Pauta Pregunta 3

Condiciones del problema:

$$(1)\quad \frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = Kx \;,\quad x \in [0,L] \,,\; t>0$$

$$(2)\quad u(0,t) = u(L,t) = 0 \;,\quad t>0$$

$$u(x,0) = \frac{\partial u}{\partial t}(x,0)$$

función auxiliar $y(x,t) = u(x,t) + \phi(x)$ tal que:

$$(3)\quad \frac{\partial^2 y}{\partial t^2} - \frac{\partial^2 y}{\partial x^2} = 0 \;,\quad x \in [0,L] \,,\; t>0$$

$$(4)\quad y(0,t) = y(L,t) = 0 \,,\; t>0 \qquad [L=1]$$

$$y(x,0) = \phi(x) \;,\quad \frac{\partial y}{\partial t}(x,0) = 0 \;,\quad x \in [0,L]$$

Primero la forma de $\phi(x)$:

$$(3) \Longrightarrow \underbrace{\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2}}_{Kx} - \frac{\partial^2 \phi}{\partial x^2} = 0$$

$$\Longrightarrow \frac{\partial^2 \phi}{\partial x^2} = Kx \Longrightarrow \phi(x) = Ax^3 + Bx + C$$

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PERO:

$$y(0,t) = \underbrace{u(0,t)}_{0\ (\text{POR } (2))} + \phi(0) = 0$$

$$y(1,t) = \underbrace{u(1,t)}_{0} + \phi(1) = 0$$

$\phi(0) = 0 \Longrightarrow C = 0$, luego $\phi(x) = Ax^3 + Bx$, $A,B \in \mathbb{R}$