Pauta P1 Examen

i) P.D.Q. $\iiint_{\Omega} (v \cdot \Delta u + \nabla u \cdot \nabla v)\, dV = \iint_{\partial\Omega} \frac{\partial u}{\partial n} \cdot v \, dS$

$$\iint_{\partial\Omega} \frac{\partial u}{\partial n} \cdot v \, dS = \iint_{\partial\Omega} (\nabla u \cdot v)\, \hat{n} \, dS \underset{\text{Teorema Divergencia}}{=} \iiint_{\Omega} div(\nabla u \cdot v)\, dV$$

1,5 ptos

$$= \iiint_{\Omega} \left(\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right] \cdot v + \left[\frac{\partial u}{\partial x}\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\frac{\partial v}{\partial y} + \frac{\partial u}{\partial z}\frac{\partial v}{\partial z} \right] \right) dV$$

$$= \iiint_{\Omega} (v \cdot \Delta u + \nabla u \cdot \nabla v)\, dV \quad /// \qquad 1{,}5 \text{ ptos}$$

ii) $\star$ $\begin{cases} \Delta u = f & \text{en } \Omega \ (a) \\ u = g & \text{sobre } \partial\Omega \ (b) \end{cases}$ $\quad w = u_1 - u_2$ con u_1, u_2 solucion de $(\star)$

claramente $\Delta u_1 = \Delta u_2 = f$ en Ω por (a)

$\Longrightarrow \Delta(u_1 - u_2) = \Delta w = 0$ en Ω (1)

0,5 ptos

además $u_1 = u_2 = g$ sobre $\partial\Omega$ por (b)

$\Longrightarrow u_1 - u_2 = w = 0$ sobre $\partial\Omega$ (2)

0,5 ptos

Ahora sea $u = v = w \in \mathcal{C}^1(\bar{\Omega}) \cap \mathcal{C}^2(\Omega)$

y apliquemos (i)