

ii) Notemos que  $\hat{N} = \frac{dT}{ds}$  y  $\hat{N} \perp \hat{T}$

$\Rightarrow \nabla F \parallel \hat{N}$  • bien existe  $\alpha(s)$  tal que  $\frac{d\hat{T}}{ds} = \alpha(s) \nabla F$ .  
(0.5)

iii) 
$$\frac{d\hat{T}}{ds} = \frac{1}{\|\nabla F\|} \left( -\frac{\partial^2 F}{\partial x \partial y} x' - \frac{\partial^2 F}{\partial y^2} y', \frac{\partial^2 F}{\partial x^2} x' + \frac{\partial^2 F}{\partial y \partial x} y' \right)$$
  

$$+ \left( -\frac{\partial F}{\partial y}, \frac{\partial F}{\partial x} \right) \cdot \frac{1}{\|\nabla F\|^3} \left[ \frac{\partial F}{\partial x} \left( \frac{\partial^2 F}{\partial x^2} x' + \frac{\partial^2 F}{\partial y \partial x} y' \right) + \frac{\partial F}{\partial y} \left( \frac{\partial^2 F}{\partial x \partial y} x' + \frac{\partial^2 F}{\partial y^2} y' \right) \right]$$

con  $x' = -\frac{\partial F}{\partial y} \cdot \frac{1}{\|\nabla F\|}$ ,  $y' = \frac{\partial F}{\partial x} \cdot \frac{1}{\|\nabla F\|}$  (Δ) (1.5)

iv) Notemos que  $\frac{d\hat{T}}{ds} = \alpha(s) \nabla F$  /  $\cdot \frac{\nabla F}{\|\nabla F\|^2}$

$$\Rightarrow \alpha(s) = \frac{d\hat{T}}{ds} \cdot \frac{\nabla F}{\|\nabla F\|^2}$$

Luego,

$$\alpha(s) = \frac{1}{\|\nabla F\|^3} \left( -\frac{\partial F}{\partial x} \frac{\partial^2 F}{\partial x \partial y} x' - \frac{\partial^2 F}{\partial y^2} y' \frac{\partial F}{\partial x} + x' \frac{\partial F}{\partial y} \frac{\partial^2 F}{\partial x^2} + \frac{\partial F}{\partial y} \frac{\partial^2 F}{\partial x \partial y} y' \right)$$
  

$$+ \underbrace{\left( -\frac{\partial F}{\partial y} \frac{\partial F}{\partial x} + \frac{\partial F}{\partial x} \frac{\partial F}{\partial y} \right)}_0 \cdot \frac{-1}{\|\nabla F\|^5} \cdot (\Delta)$$

Reemplazando  $x'$  e  $y'$ :

$$\alpha(s) = \frac{-1}{\|\nabla F\|^4} \left( \left( \frac{\partial F}{\partial x} \right)^2 \frac{\partial^2 F}{\partial y^2} - 2 \frac{\partial F}{\partial x} \frac{\partial F}{\partial y} \frac{\partial^2 F}{\partial x \partial y} + \left( \frac{\partial F}{\partial y} \right)^2 \frac{\partial^2 F}{\partial x^2} \right)$$

Por ultimo

$$K = \left\| \frac{d\hat{T}}{ds} \right\| = \|\alpha(s) \nabla F\| = |\alpha(s)| \cdot \|\nabla F\|$$

$$\Rightarrow K = \frac{\left| \left( \frac{\partial F}{\partial x} \right)^2 \frac{\partial^2 F}{\partial y^2} - 2 \frac{\partial F}{\partial x} \frac{\partial F}{\partial y} \frac{\partial^2 F}{\partial x \partial y} + \left( \frac{\partial F}{\partial y} \right)^2 \frac{\partial^2 F}{\partial x^2} \right|}{\|\nabla F\|^3}$$
 (1.5)