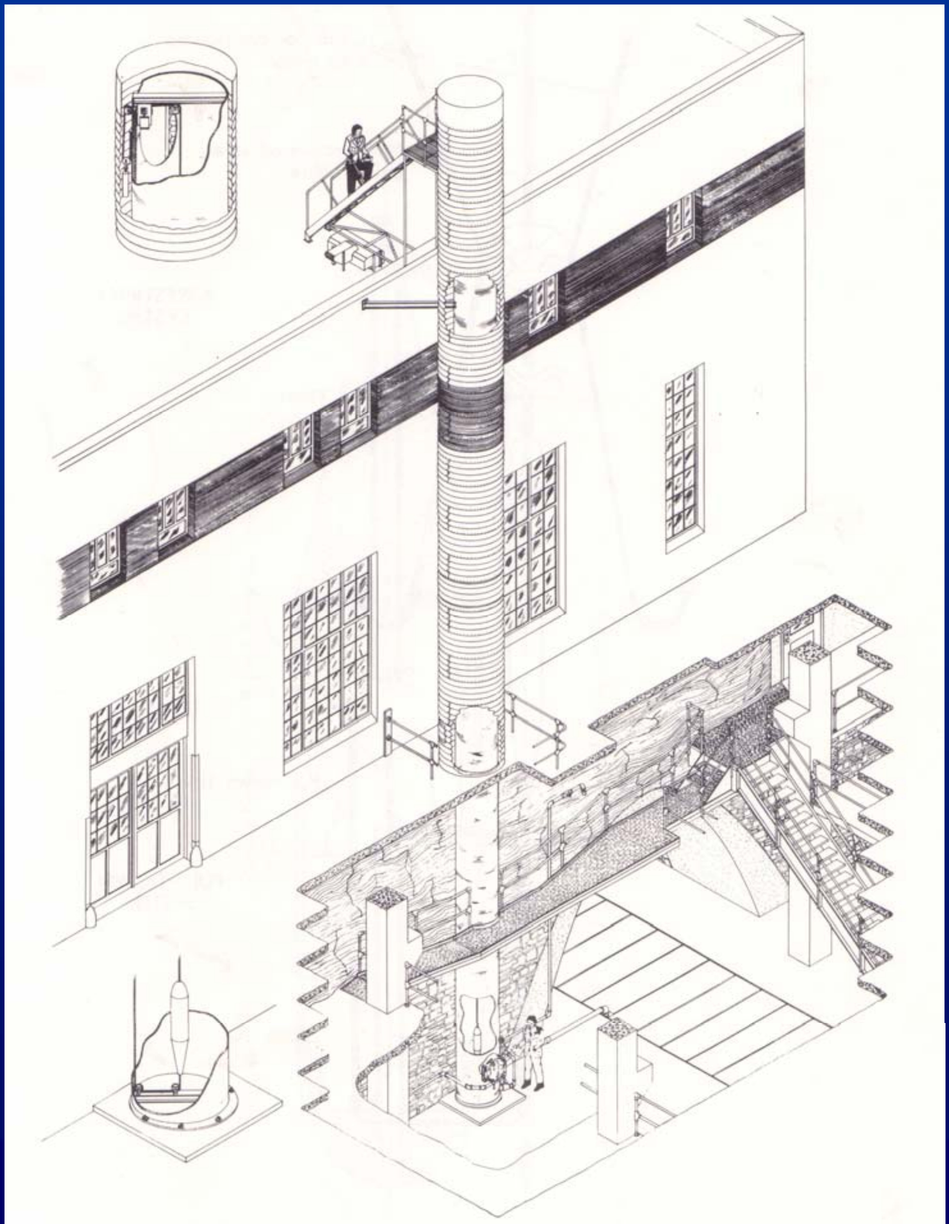
**Wilfley Weber Membrane Diffuser**
**3 - 3.5 mm bubbles at low flow rates**



**Coarse Bubble Diffuser**
**3 - 6 mm bubbles, up to 40 mm**

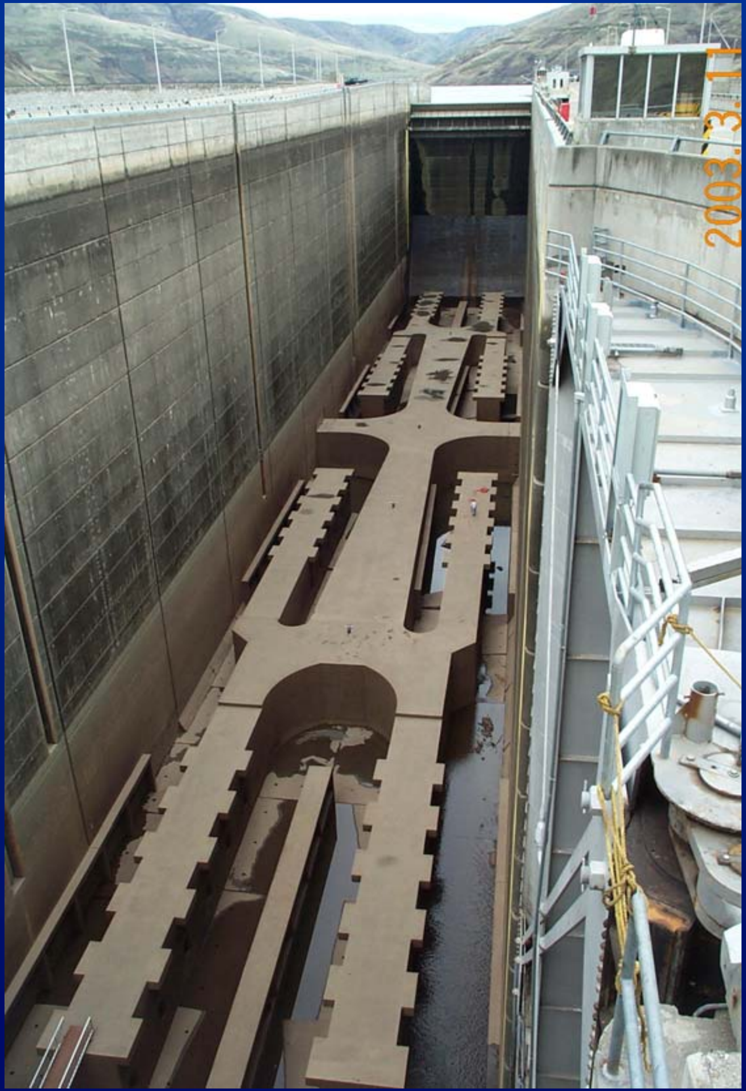
# Deep Bubble Facility Experiments

- 26 meters tall
- 1.06 meter diameter
- Maximum water level: 24 meters



# Lock Experiments

## 32 m in depth

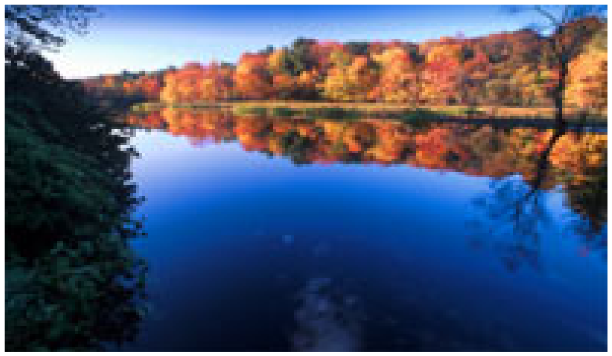



LAKES

WASTEWATER TREATMENT

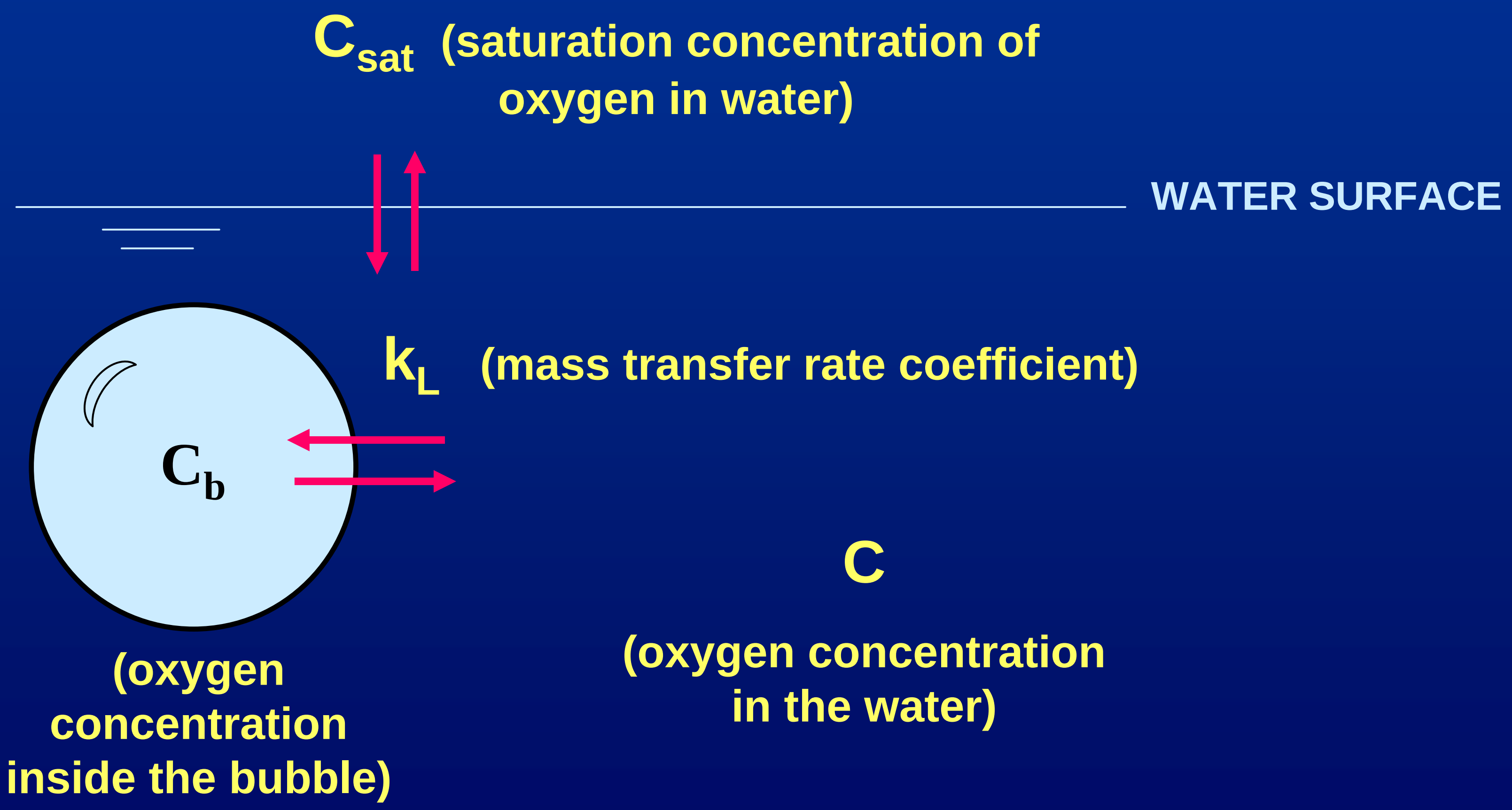
**Applications of Aeration Systems**

RESERVOIRS

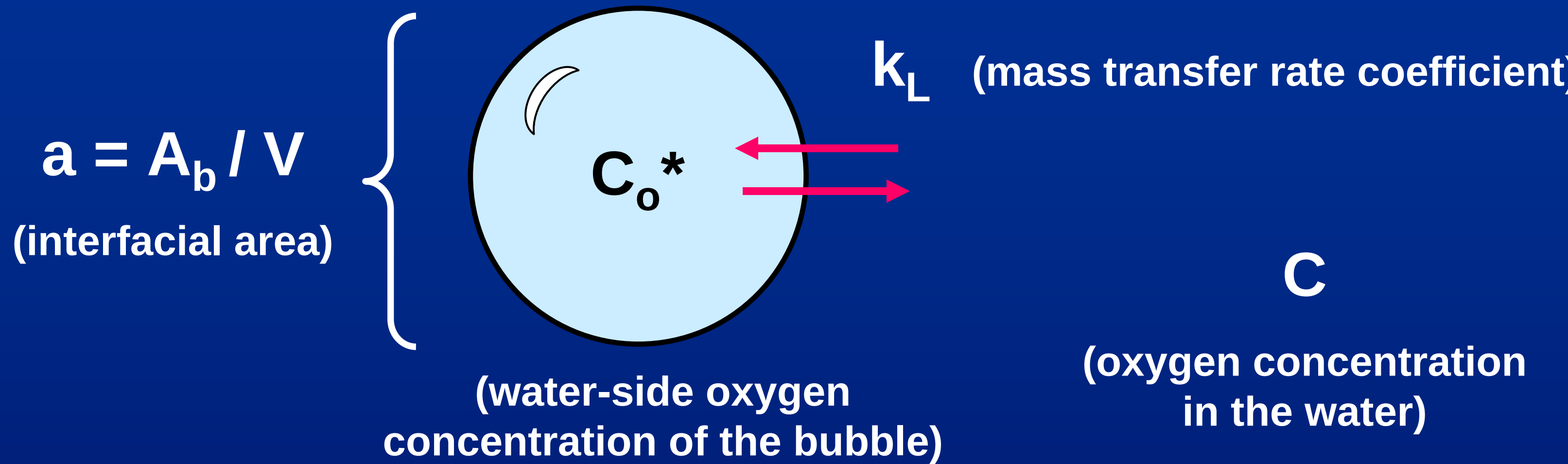


RIVERS

# Mass Conservation for Aeration Systems



# Description of Oxygen Transfer Across the Bubble Interface




$$F_b = \int_V k_L a_b \left( C_o^* - C \right) dV$$

# Description of Oxygen Transfer Across the Air-Water Interface

**$C_{sat}$** (saturation concentration of oxygen in water)

WATER SURFACE

**$k_L$** (mass transfer rate coefficient)

**$a = A_s / V$**

(interfacial area)

**C** (oxygen concentration in the water)

$$F_s = \int_{A_s} k_{Ls} \left( C_{sat} - C \right) dA_s$$

# Mass Conservation Equation

$$V\, dC/dt = F_b + F_s$$

Or

$$V\, dC/dt = \int_V k_L a_b \left( C_o^* - C \right) dV + \int_{A_s} k_{Ls} \left( C_{sat} - C \right) dA_s$$

$C = C(t)$

$C^* = C^*(z, t) \Rightarrow C^*(z)_{ss}$

Solve for $C^*(z)_{ss}$ at each time step to compute the next $C(t)$

# Oxygen Transfer Equation

**Total rate of oxygen addition due to aeration ($F_a$)**

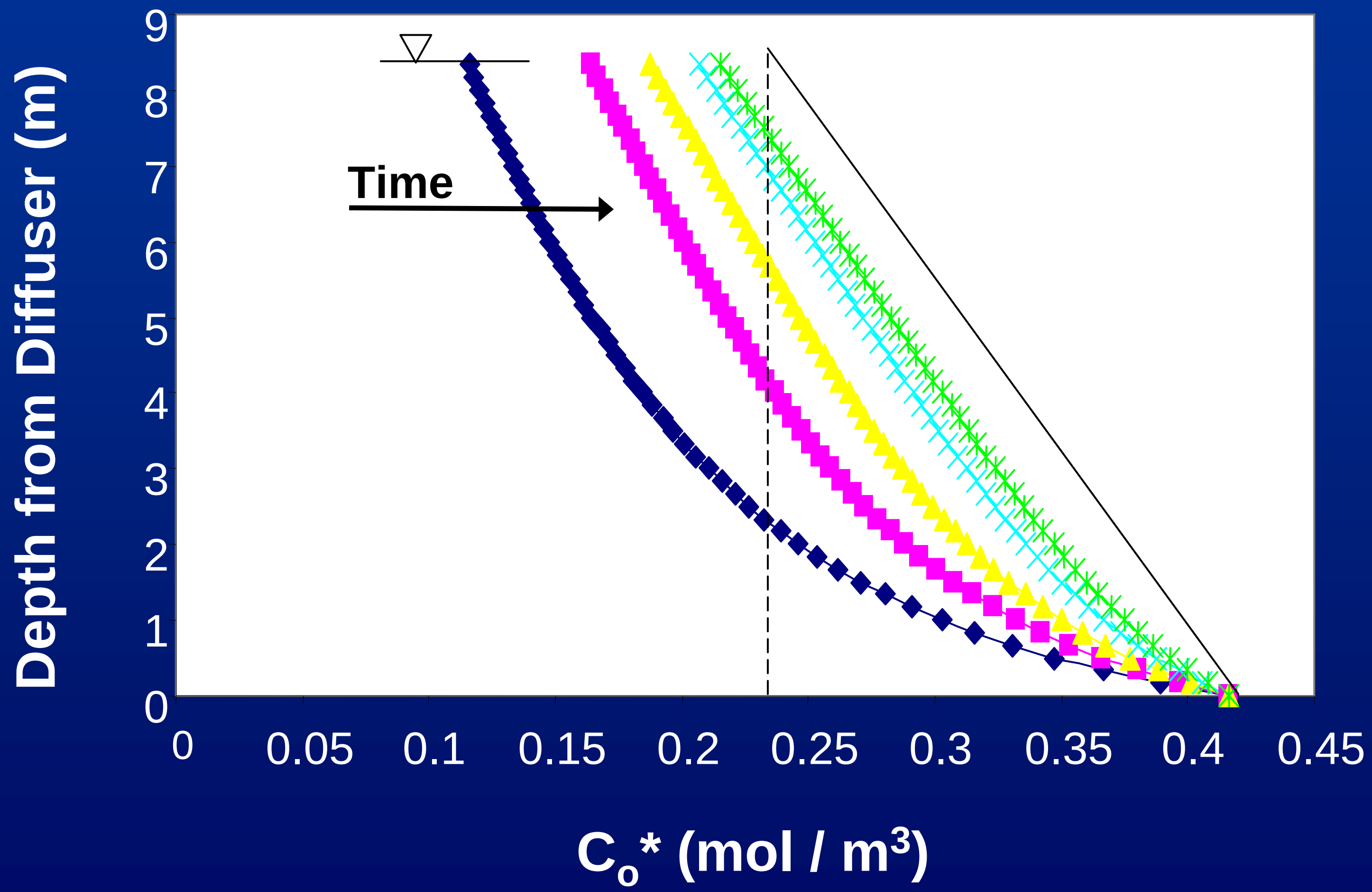
**COMMON DESCRIPTION**

$$F_a = k_L a \left(C_{ss} - C\right)$$

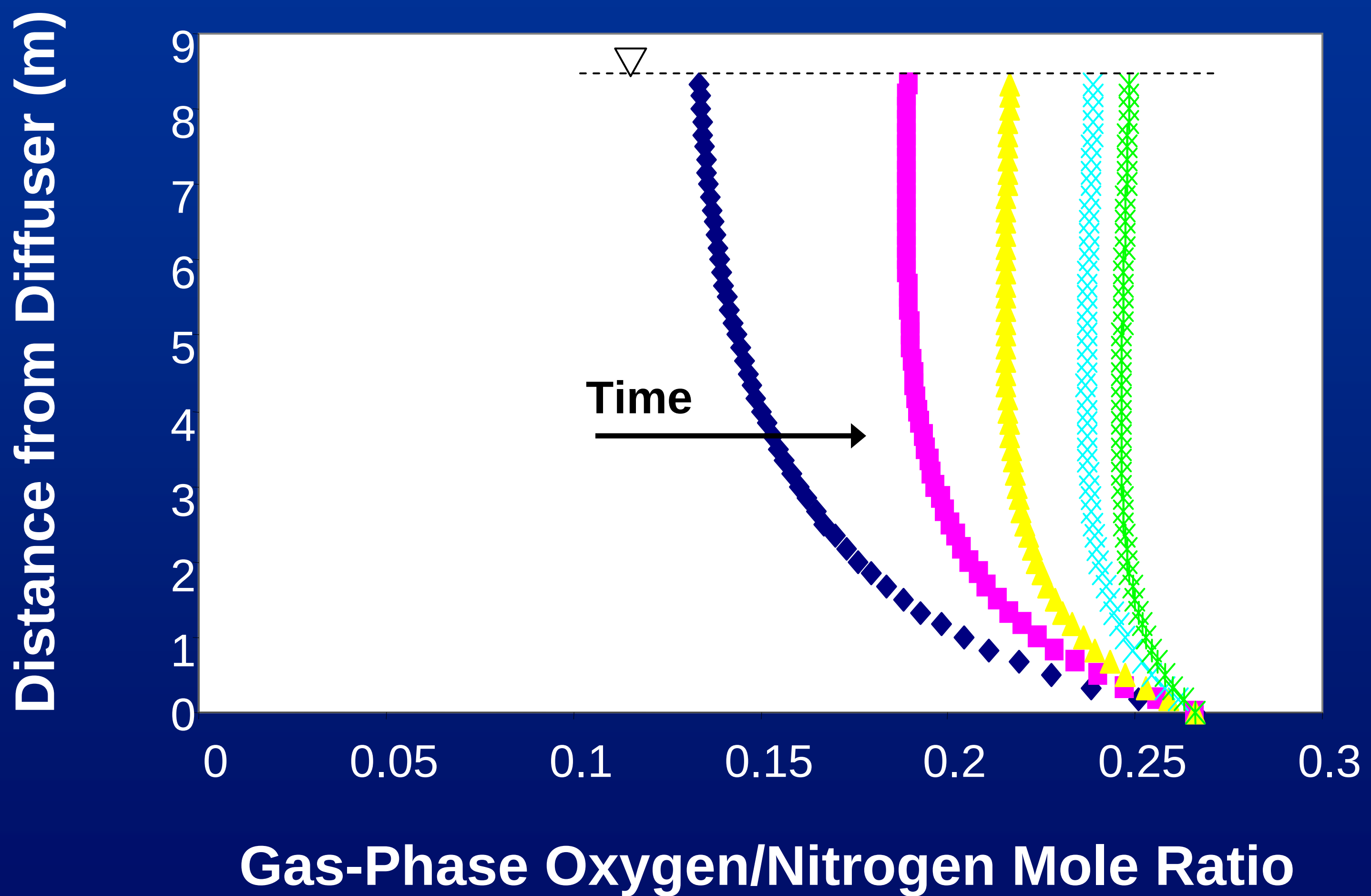
**MORE ACCURATE DESCRIPTION**

$$F_a = \int_V k_L a_b \left(C_o^* - C\right) dV + \int_{A_s} k_{Ls} \left(C_{sat} - C\right) dA_s$$

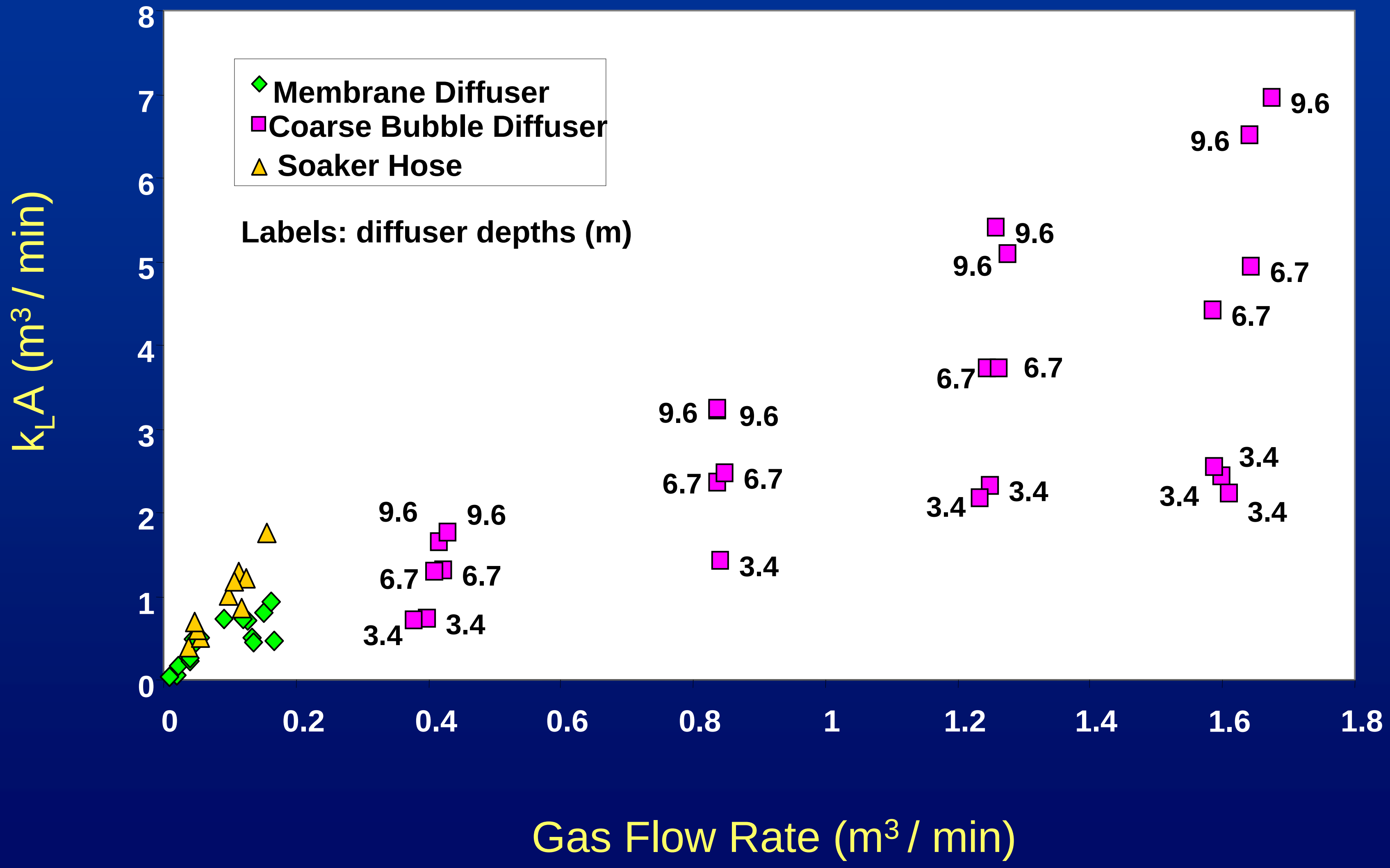
# Oxygen Concentration Inside Bubbles



# Fraction of Oxygen / Nitrogen Inside Bubbles



# Comparison of Total Mass Transfer Coefficients



# Prediction of $K_LA_b$

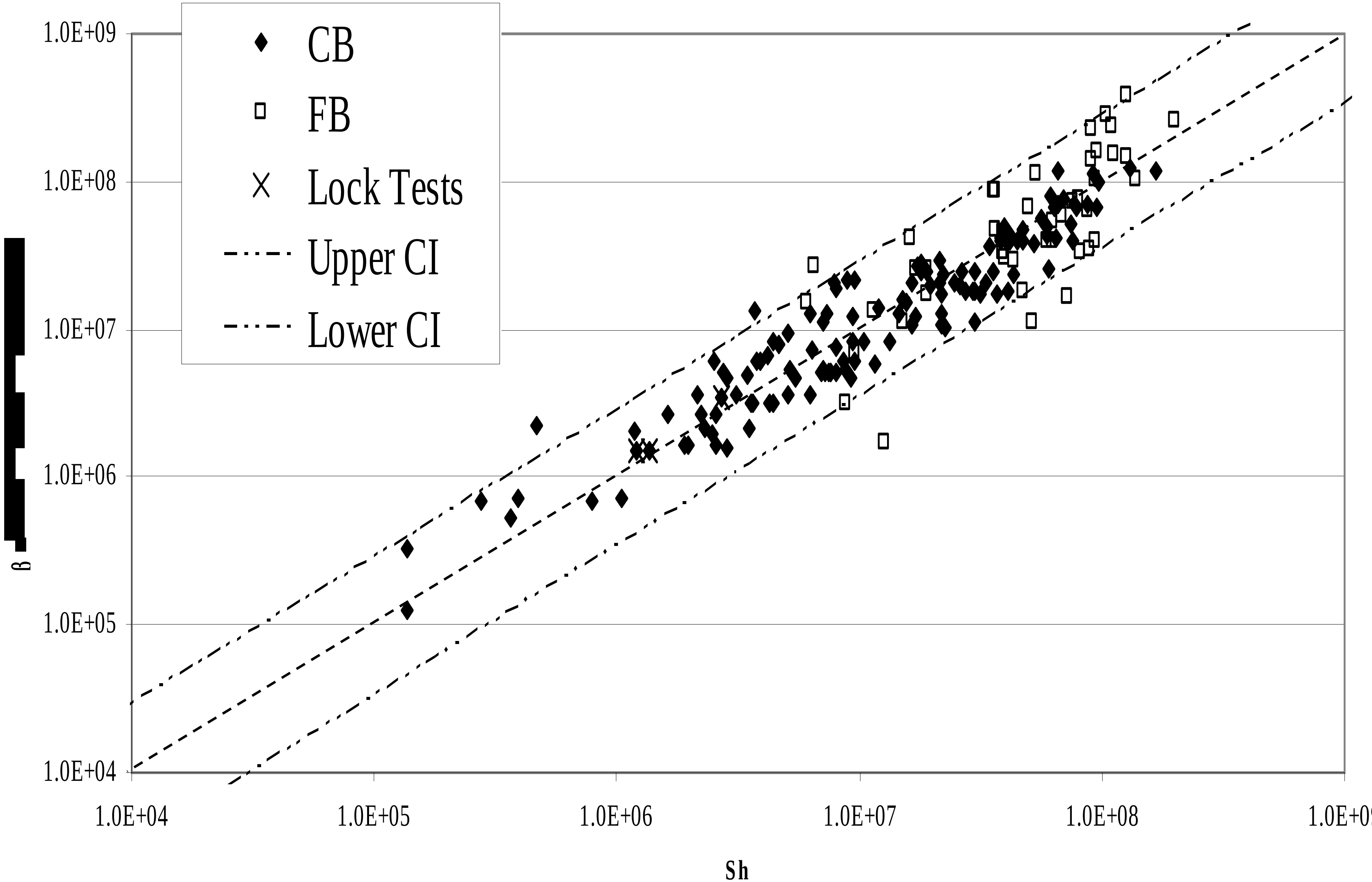



**Figure 8 - Correlation of the bubble mass transfer coefficients to dimensionless parameters for all tests, separated by coarse bubble and fine**

# Prediction of $K_{Ls}$

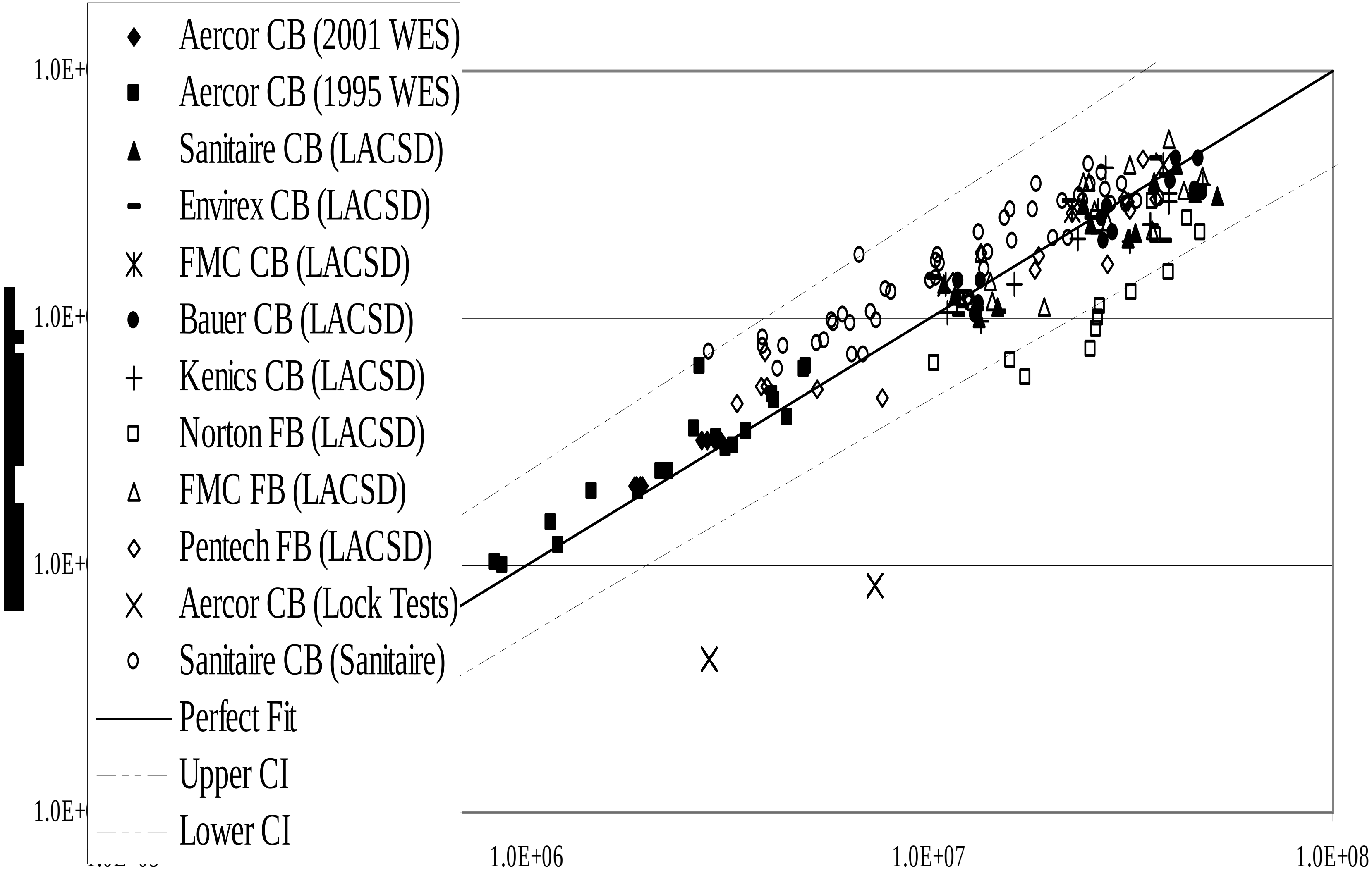



Figure 9 - Correlation of the surface mass transfer coefficients to dimensionless parameters for all diffusers. Included is the 95%

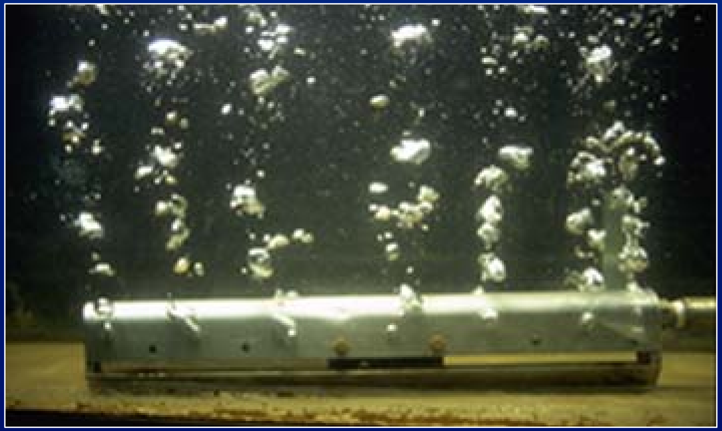
# Diffuser Types

**Wilfley Weber Membrane Diffuser**

**3 - 3.5 mm bubbles at low flow rates**


**Soaker Hose**

**2 – 3 mm fine bubbles**



**Coarse Bubble Diffuser**

**3 - 6 mm bubbles, up to 40 mm**

# Diffuser Comparison

## COARSE BUBBLE DIFFUSER

- Higher air discharge
- Lower SAE
- Should function well for depths >10 m

## SOAKER HOSE (1.1 m section)

- Low air discharge
- High SOTR
- High SAE

} 4.9 m of SH = 1 CBD

## MEMBRANE DIFFUSER

- Low discharges → soaker hose
- High discharges → coarse bubble 10 MD = 1 CBD
- Functions well in shallow depths

# In Our Deep Tanks (51-76 $m^3$/hr, coarse bubble diffuser)

- ~ Bubble transfer coefficient, $k_L a_b$, is dominant.
- ~ Bubble transfer concentration gradient, $(C_o^* - C)$, is dominant.

**Bubble transfer is the principal oxygen transfer location.**

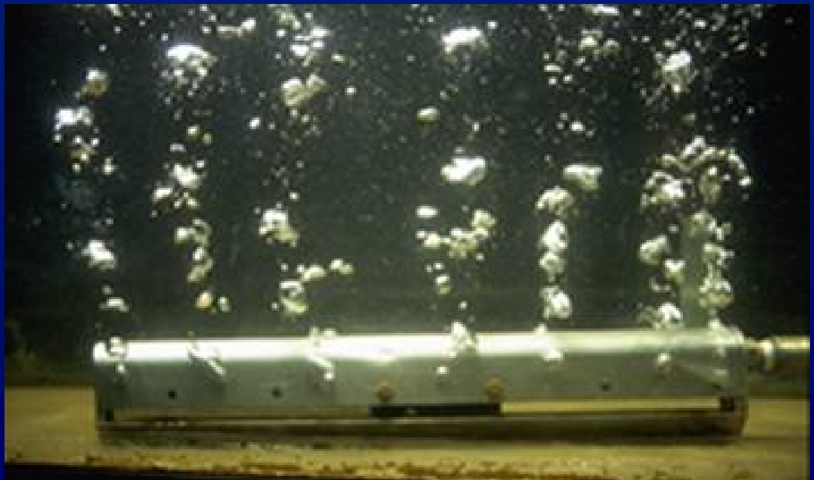
# What We Learned

1. Mass conservation is crucial to environmental engineering.
2. Increasing diffuser depth increases oxygen transfer.
3. Total mass transfer coefficient, $k_L a$, cannot be reliably extrapolated to deeper depths.

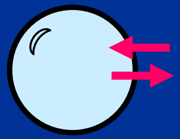
# Insights

- ~ The Standard method "works" for shallow water systems. However, $k_La$ and $C_{ss}$ have mixed physical meaning
  - ~ The Standard method cannot be applied to deep water systems.
    - ~ Water surface and bubble surface both provide oxygen transfer.
      - ~ Fluxes across water surface and bubbles must be calculated separately to predict the oxygen transfer from a diffuser in a deep water system.

# Questions?

# Gas Transfer Across Bubble Surface



**Azbel (1981), Hinze (1955), and probable bubble size distribution:**

$$k_L a_b = \beta_1 \frac{\phi}{\left[\left(\frac{\sigma}{\rho}\right)^{3/5} \varepsilon^{-2/5}\right]} \frac{(1-\phi)^{1/2}}{(1-\phi^{5/3})^{1/4}} \frac{D^{1/2} U^{3/4}}{(L\nu)^{1/4}}$$

$\phi$ = gas void ratio (air bubble volume / liquid volume)

$\beta_1$ = bubble transfer coefficient

$\varepsilon$ = turbulent energy dissipation rate

D = diffusivity coefficient

L = characteristic length

$\sigma$ = surface tension

$\nu$ = kinematic viscosity

$\rho$ = density

U = characteristic velocity

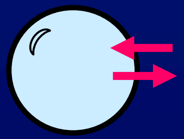
# Transfer Coefficients

$$k_L = \frac{D}{\delta_c}$$

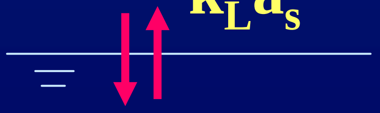
$k_L$ = mass transfer rate coefficient
D = diffusion coefficient of the gas
$\delta_c$ = f (D, turbulence) = instantaneous thickness of the boundary layer

$$a = \frac{A_s}{V}$$

a = specific area
$A_s$ = interfacial surface area
V = volume of liquid

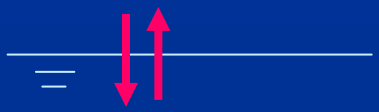


$k_L a_b$



$k_L a_s$

# Gas Transfer Across Water Surface



**$k_L$ relationship based on Lamont's small eddy model (1970):**

$$k_L = \beta_2 \left(\frac{\nu}{D}\right)^{-1/2} (\varepsilon\nu)^{1/4}$$

**where, $\beta_2$ = surface transfer coefficient $\nu$ = kinematic viscosity**
**D = diffusivity coefficient $\varepsilon$ = energy dissipation rate**

$$a_s = \frac{A_{\text{water surface}}}{V_{\text{water body}}}$$

# “Champagne Bottle”:  Dimensions

- 26 meters tall
- 1.06 meter diameter
- Maximum water level:  24 meters
- 124 steps from bottom to top (~ 78 ft)
- Maximum water capacity:  21,180 L (5,600 gal)