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# Airline Schedule Planning: Integrated Models and Algorithms for Schedule Design and Fleet Assignment

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Constructing a profitable schedule is of utmost importance to an airline because its profitability is critically influenced by its flight offerings. We focus our attention on the steps of the airline schedule planning process involving schedule design and fleet assignment. Airline schedule design involves determining when and where to offer flights such that profits are maximized, and fleet assignment involves assigning aircraft types to flight legs to maximize revenue and minimize operating cost. We present integrated models and solution algorithms that simultaneously optimize the selection of flight legs for and the assignment of aircraft types to the selected flight legs. Preliminary results, based on data from a major U.S. airline, suggest that significant benefits can be achieved.

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In scheduled passenger air transportation, airline profitability is critically influenced by the airline's ability to construct flight schedules containing flights at desirable times in profitable markets (defined by origin-destination pairs). To produce operational schedules, airlines engage in a complex decision-making process, generally referred to as *airline schedule planning*. Because it is impossible to simultaneously represent and solve the entire airline schedule planning problem, the many decisions required in airline schedule planning have historically been compartmentalized and optimized in a sequential manner.

The *schedule planning* step typically begins 12 months prior to operation of the schedule and lasts approximately 9 months. It begins with *route development*, in which the airline decides which city pairs it wants to serve, based primarily on systemwide demand information. Most of the time, the schedule planning step starts from an existing schedule, with a well-developed route structure. Changes are introduced to the existing schedule to reflect changing demands and environment; this is referred to as *schedule development*. There are three major components in the schedule development step.

The first step of schedule development, *schedule design*, is arguably the most complicated step of all and traditionally has been decomposed into two sequential steps:

(1) *frequency planning*, in which planners determine the appropriate service frequency in a market; and

(2) *timetable development*, in which planners place the proposed services throughout the day, subject to *network considerations* and other constraints.

The purpose of the second step of schedule development, *fleet assignment*, is to assign available aircraft to flight legs such that seating capacity on an assigned aircraft matches closely with flight demand. If too small an aircraft is assigned to a flight, many potential passengers are turned away, or *spilled*, resulting in lost revenue. If too big an aircraft is assigned to a flight, many empty seats, which can be utilized more profitably elsewhere, are flown.

The objective of the third step of schedule development, *aircraft rotation*, is to find a *maintenance feasible rotation* (or routing) of aircraft, given a fleeted schedule and the available number of aircraft of each type. A *maintenance feasible rotation*, a sequence of connected flight legs assigned to a specific aircraft respecting the maintenance rules of the airline and regulatory agencies, begins and ends at the same location.

Subsequent steps in the airline schedule planning process include revenue management and crew scheduling. For a detailed overview, interested readers are referred to Lohatepanont (2001).

Today, advanced technologies and a better understanding of the problems have allowed operations researchers to begin integrating and globally optimizing these decisions. In this paper, we present integrated models and algorithms for *airline schedule design* and *fleet assignment*. The schedule design problem involves determining when and where to offer flights so that profits are maximized, while the fleet assignment problem involves assigning aircraft types to flight legs to maximize revenue and minimize operating costs. Our integrated models simultaneously optimize the process of selecting flight legs to include in the schedule and assigning aircraft types to these legs. We present our computational experiences using data from a major U.S. airline.

Our contributions in this paper include:

(1) a framework for considering demand and supply interactions in the context of airline schedule design;

(2) two new integrated models and solution algorithms for airline schedule design and fleet assignment, namely,

• the integrated schedule design and fleet assignment model and

• the approximate schedule design and fleet assignment model;

(3) a proof of concept of our approach, with limited computational results for problem instances provided by a major U.S. airline.

In §1, we describe the nature of air travel demand, supply, and their interactions in the context of airline schedule design. We give an overview of our integrated models for schedule design and fleet assignment in §2. We present integrated models and algorithms for schedule design and fleet assignment together with computational results in §§3 and 4.

## 1. Demand and Supply Interactions

A crucial element in the construction of an airline schedule is the interaction between demand and supply. In this section, we review relevant literature on this issue. As will be seen, such understandings are essential in developing a flight schedule.

**Demand.** For the purpose of schedule design and fleet assignment, a *market* is defined by an origin and destination pair. For example, Boston–Los Angeles is a market, and Los Angeles–Boston is another distinct market, referred to as an *opposite market*. Boston–San Francisco and Boston–Oakland are examples of *parallel markets* because San Francisco and Oakland are close enough to each other that passengers are often indifferent as to which location is preferable.

For the purpose of schedule design for a given airline, we are interested in its *unconstrained market demand* (or more succinctly, market demand), that is, the maximum demand the airline is able to capture. Unconstrained market demand is allocated to *itineraries*, sequences of connecting flight legs, in each market to determine *unconstrained itinerary demand*. The term *unconstrained* is used to denote that the quantity of interest is measured or computed without taking into account capacity restrictions. For example, *unconstrained revenue* is the maximum revenue attainable by an airline, based only on unconstrained demand and not on capacity offered; while *constrained revenue* is the achievable revenue subject to capacity constraints.

Simpson and Belobaba (1992a) illustrate that the unconstrained market demand for a carrier is affected by its flight schedule (with frequency of service being one critical element). They also show that *total* market demand can change as a result of changes in the flight schedule. Specifically, additional demand can be *stimulated* or, more precisely, diverted from other modes of transportation when the number of itineraries/flights (that is, supply) is increased (given that the demand has not yet reached the maximum demand) and vice versa. It is also true that supply is a function of demand: The carrier purposefully designs its schedule to capture the largest demands.

There are a number of demand forecasting models available. Most of these models are logit based (Ben Akiva and Lerman 1985), while some are *Quality Service Index* (*QSI*) based (as described in §3).

**Supply.** The airline develops its flight network to compete for market demand. The first step in developing the flight network is to adopt an appropriate network structure. Unlike other modes of transportation in which the routes are restricted by geography, most of the time the route structures for air transportation are more flexible. Simpson and Belobaba (1992b) present three basic network structures, namely, *a linear network*, *a point-to-point* (*complete*) *network*, and *a hub-and-spoke network*.

A pure linear network is one in which all flight legs connect all cities in one large uni- or bi-directional tour. A pure point-to-point network is a complete network, where there are services from every city to every other city. A hub-and-spoke network contains at least one hub airport. All spoke cities have flights departing from and arriving to the hub airport(s). This is the most prevalent network type for most major U.S. carriers. Its main advantage derives from connecting opportunities at the hub airport(s), enabling airlines to consolidate demand from several markets onto each flight. Simpson and Belobaba (1992b) also note that the hub-and-spoke network structure creates more stable demand at the flight-leg level. By mixing and consolidating demands from different markets on each flight leg, the hub-and-spoke network can reduce variations in the number of passengers at the flight-leg level because markets have different demand distributions.

**Demand and Supply Interactions.** Hub-and-spoke networks illustrate demand and supply interactions.

To see this, consider removing a flight leg from a *connecting bank or complex*. A bank or complex is a set of flights arriving or departing a hub airport in some period of time. Banks are typically designed with a set of flight arrivals to the hub followed by a sequence of departures from the hub to facilitate passenger connections. The removal of a flight from a hub can have serious ramifications on passengers in many markets throughout the network. This occurs because in addition to carrying local passengers from the flight's origin to its destination, the removed flight carries a significant number of passengers from many other markets containing that leg. From the viewpoint of the passengers in those markets, the quality of service is deteriorated because the frequency of service is decreased. The result can then be that the carrier experiences a decrease in its unconstrained market demands in the affected markets. The situation is the opposite when a flight leg is added to the bank. For air transportation, Teodorovic and Krcmar-Nozic (1989) show that flight frequencies and departure times are among the most important factors that determine passengers' choices of an air carrier when there is a high level of competition.

**An Example of Demand and Supply Interactions.** In this section, we present an example of how changing the flight schedule of an individual airline can affect demand for that airline. Consider the example in Figure 1(a), a market with four itineraries. Let the first itinerary be in the morning and the last three itineraries be in the afternoon.

Above each figure, the airline's market demand is specified for the case when all itineraries in the figure are flown. These market demands are unconstrained, which can be allocated to all itineraries of the airline as shown. Specifically, in Figure 1(a), each itinerary has 100 requests except the second itinerary, which has 150 requests. (We can view the second itinerary as a nonstop itinerary and the rest as connecting itineraries.)

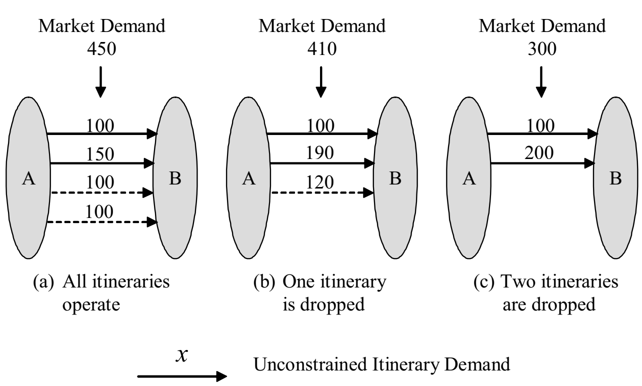


**Figure 1** **An Example of Demand and Supply Interactions**

In Figure 1(b), we assume that one of the optional flights is deleted and, as a result, the last itinerary no longer exists. Based on this new level of service, the airline market demand reduces from 450 requests in Figure 1(a) to 410 requests in Figure 1(b). These 410 requests can be allocated to the remaining three itineraries as shown in Figure 1(b). Specifically, some of the 100 potential customers previously on the deleted itinerary will go to other airlines, and some will remain with the airline. Those that remain with the airline will request other itineraries still in the schedule (Itineraries 1–3 in this case). We assume that 40 and 20 of the 100 requests previously on the fourth itinerary will request the second and third itineraries, respectively. The first itinerary does not receive any additional requests because it is in the morning and the 100 passengers prefer itineraries that depart close to the time (the afternoon) of their original itinerary. The second itinerary receives more requests than the third because the former is nonstop while the latter is connecting.

In Figure 1(c), we assume that two itineraries are deleted. The same phenomenon occurs. Demand of the airline reduces from 450 requests to 300 requests as a result of deleting two optional flight legs. The lost requests when two itineraries are deleted more than double those when only one itinerary is deleted, illustrating the nonlinear relationship between demand and supply.

## 2. Integrated Models for Schedule Design and Fleet Assignment: An Overview

Previous efforts to improve profitability of airline schedules are described in, for example, Soumis et al. (1980), Dobson and Lederer (1993), Marsten et al. (1996), Berge (1994), Erdmann et al. (1999), and for a survey: Etschmaier and Mathaisel (1984).

Marsten et al. (1996) present a framework for incremental schedule design. In their approach, they enumerate potential combinations of flight additions and deletions. Given a schedule, demands are estimated using a schedule evaluation model. Then, the fleet assignment problem is solved on the given schedule with the corresponding estimated demands. Revenues are computed based on the passengers transported, and costs are computed based on flight operating costs of the fleeted schedule. The profits from several sets of proposed additions and deletions are then compared to identify the best set. In this paper, we present an alternative approach, in which flight-leg selection and fleet assignment are simultaneously optimized.

### 2.1. Framework

In developing our integrated models for airline schedule design and fleet assignment, we assume our schedule is daily; that is, the schedule repeats everyday. Because conservation of aircraft, or balance, is always maintained, we can count the number of aircraft in the network by taking a snapshot of the network at a prespecified point in time, defined as the *count time* (for example, 3:00 a.m.) and counting the number of aircraft both in the air and on the ground at stations.

Our approach to schedule design is incremental; that is, we do not attempt to build a schedule from scratch. Instead a number of modifications are introduced to a *base schedule*, that is, a schedule from the current or previous season. This is, in fact, the practice in the industry—planners build a schedule for the new season by making changes to the current schedule. There are a few reasons for this:

(1) Building an entirely new schedule requires data which might or might not be available to the airline;

(2) building an improved new schedule from scratch is operationally impractical and computationally difficult, if not intractable;

(3) frequently changing network structures require significant investment at airport stations (for example, gates, check-in counters); and

(4) airlines prefer a degree of consistency from one season to the next, especially in business markets in which reliability and consistency are highly valued by travelers.

By building the next season's schedule from the previous one, (1) airlines are able to use historical booking and other important traffic forecast data; (2) required planning efforts and time are manageable; (3) fixed investments at stations can be utilized efficiently; and lastly, (4) consistency can be easily maintained by introducing a limited number of changes to the schedule.

Our models take as input a *master flight list*, that is, a list of flight legs composed of: (1) mandatory flights and (2) optional flights.

*Mandatory flights* are flight legs that have to be included in the flight schedule, while *optional flights* are flight legs that may, but need not, be included. Figure 2 depicts the construction of a master flight list. We take as our starting point a *base schedule* (Figure 2a), which is often the schedule from the previous season. The potential modifications include:

(1) Existing flight legs in the base schedule can be deleted (Figure 2b), and

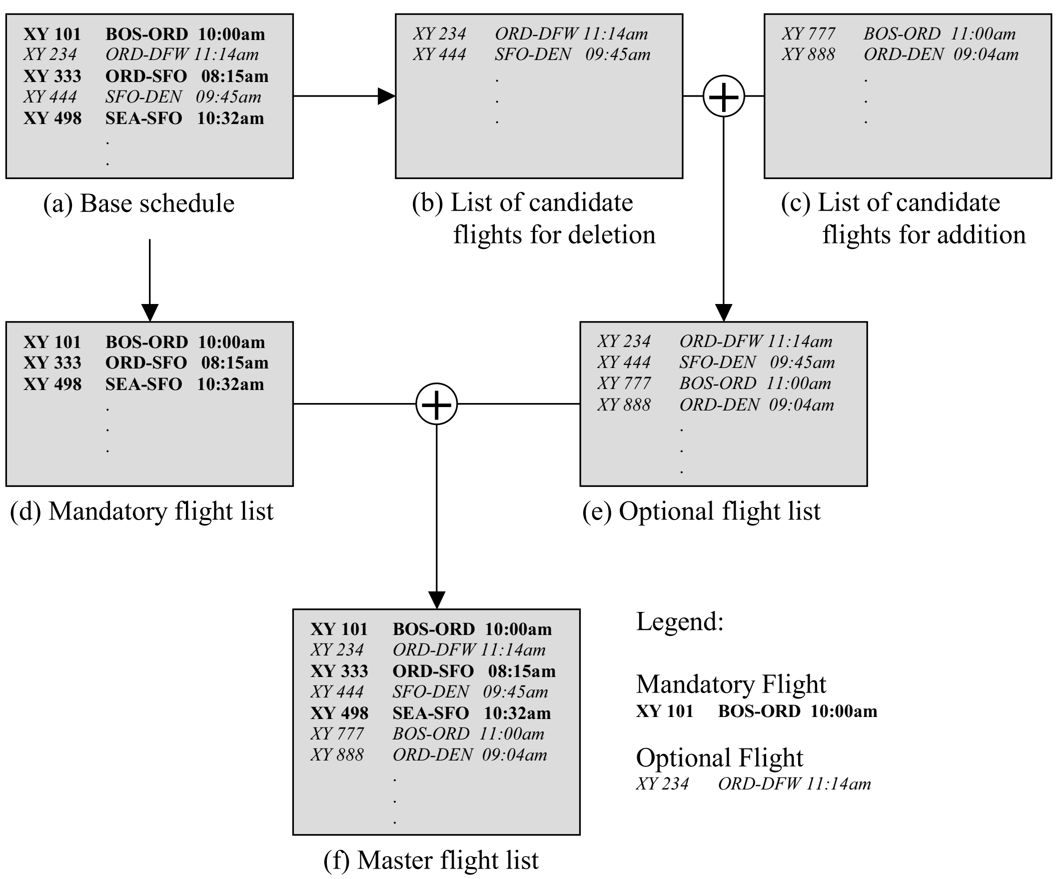


**Figure 2** **Master Flight List Construction Diagram**

(2) new flight legs can be added to the base schedule (Figure 2c).

Flights identified in Figures 2(b) and 2(c) comprise the optional flight list (Figure 2e). The mandatory list contains those flight legs in the base schedule not marked as optional (Figure 2d). The master flight list (Figure 2f) is the combination of the mandatory and optional flight lists. A master flight list must be prepared by planners for input to our integrated models.

Apart from a master flight list, another crucial input is the *average unconstrained itinerary demands* associated with operating *all* of the flights in the master flight list. We estimate these demands for a given schedule using a *schedule evaluation model*. There are several schedule evaluation models used in the industry, for example, the Sabre® Airline Profitability Model, and United Airlines' Profitability Forecasting Model. Details of these models, however, are often difficult to obtain due to their proprietary nature.

The output of our models is a list of recommended flight legs to include in a new schedule and an associated fleet assignment.

Our models integrate the schedule design and fleet assignment steps in the schedule development subprocess. Notice, also, that we depart from the traditional approach of sequentially performing frequency planning and timetable development. In our approach, market service frequency, departure times (given a prespecified list of candidate flight departures), and fleet assignments are all determined simultaneously.

## 2.2. Underlying Assumptions

**2.2.1. Demand Issues and Assumptions.** There are a number of assumptions underlying most fleeting models, namely:

(1) *Fare Class Differentiation*: Most fleeting models require that different fare class demands be aggregated into one fare class. Kniker (1998) shows that demands differentiated by fare class can be utilized in fleet assignment models. To our knowledge, however, there have been no attempts to solve fare class differentiated fleet assignment models on large-scale problems.

(2) *Demand Variation*: Because fleeting models typically solve the *daily* problem (that is, the problem in which the schedule is assumed to repeat each day of the week), a *representative daily demand* is input into the fleet assignment model. This requires that demands from different days of the week (demand varies by day of week) must be aggregated into a representative demand.

(3) *Observed vs. Unconstrained Demand*: Fleet assignment models take as input some form of *unconstrained demand*, that is, the maximum demand for air travel that an airline can experience regardless of the airline's network capacity. All of the observed demand data, however, is *constrained demand*, reflecting network capacity. Hence, in practice, the demand distribution for each leg is truncated at the capacity of the fleet type assigned to that leg. Therefore, using assumptions about demand distributions, these observed data must be "unconstrained."

(4) *Demand Recapture*: Recapture occurs when passengers spilled from their desired itineraries are accommodated on alternate itineraries in the system. Most fleet assignment models ignore recapture, partly because it is difficult to observe. In this paper, we employ the approach described in Barnhart et al. (2001) to model recapture in the fleeting process.

(5) *Flight-Leg Independence*: Most fleet assignment models assume *flight-leg independence*, that is, that demand on one leg is independent of all other legs and moreover, that flight revenue is leg specific. A significant proportion of passengers, especially in the U.S., travel on multileg itineraries, thus creating flight-leg interdependencies. These interdependencies introduce revenue estimation errors in the fleet assignment models assuming leg independence. Barnhart et al. (2001) discuss this assumption and its ramifications and provide quantitative evidence that fleet assignment models assuming independence miss an important aspect of the fleet assignment problem. To capture flight-leg interdependencies, or network effects, they introduce an itinerary-based fleet assignment approach, which we use in our integrated schedule planning models.

Barnhart et al. (2001) test the effects of several of the above assumptions using their *itinerary-based fleet assignment model* (*IFAM*). They show that using average demand data, ignoring the associated uncertainty (or distribution), and accounting for network effects, IFAM produces better assignments than the basic (leg-based) fleet assignment models. Their experiments also show that IFAM produces relatively consistent fleetings over a limited range of recapture values. Hence, IFAM is the core around which we build the integrated schedule planning model presented in this paper.

## 2.3. Notation

To facilitate the discussion of our integrated model, we first list all notation in this section.

**Parameters/Data**

$CAP_i$: the number of seats available on flight-leg $i$ (assuming fleeted schedule).

$SEATS_k$: the number of seats available on aircraft of fleet type $k$.

$N_k$: the number of aircraft in fleet-type $k$.

$N_q$: the number of flight legs in itinerary $q$.

$D_p$: the unconstrained demand for itinerary $p$, that is, the number of passengers requesting itinerary $p$.

$Q_i$: the unconstrained demand on leg $i$ when all itineraries are flown.

$fare_p$: the fare for itinerary $p$.

$\widetilde{fare}_p$: the carrying cost-adjusted fare for itinerary $p$.

$b_p^r$: recapture rate from $p$ to $r$; the fraction of passengers spilled from itinerary $p$ that the airline succeeds in redirecting to itinerary $r$.

$$\delta_i^p := \begin{cases} 1 & \text{if itinerary } p \text{ includes flight leg } i; \\ 0 & \text{otherwise.} \end{cases}$$

$\Delta D_q^p$: demand correction term for itinerary $p$ as a result of cancelling itinerary $q$.

**Sets**

$P$: the set of itineraries in a market including the null itinerary, indexed by $p$ or $r$.

$P^O$: the set of itineraries containing optional flight legs, indexed by $q$.

$A$: the set of airports, or stations, indexed by $o$.

$L$: the set of flight legs in the flight schedule, indexed by $i$.

$L^F$: the set of mandatory flights, indexed by $i$.

$L^O$: the set of optional flights, indexed by $i$.

$K$: the set of fleet types, indexed by $k$.

$T$: the sorted set of all event (departure or availability) times at all airports, indexed by $t_j$. The event at time $t_j$ occurs before the event at time $t_{j+1}$. Suppose $|T| = m$; then $t_1$ is the time associated with the first event after the count time and $t_m$ is the time associated with the last event before the next count time.

$N$: the set of nodes in the timeline network indexed by $\{k, o, t_j\}$.

$CL(k)$: the set of flight legs that pass the count time when flown by fleet-type $k$.

$I(k, o, t)$: the set of inbound flight legs to node $\{k, o, t_j\}$.

$O(k, o, t)$: the set of outbound flight legs from node $\{k, o, t_j\}$.

$L(q)$: the set of flight legs in itinerary $q$.

**Decision Variables**

$t_p^r$: the number of passengers requesting itinerary $p$ that the airline attempts to redirect to itinerary $r$.

$$f_{k,i} := \begin{cases} 1 & \text{if flight leg } i \text{ is assigned to fleet type } k; \\ 0 & \text{otherwise.} \end{cases}$$

$$Z_q := \begin{cases} 1 & \text{if itinerary } q \text{ is included in the flight network;} \\ 0 & \text{otherwise.} \end{cases}.$$

$y_{k,o,t_j^+}$: the number of fleet-type $k$ aircraft that are on the ground at airport $o$ immediately after time $t_j$.

$y_{k,o,t_j^-}$: the number of fleet-type $k$ aircraft that are on the ground at airport $o$ immediately before time $t_j$. If $t_1$ and $t_2$ are the times associated with adjacent events, then $y_{k,o,t_1^+} = y_{k,o,t_2^-}$.

# 3. Integrated Schedule Design and Fleet Assignment Model

## 3.1. Demand Correction Terms

In this section we present, the *integrated schedule design and fleet assignment model* (*ISD-FAM*). ISD-FAM is built upon the itinerary-based fleet assignment model (IFAM) by Barnhart et al. (2001). We assume that markets are independent of one another; that is, demands in a market do not interact with demands in any other markets. This enables us to adjust demand for each market only if the schedule for that market is altered. ISD-FAM adjusts demand as changes are made to the schedule, using *demand correction terms*. For example, the demand correction term $\Delta D_q^p$ corrects demand on itinerary $p$ when itinerary $q$ is deleted (that is, one or more flight legs in $q$ are deleted) from the flight network. Consider the example in Figure 3. Suppose that the first and second itineraries are *mandatory itineraries*; that is, they contain only mandatory flights. The third and fourth itineraries are *optional itineraries*; that is, each of these itineraries contains at least one optional flight. Suppose that the fourth itinerary in Figure 3(b) is deleted. Then, the increase of 40 passengers on the second itinerary and 20 passengers on the third itinerary represent the demand correction terms on the second and third itineraries, respectively. Mathematically, $\Delta D_4^2 = +40$, $\Delta D_4^3 = +20$, and $\Delta D_4^1 = 0$. Thus, our demand correction terms can capture the changes in itinerary demands when one optional itinerary is deleted in a market. Assuming that the effect is the same if only the third itinerary is deleted, then $\Delta D_3^1 = 0$, $\Delta D_3^2 = +40$, and $\Delta D_3^4 = +20$. When the third and fourth itineraries are deleted simultaneously, the total demand increase on the first itinerary is estimated to be $\Delta D_3^1 + \Delta D_4^1 = 0$ and on the second itinerary is estimated to be $\Delta D_3^2 + \Delta D_4^2 = +80$. The resulting market demand is estimated to be 330, however, the actual market demand when two itineraries are deleted is 300. To estimate these new demands accurately, a *second-order degree correction*

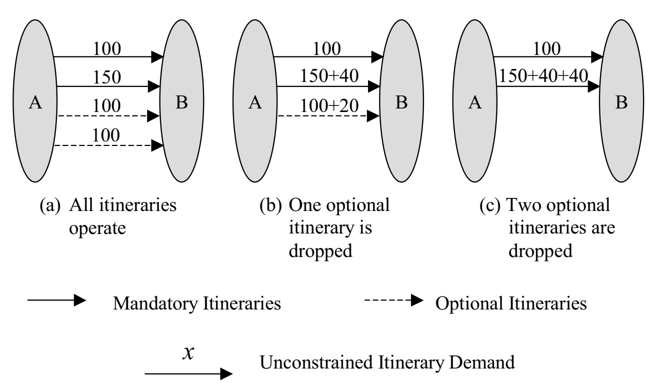


**Figure 3** Our Approach for Capturing Demand and Supply Interactions

*term*, $\Delta D_{3,4}^2 = -30$, is needed to adjust the demand on the second itinerary when both the third and fourth itineraries are deleted. In general, *n*th *degree correction terms* might be needed to correct demand exactly when *n* optional itineraries are deleted simultaneously from a market. In our current implementation, however, these higher-order correction terms are omitted for tractability.

In our ISD-FAM formulation, demand correction terms, $\Delta D_q^p$, $\forall p \in P$ are applied only if itinerary $q$ is deleted from the flight network. $Z_q$, the *itinerary status variable*, indicates whether or not itinerary $q$ is operated, with $Z_q$ equal to one if itinerary $q$ is operated and zero otherwise. Notice that itinerary $q$ is operated only when all flight legs contained in $q$ are operated.

### 3.2. Objective Function

In this formulation, average unconstrained itinerary demands are computed with a schedule evaluation model, for the schedule with *all* optional flights flown. The objective of ISD-FAM is to maximize *schedule contribution*, defined as *revenue* generated less *operating cost* incurred. The operating cost of a schedule, denoted **O**, can be computed as $\sum_{i\in L}\sum_{k\in K} C_{k,i} f_{k,i}$ for all fleet-flight assignments. The total revenue of a schedule can be computed from the following components:

(1) *Initial unconstrained revenue* (**R**):

$$\mathbf{R} = \sum_{p\in P} fare_p D_p. \tag{1}$$

(2) *Changes in unconstrained revenue due to market demand changes* ($\Delta\mathbf{R}$):

$$\Delta\mathbf{R} = \sum_{q\in P^O}\left(fare_q D_q - \sum_{p\in P:\, p\neq q} fare_p \Delta D_q^p\right)\cdot(1 - Z_q). \tag{2}$$

(3) *Lost revenue due to spill* (**S**):

$$\mathbf{S} = \sum_{p\in P}\sum_{r\in P} fare_p t_p^r. \tag{3}$$

(4) *Recaptured revenue from recapturing spilled passengers* (**M**):

$$\mathbf{M} = \sum_{p\in P}\sum_{r\in P} b_p^r fare_r t_p^r. \tag{4}$$

Equation (1) is the initial unconstrained revenue for the schedule containing all optional legs and the associated unconstrained demands. The term $fare_q D_q$ in Equation (2) is the total unconstrained revenue of itinerary $q$. The term $\sum_{p\in P:\, p\neq q} fare_p \Delta D_q^p$ is the total change in unconstrained revenue on all other itineraries $p$ ($\neq q$) in the same market due to deletion of itinerary $q$. Recall that $Z_q$ equals one if $q$ is flown and zero otherwise. Thus, Equation (2) is the change in unconstrained revenue if itinerary $q$ is deleted.

Equations (3) and (4) measure the changes in revenue due to spill and *recapture*, respectively. Recall that spill occurs when passengers cannot be accommodated on their desired itineraries due to insufficient capacity. Some of these passengers are redirected to alternate itineraries within the airline network. Recapture occurs when some number of these redirected passengers are accommodated on alternate itineraries of that airline. In our work, the recapture rate ($b_p^r$), the successful rate of redirecting passengers to itinerary $r$ when itinerary $p$ is capacitated, is computed based on the *Quality of Service Index* (*QSI*) (Kniker 1998). QSI is an industry measure of the "attractiveness" of an itinerary relative to that of all other itineraries (including competing airlines) in that market. The sum of the QSI values corresponding to all itineraries (including competitors) in a market is equal to one. The sum of the QSI for one airline is an approximate measure of its market share for that specific market. Let $q_p$ denote the QSI value of itinerary $p$. Let $Q_m$ represent the sum of all QSI values in market $m$ for the particular airline; that is, $Q_m = \sum_{p\in m} q_p$. Then, the *recapture rate*, $b_p^r$ is

$$b_p^r = \begin{cases} 1.0 & \text{if } p = r; \\ \dfrac{q_r}{1 - Q_m + q_r} & \text{otherwise.} \end{cases} \tag{5}$$

This recapture rate is a measure of the probability that a passenger spilled from itinerary $p$ will accept itinerary $r$ as an alternative. (For more details, see Barnhart et al. 2001.)

Mathematically, $t_p^r$ is the number of passengers being redirected from itinerary $p$ to itinerary $r$ and $b_p^r t_p^r$ is the number of passengers who are recaptured from itinerary $p$ onto itinerary $r$. We denote $t_p^-$ as spill from itinerary $p$ to a *null itinerary* and assign its associated recapture rate, $b_p^-$, a value of one. Passengers spilled from itinerary $p$ onto a null itinerary are not recaptured on any other itinerary of the airline and are *lost* to the airline.

The contribution maximizing objective function of ISD-FAM is therefore

$$\text{Max}\,\mathbf{R} - \Delta\mathbf{R} - \mathbf{S} + \mathbf{M} - \mathbf{O}. \tag{6}$$

Given unconstrained demands for the schedule, the initial unconstrained revenue (**R**) is a constant and we can remove it from the objective function. If we reverse the signs of the rest of the terms in Equation (6), we obtain an equivalent cost-minimizing objective function

$$\text{Min}\,\mathbf{O} + (\mathbf{S} - \mathbf{M}) + \Delta\mathbf{R}. \tag{7}$$

Additional cost items that can be included are passenger-related costs, which are composed of, but not limited to,

(1) passenger carrying costs (for example, meals, luggage handling) and

(2) cost per revenue dollar (for example, reservation commission).

These cost items can be incorporated into the model by deducting them from the revenue (or fare) obtained from a passenger. Thus, in the model, instead of using $fare_p$ to denote revenue obtained from a passenger traveling on itinerary $p$, we use $\widetilde{fare}_p$, the *net revenue* from a passenger traveling on itinerary $p$.

### 3.3. Formulation

ISD-FAM can be formulated as shown in (8)–(20):

$$\text{Min} \sum_{i\in L}\sum_{k\in K} C_{k,i} f_{k,i} + \sum_{p\in P}\sum_{r\in P} (\widetilde{fare}_p - b_p^r \widetilde{fare}_r) t_p^r + \sum_{q\in P^O} \left( \widetilde{fare}_q D_q - \sum_{p\in P: p\neq q} \widetilde{fare}_p \, \Delta D_q^p \right) \cdot (1 - Z_q) \tag{8}$$

subject to

$$\sum_{k\in K} f_{k,i} = 1 \quad \forall i \in L^F, \tag{9}$$

$$\sum_{k\in K} f_{k,i} \le 1 \quad \forall i \in L^O, \tag{10}$$

$$y_{k,o,t^-} + \sum_{i\in I(k,o,t)} f_{k,i} - y_{k,o,t^+} - \sum_{i\in O(k,o,t)} f_{k,i} = 0 \quad \forall \{k,o,t\} \in N, \tag{11}$$

$$\sum_{o\in A} y_{k,o,t_n} + \sum_{i\in CL(k)} f_{k,i} \le N_k \quad \forall k \in K, \tag{12}$$

$$\sum_{p\in P}\sum_{q\in P^O} \delta_i^p \Delta D_q^p (1 - Z_q) + \sum_{k\in K} CAP^k f_{k,i} + \sum_{r\in P}\sum_{p\in P} \delta_i^p t_p^r - \sum_{r\in P}\sum_{p\in P} \delta_i^p b_r^p t_r^p \ge Q_i \quad \forall i \in L, \tag{13}$$

$$\sum_{q\in P^O} \Delta D_q^p (1 - Z_q) + \sum_{r\in P} t_p^r \le D_p \quad \forall p \in P, \tag{14}$$

$$Z_q - \sum_{k\in K} f_{k,i} \le 0 \quad \forall i \in L(q), \tag{15}$$

$$Z_q - \sum_{i\in L(q)}\sum_{k\in K} f_{k,i} \ge 1 - N_q \quad \forall q \in P^O, \tag{16}$$

$$f_{k,i} \in \{0,1\} \quad \forall k \in K, \forall i \in L, \tag{17}$$

$$Z_q \in \{0,1\} \quad \forall q \in P^O, \tag{18}$$

$$y_{k,o,t} \ge 0 \quad \forall \{k,o,t\} \in N, \tag{19}$$

$$t_p^r \ge 0 \quad \forall p, r \in P. \tag{20}$$

Constraints (9), the cover constraints for mandatory flights, ensure that every mandatory flight is assigned to a fleet type. Constraints (10) are cover constraints for optional flights allowing the model to choose whether or not to fly flight $i$ in the schedule. If flight $i$ is selected, a fleet type has to be assigned to it. Constraints (11) ensure conservation of aircraft flows. Constraints (12) are count constraints guaranteeing that only available aircraft are used. Constraints (13) are capacity constraints ensuring for each flight $i$ that the number of passengers on $i$ does not exceed the capacity assigned to $i$. Constraints (14) are demand constraints restricting the number of passengers spilled from an itinerary to the demand for that itinerary. The term $\sum_{q\in P^O} \Delta D_q^p (1 - Z_q)$ in constraints (14) corrects the unconstrained demand for itinerary $p \in P$ when optional itineraries $q \in P^O$ are deleted. Similarly, the term $\sum_{p\in P}\sum_{q\in P^O} \delta_i^p \Delta D_q^p (1 - Z_q)$ in constraints (13) represents corrected demand but at the flight level. Constraints (15)–(16) are *itinerary status constraints* that control the $\{0,1\}$ variable, $Z_q$, for itinerary $q$. Specifically, constraints (15) ensure that $Z_q$ takes on value zero if at least one leg in $q$ is not flown, and constraints (16) ensure that $Z_q$ takes on value one if all legs in $q$ are flown.

**Passenger Flow Adjustment.** Notice that ISD-FAM employs two mechanisms to adjust passenger flows: (1) demand correction terms and (2) recapture rates.

Both mechanisms accomplish the objective of reaccommodating passengers on alternate itineraries when desired itineraries are not available, but with different underlying assumptions. To illustrate, consider itineraries $p \in P$, $q \in P^O$, and $r \in P$ in a market $m$. With demand correction terms, ISD-FAM attempts to capture demand and supply interactions by adjusting the unconstrained demand on alternate itineraries $p \in P$ in market $m$ when an optional itinerary $q \in P^O$ ($p \neq q$) is deleted, utilizing demand correction terms $\Delta D_q^p$'s. In so doing, the total unconstrained demand of the airline in market $m$ is altered by $D_q - \sum_{p\in P: p\neq q} \Delta D_q^p$ when itinerary $q$ is deleted.

With recapture rates, ISD-FAM attempts to reallocate passengers on alternate itineraries $r \in P$ in market $m$ when an itinerary $p \in P$ is capacitated, utilizing the recapture rates $b_p^r$'s. This mechanism, however, does not affect the total unconstrained demand of the airline in market $m$.

An understanding of this difference is important to the introduction of our model in the next section.

### 3.4. Solution Approach

ISD-FAM takes as input the master flight list, recapture rates, demand data, demand correction terms, and fleet composition and size. In theory, using a schedule evaluation model, we could estimate exactly all correction terms and include all of them in ISD-FAM. Then, by solving ISD-FAM once, an optimal schedule is determined. This strategy is impractical, however, because exponentially many runs of the schedule evaluation model (one run for each possible combination of flight additions and deletions) are necessary to estimate the correction terms.

Consequently, we adopt the solution algorithm, outlined in Figure 4. Instead of trying to estimate all demand correction terms at the outset, we use rough estimates of these terms and revise them iteratively as needed. Specifically, we initially obtain demand estimates for the full schedule (containing all flights from the master flight list) using a schedule evaluation model. In Step 1 of our approach in Figure 4, we solve ISD-FAM (detailed in §3.4.1) to obtain a fleeted schedule. In Step 2, a schedule evaluation model is called to determine the new set of demands for the schedule resulting from Step 1. Given these demand estimates and the fleeted schedule, in Step 3 we solve the *passenger mix model* (*PMM*), presented by Kniker (1998). (The PMM formulation, cast as a multicommodity flow problem with specialized variables is detailed in the Appendix.) To solve PMM, column and row generation techniques are used to manage problem size. PMM takes as input a fleeted schedule and deterministic unconstrained itinerary demands and outputs a revenue-maximizing flow of passengers. The objective function value of the model is referred to as *PMM revenue*. It represents a maximum, not expected, schedule

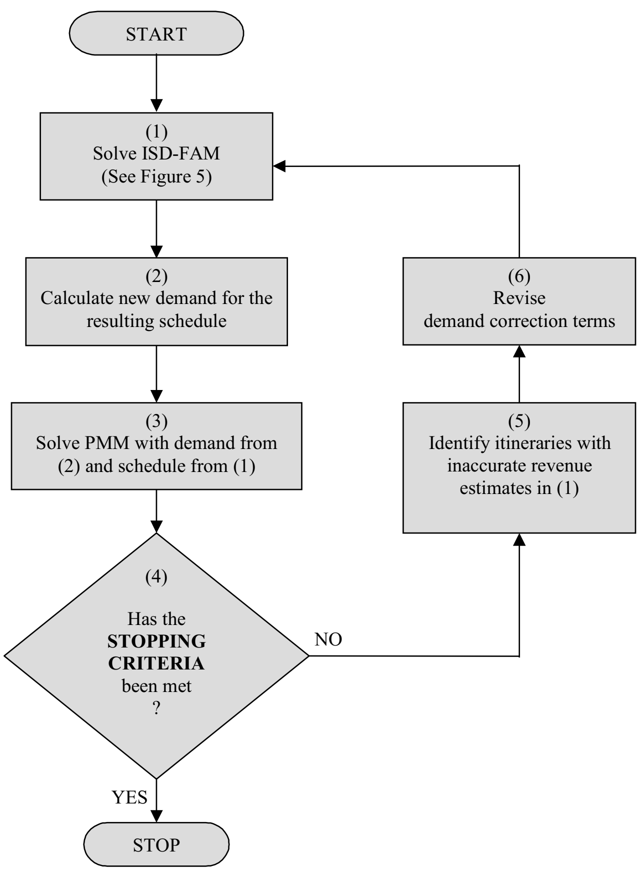


**Figure 4** The Solution Approach for ISD-FAM, Estimating Demand Correction Terms as Needed

revenue (the underlying assumptions do not always hold). In Step 4, we compute:

(1) the difference between our current and previous solution and

(2) the closeness of the revenue estimates of Steps 1 and 3.

If either of these computed values are beneath our defined threshold values, the algorithm is terminated; otherwise itineraries with inaccurate revenue estimates in the ISD-FAM solution are identified in Step 5 and their associated demand correction terms are updated (in Step 6) using the demand information obtained from solving the schedule evaluation model in Step 2. ISD-FAM is then resolved and the procedure repeats.

At every iteration, however, existing demand correction terms are updated and/or new ones are introduced, hence, capturing more exactly demand and supply interactions. Thus, if higher-order correction terms are included, the algorithm will converge in a finite number of iterations. This, however, can take many iterations if the model's solution is highly sensitive to the values of the demand correction terms.

**3.4.1. Solution Algorithm for ISD-FAM.** The algorithm for solving ISD-FAM is depicted in Figure 5. We first construct a *restricted master problem* (*RMP*) excluding constraints (14) and including only spill variables corresponding to null itineraries. Then, the LP relaxation of the RMP is solved using column and row generation. Negative reduced-cost columns corresponding to spill variables and violated constraints (14) are added to the RMP, and the RMP is resolved until the ISD-FAM LP relaxation is solved. Given the solution to the ISD-FAM LP relaxation, branch-and-bound is invoked to find an integer solution. Because column generation at nodes within the branch-and-bound tree is nontrivial to implement using available optimization software, we employ a heuristic IP solution approach in which branch-and-bound allows column and row generation only at the root node and after the integer solution is found. Row (and column) generation occurs after an integer solution is obtained if any constraints are violated by the current solution. In this case, column and row generation allows the best passenger flow decisions to be determined, for the selected flight schedule and fleeting.

### 3.5. Computational Results

We perform computational tests for ISD-FAM as outlined in Figure 6. Using a Period I schedule as our base schedule, planners provide a master flight list including both mandatory and optional flight legs. Using this master flight list, planners generate a proposed schedule for Period II, the *planners' schedule*,

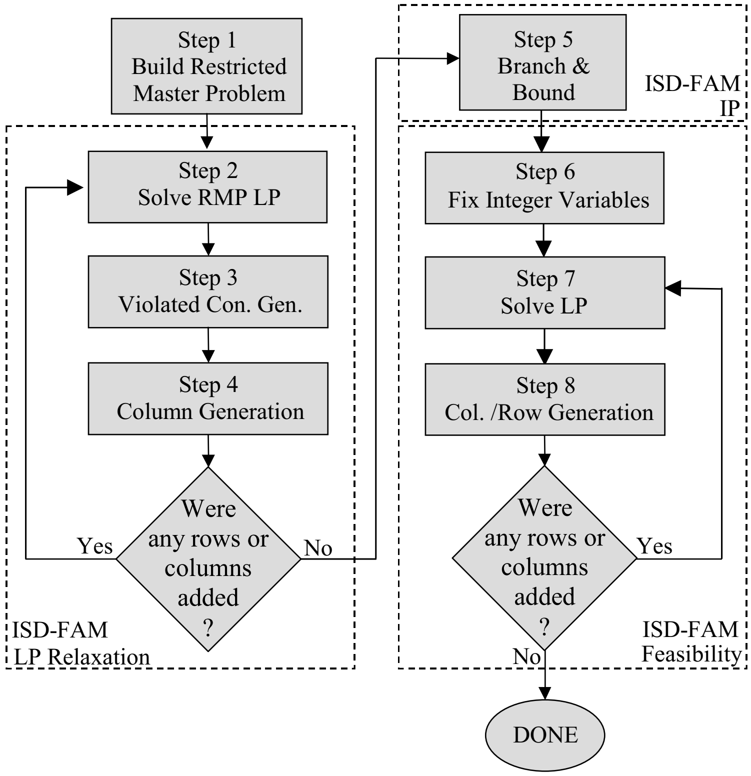


**Figure 5** **The ISD-FAM Solution Algorithm (Step 1 in Figure 4) for Specified Demand Correction Terms**

which is compared to the ISD-FAM-generated schedule. A schedule evaluation model is used to generate a new set of demands for both the planners' schedule and the ISD-FAM-generated schedule. The PMM is then solved to obtain PMM revenues for the Period II schedules. Finally, *PMM contributions* are computed by subtracting operating costs from PMM revenues. In most cases, PMM contributions for an ISD-FAM-generated schedule differs from the ISD-FAM objective function value, called *ISD-FAM contribution*. The difference between PMM and ISD-FAM contributions for a schedule measures the *revenue discrepancy* resulting from using demand estimates based on a flight schedule that does not match the resulting ISD-FAM flight schedule.

We perform our experiments on actual data, including the planners' schedules, provided by a major U.S. airline. Table 1 shows the characteristics of the networks in our two data sets. All runs are performed on an HP C-3000 workstation computer with two GB RAM, running CPLEX 6.5.

Tables 2 and 3 show the problem sizes and ISD-FAM run-times, and solution results for data sets D1 and D2, respectively. For data set D1, ISD-FAM generates a considerably improved schedule compared to the planners' schedules, achieving an increase in the daily contribution of $561,776. Assuming that unconstrained demand is an average of daily demands and that the airline operates this schedule 365 days, these daily improvements translate to over $200 million per year. The revenue discrepancy for data set D1 is $187,263 per day, suggesting that more iterations of the algorithm, leading to more accurate estimates of demand, could result in even greater improvements.

For data set D1, ISD-FAM achieves significant improvements through savings in operating costs (from operating fewer flights) rather than by generating higher revenues (Table 4). Additionally, the ISD-FAM solution requires significantly fewer aircraft to operate the schedule. Each aircraft removed from the schedule can represent substantial savings to the airline because the airline can increase profitability by employing the aircraft elsewhere, such as in new markets.

Although ISD-FAM captures interactions between demand and supply through demand correction terms, it suffers from tractability issues for larger size problems. For data set D2, while the LP relaxation could be solved in 66 hours (46 iterations of column and row generations), ISD-FAM ran for three plus days without achieving an integer solution. In the next section, we explore an approximate schedule design model that does not continuously adjust demand as the schedule changes.

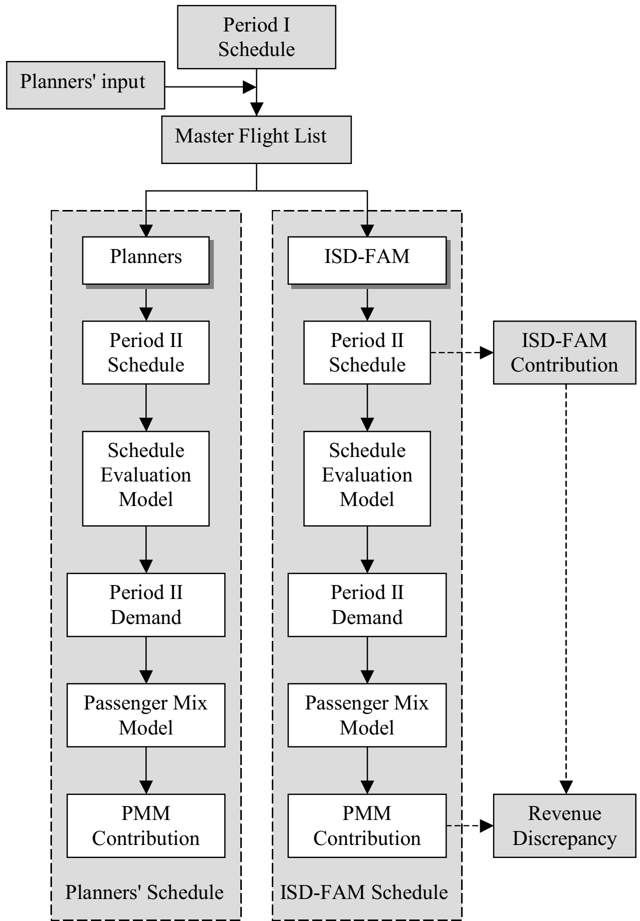


**Figure 6** **Testing Methodology**

# 4. Approximate Schedule Design and Fleet Assignment Model

## 4.1. An Approximate Treatment of Demand and Supply Interactions

In this section, we present the fundamental concept, model formulation, and solution algorithm of our *approximate schedule design and fleet assignment model* (*ASD-FAM*). ASD-FAM utilizes recapture rates to approximately capture the interactions between demand and supply. We demonstrate that although recapture rates do not alter total unconstrained demand, capacity constraints on other itineraries and recapture rates indirectly dictate the maximum number of passengers the airline can reaccommodate within the system.

To illustrate, consider Market A–B in Figure 7. Suppose that there are two nonstop flights, $i$ and $j$, and two one-leg itineraries, $p$ and $r$, where $p$ is on $i$ and $r$ is on $j$. Thus, $t_p^r$ denotes the number of passengers redirected from flight-leg $i$ to flight-leg $j$. Suppose further that the average unconstrained demand on itinerary $p$ is 70 and that there are 20 empty seats available on flight-leg $j$. If ASD-FAM deletes flight-leg $i$ (and therefore itinerary $p$) from the schedule, 70 passengers previously on itinerary $p$ need reaccommodation. Suppose that the recapture rate is 0.5. Because there are 20 seats available on flight-leg $j$, ASD-FAM will attempt to redirect 40 passengers from $p$ to $r$ (20 of whom will

**Table 1** **Data Characteristics**

| Data Set | No. of Flight Legs: Mand. | Opt. | Total | No. of Itineraries | No. of Fleets | No. of Aircraft |
|---|---|---|---|---|---|---|
| D1 | 645 | 108 | 753 | 60,347 | 4 | 166 |
| D2 | 588 | 260 | 848 | 67,805 | 4 | 166 |

**Table 2** **Problem Sizes (RMPs) and Solution Times**

| | Data Set D1 | Data Set D2 |
|---|---|---|
| No. of columns | 48,742 | 65,447 |
| No. of rows | 30,206 | 60,910 |
| No. of nonzeros | 164,676 | 344,517 |
| No. of iterations to solve LP relaxation | 34 | 46 |
| LP relaxation solution time | 11.1 hrs | 66 hrs |
| Solution time | 12.7 hrs | 3+ days* |

*Solution is not available.

**Table 3** **Contribution Comparison**

| | Planners' Schedule ($/day) | ISD-FAM Schedule ($/day) | ISD-FAM Schedule Daily Improvement |
|---|---|---|---|
| Data Set D1 | | | |
| ISD-FAM contribution | N/A | 1,908,867 | |
| PMM contribution | 1,159,828 | 1,721,604 | 561,776 |
| Revenue discrepancy | N/A | 187,263 | |

**Table 4** **Resulting Schedule Characteristics**

| | Planners' Schedule | Data Set D1 |
|---|---|---|
| No. of flights flown | 717 | 668 |
| No. of optional flights not flown | 0 | 85 |
| No. of aircraft used | 166 | 157 |
| No. of aircraft not used | 0 | 9 |

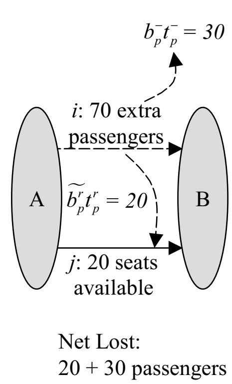


**Figure 7** **An Example of Reduced Effective Market Share Due to Modified Recapture Rates**

be successfully reaccommodated) and will spill 30 to the null itinerary. Thus, in this example, the demand is effectively reduced by $20 + 30 = 50$ passengers because of the deletion of flight-leg $i$. This example illustrates that recapture rates and capacity restrictions can cause effective market demand to be less than or equal to the initial market demand, even when deleting flight-leg $i$ is assumed to have no direct effect on market demand. Notice, however, that the recapture rates employed in this manner are for the purpose of shifting demands from deleted itineraries to remaining itineraries. Thus, these recapture rate values are not likely to equal those in IFAM, which are strictly for the purpose of reaccommodating passengers when flight capacities are constrained. For this reason, we refer to recapture rates employed in ASD-FAM as *modified recapture rates*, $\widetilde{b}_p^r$.

### 4.2. Formulation

The approximate schedule design and fleet assignment model is formulated as

$$\text{Min} \sum_{i\in L}\sum_{k\in K} C_{k,i} f_{k,i} + \sum_{p\in P}\sum_{r\in P} (\widetilde{fare}_p - \widetilde{b}_p^r \widetilde{fare}_r) t_p^r \tag{21}$$

subject to

$$\sum_{k\in K} f_{k,i} = 1 \quad \forall i \in L^F, \tag{22}$$

$$\sum_{k\in K} f_{k,i} \le 1 \quad \forall i \in L^O, \tag{23}$$

$$y_{k,o,t^-} + \sum_{i\in I(k,o,t)} f_{k,i} - y_{k,o,t^+} - \sum_{i\in O(k,o,t)} f_{k,i} = 0 \quad \forall \{k,o,t\} \in N, \tag{24}$$

$$\sum_{o\in A} y_{k,o,t_n} + \sum_{i\in CL(k)} f_{k,i} \le N_k \quad \forall k \in K, \tag{25}$$

$$\sum_{k\in K} CAP^k f_{k,i} + \sum_{r\in P}\sum_{p\in P} \delta_i^p t_p^r - \sum_{r\in P}\sum_{p\in P} \delta_i^p \widetilde{b}_r^p t_r^p \ge Q_i \quad \forall i \in L, \tag{26}$$

$$\sum_{r\in P} t_p^r \le D_p \quad \forall p \in P, \tag{27}$$

$$f_{k,i} \in \{0,1\} \quad \forall k \in K, \ \forall i \in L, \tag{28}$$

$$y_{k,o,t} \ge 0 \quad \forall \{k,o,t\} \in N, \tag{29}$$

$$t_p^r \ge 0 \quad \forall p, r \in P. \tag{30}$$

ASD-FAM is a special case of ISD-FAM, in which all demand correction terms are removed. As a result, the objective function (Equation 21) and constraints (26) and (27) are modified. To offset this, a set of modified recapture rates ($\widetilde{b}_p^r$) is employed in ASD-FAM to approximate changes in market demand as the flight schedule is altered.

**Table 5 Problem Sizes (RMPs) and Solution Times**

| | Data Set D1 | Data Set D2 |
|---|---|---|
| No. of columns | 35,633 | 38,837 |
| No. of rows | 2,999 | 3,326 |
| No. of nonzeros | 55,039 | 61,655 |
| No. of iterations to solve LP relaxation | 31 | 28 |
| LP relaxation solution time | 23 mins | 41 mins |
| Solution time | 39 mins | 78 mins |

### 4.3. Solution Approach

The accuracy of ASD-FAM is critically linked to the *modified recapture rates* ($\widetilde{b}_p^r$) because they are the only mechanism through which market demand is adjusted and through which passengers can be reallocated. Finding the set of modified recapture rates that accurately capture supply and demand interactions might require several iterations or, even worse, might be impossible. In our experiments, we let $\widetilde{b}_p^r = b_p^r$ for the case when all itineraries in a market are operated.

The solution algorithm for ASD-FAM is the ISD-FAM algorithm, outlined in Figure 4, with some modifications. Initially, demand estimates for the full schedule (containing all flights from the master flight list) are obtained using a schedule evaluation model. In Step 1, ASD-FAM is solved using the ISD-FAM algorithm (outlined in §3.4.1) to obtain a fleeted schedule. In Step 2, the new set of demands for the schedule resulting from Step 1 is obtained from a schedule evaluation model. Given these demand estimates and the fleeted schedule, in Step 3 we solve PMM to obtain the PMM revenue. In Step 4, the stopping criteria, as used in ISD-FAM, are evaluated. If the stopping criteria are not met, itineraries with inaccurate revenue estimates in the ASD-FAM solution are identified in Step 5, and their modified recapture rates are revised (in Step 6) using the demand information from Step 2. ASD-FAM is then resolved and the procedure repeats.

### 4.4. Computational Results

We evaluate ASD-FAM on the two medium-size schedules used for testing ISD-FAM. (Table 1 describes the characteristics of the data sets.) The testing procedure is as outlined in Figure 6. A Period I schedule serves as our basis for constructing a master flight list. ASD-FAM is solved on this master flight list to obtain an ASD-FAM-generated Period II schedule. A schedule evaluation model is used to generate a new set of demands for the ASD-FAM-generated schedule. PMM is then solved to obtain the PMM contributions, which are compared with the ASD-FAM contributions.

The problem sizes and ASD-FAM solution times for the two problems are reported in Table 5. From Table 6, it can be seen that ASD-FAM solutions

**Table 6 Contribution Comparision**

| | Planners' Schedule | ASD-FAM Schedule | |
|---|---|---|---|
| | ($/day) | ($/day) | Improvement |
| Data Set D1 | | | |
| ASD-FAM contribution | N/A | 1,999,094 | |
| PMM contribution | 1,159,828 | 1,742,349 | 582,521 |
| Revenue discrepancy | N/A | 256,745 | |
| Data Set D2 | | | |
| ASD-FAM contribution | N/A | 2,079,504 | |
| PMM contribution | 1,159,828 | 1,888,948 | 729,120 |
| Revenue discrepancy | N/A | 190,556 | |

are significantly improved compared to the planners' solutions. In data set D1 and D2, daily improvements of $582,521 and $729,120 are achieved, respectively. Assuming schedules repeat daily with similar demand for every day of the year, these improvements translate into annual improvements of approximately $210 and $270 million, respectively. The revenue discrepancy of $256,745 per day for data set D1 exceeds that of the ISD-FAM solution, as expected.

Table 7 presents the characteristics of the resulting schedules. Similar to ISD-FAM, ASD-FAM achieves significant improvements through savings in operating costs (from operating fewer flights) rather than through the generation of higher revenues. We also note that ASD-FAM solutions require significantly fewer aircraft to operate the schedule.

Finally, we note that ASD-FAM outperforms ISD-FAM, a couterintuitive result given that ASD-FAM is an approximation of ISD-FAM. The improved ASD-FAM performance results from its reduced complexity (relative to ISD-FAM) and, hence, our ability to generate more integer solutions.

### 4.5. Full-Size Problems

Unlike ISD-FAM that cannot be solved for data set D2, ASD-FAM is solvable for full-size schedules. Table 8 gives the characteristics of two additional data sets, each accompanied by the associated planners' schedules. Notice that in both data sets, all flights are optional. They represent full-size problems at a major U.S. airline. The runs are performed at the airline using their implementation of our prototype ASD-FAM model, on a six-processor computer, with two GB RAM, running Parallel CPLEX 6.

**Table 7 Resulting Schedule Characteristics**

| | Planners' Schedule | Data Set D1 | Data Set D2 |
|---|---|---|---|
| No. of flights flown | 717 | 660 | 614 |
| No. of optional flights not flown | 0 | 93 | 234 |
| No. of aircraft used | 166 | 154 | 148 |
| No. of aircraft not used | 0 | 12 | 18 |

**Table 8 Data Characteristics**

| | No. of Flight Legs | | | | | |
|---|---|---|---|---|---|---|
| Data Set | Mand. | Opt. | Total | No. of Itineraries | No. of Fleets | No. of Aircraft |
| F1 | — | 1,993 | 1,993 | 179,965 | 8 | 350 |
| F2 | — | 1,988 | 1,988 | 180,114 | 8 | 350 |

**Table 9 Contribution Comparison**

| | Planners' Schedule | ASD-FAM Schedule | |
|---|---|---|---|
| | ($/day) | ($/day) | Improvement |
| Data Set F1 | | | |
| ASD-FAM contribution | N/A | 36,932,000 | |
| PMM contribution | 36,387,000 | 37,375,000 | 988,000 |
| Revenue discrepancy | N/A | −443,000 | |
| Data Set F2 | | | |
| ASD-FAM contribution | N/A | 36,124,000 | |
| PMM contribution | 35,668,000 | 36,072,000 | 404,000 |
| Revenue discrepancy | N/A | 52,000 | |

**Table 10 Resulting Schedule Characteristics**

| | Data Set F1 | | Data Set F2 | |
|---|---|---|---|---|
| | Planners' Schedule | ASD-FAM | Planners' Schedule | ASD-FAM |
| No. of flights flown | 1,619 | 1,545 | 1,591 | 1,557 |

**Table 11 Problem Sizes (RMPs) and Solution Times**

| | Data Set F1 | Data Set F2 |
|---|---|---|
| Solution time | 16 hrs | 19 hrs |

Table 9 summarizes the results on these full-size problems. In data set F1, ASD-FAM achieves a daily improvement of $988,000 over the planners' schedule. In data set F2, the improvement is smaller at $404,000 per day. In both of these data sets, ASD-FAM produces schedules that operate fewer flight legs (Table 10).

We can clearly appreciate the complexity of these problems when we look at their solution times, reported in Table 11. Note that these are run-times on a six-processor workstation, running parallel CPLEX.

### Acknowledgments

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## Appendix. The Passenger Mix Model

The *passenger mix model* (*PMM*) (Kniker 1998) takes a *fleeted schedule* (that is, each flight leg is assigned one fleet type) and *unconstrained* *itinerary demand* as input and finds a flow of passengers over this schedule maximizing fleeting contribution, or equivalently minimizing assignment cost. Because the schedule is fleeted, flight operating costs are fixed and only passenger carrying and spill costs are minimized. The objective of the model, then, is to identify the best mix of passengers from each itinerary on each flight leg. The solution algorithm spills passengers on less profitable itineraries to accommodate passengers on more profitable itineraries. Hence, the PMM is

> *Given a fleeted flight schedule and the unconstrained itinerary demands, find the flow of passengers over the network that minimizes carrying plus spill cost, such that* (1) *the total number of passengers on each flight does not exceed the capacity of the flight, and* (2) *the total number of passengers on each itinerary does not exceed the unconstrained demand of that itinerary.*

Using previously defined notations, the mathematical formulation for PMM is

$$\text{Min} \sum_{p\in P}\sum_{r\in P}\left(\widetilde{fare}_p - b_p^r\,\widetilde{fare}_r\right) t_p^r \tag{A.1}$$

subject to

$$\sum_{p\in P}\sum_{r\in P}\delta_i^p t_p^r - \sum_{r\in P}\sum_{p\in P}\delta_i^p b_r^p t_r^p \geq Q_i - CAP_i \quad \forall\, i\in L, \tag{A.2}$$

$$\sum_{r\in P} t_p^r \leq D_p \quad \forall\, p\in P, \tag{A.3}$$

$$t_p^r \geq 0 \quad \forall\, p, r\in P. \tag{A.4}$$

The objective function minimizes the difference of spill and recapture. Constraints (A.2) are the capacity constraints. For leg $i$, the term $\sum_{r\in P}\sum_{p\in P}\delta_i^p t_p^r - \sum_{p\in P}\delta_i^p t_p^p$ can be viewed as the number of passengers who are spilled from their desired itinerary $p$. For leg $i$, the term $\sum_{r\in P}\sum_{p\in P}\delta_i^p b_r^p t_r^p - \sum_{p\in P}\delta_i^p b_p^p t_p^p$ is the number of passengers who are recaptured by the airline. (Note that we assume $b_p^p = 1$.) $CAP_i$ is the capacity of the aircraft assigned to leg $i$, and $Q_i$ is the unconstrained demand on flight-leg $i$, namely,

$$Q_i = \sum_{p\in P}\delta_i^p D_p. \tag{A.5}$$

Constraints (A.3) are the demand constraints restricting the total number of passengers spilled from itinerary $p$ to the unconstrained demand for itinerary $p$. $t_p^r$ must be greater than zero but need not be integer because we model the problem based on average demand data, which can be fractional.

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