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Output, the Stock Market, and Interest Rates

By OLIVIER J. BLANCHARD*

This paper develops a simple model of the determination of output, the stock market and the term structure of interest rates. The model is an extension of the *IS-LM* model and borrows from it the assumption that output is determined by aggregate demand and that the price level can only adjust over time to its equilibrium value. However, whereas the *IS-LM* emphasizes the interaction between "the interest rate" and output, this model emphasizes the interaction between asset values and output. Asset values, rather than the interest rate, are the main determinants of aggregate demand and output. Current and anticipated output and income are in turn the main determinants of asset values. It is this interaction that the model intends to capture; its goal is to characterize the joint response of asset values and output to changes in the environment, such as changes or announcement of changes in monetary and fiscal policy. As the above brief description makes clear, anticipations are central to the story; the assumption made in this paper will be one of rational expectations.

The paper is organized as follows. Section I describes the model, and Sections II–IV characterize the behavior of the economy under the extreme but convenient assumption that prices are fixed forever. Sections V and VI extend the analysis to the case where prices adjust over time to their equilibrium value.

I. The Model

Let us assume that the economy is closed and that the physical capital stock is constant. There is one good and four marketable assets. These are shares which are titles to the physical capital, private short- and

long-term bonds issued and held by individuals, and outside money.

A. Equilibrium in the Goods Market

We shall assume that there are three main determinants of spending. The first is the value of shares in the stock market—the stock market for short; being part of wealth, it affects consumption; determining the value of capital in place relative to its replacement cost, it affects investment¹ (see James Tobin). The second is current income which may affect spending independently of wealth if consumers or firms are liquidity constrained. The third is fiscal policy, both through public spending and taxes; fiscal policy will be summarized by an index rather than by an explicit treatment of both taxes and spending. A change in the index may be thought of as a balanced budget change in public spending. Total spending is expressed as

$$d = aq + \beta y + g; \quad a > 0; \quad \beta \in [0, 1)$$

All variables are real, d denotes spending, q is the stock market value, y is income, and g is the index of fiscal policy.

Output adjusts to spending over time:

$$(1) \quad \dot{y} = \sigma(d - y) \quad \sigma > 0 \\ = \sigma(aq + g - by) \quad b \equiv 1 - \beta > 0$$

where a dot denotes a time derivative.

There are two interpretations of (1), leading to the same functional form. The first is the one given implicitly above which assumes that inventories are decumulated after

¹A more detailed specification of aggregate demand would distinguish between the average value of capital which affects consumption and the marginal value which determines investment. It would also possibly introduce human wealth and outside money as other components of wealth.

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an increase in aggregate demand until production is increased to meet demand.² The second is that spending is always equal to production but that actual spending adjusts slowly to desired spending d . The first emphasizes the costs of adjusting production, the second the slow adjustment of spending. The interpretations are not mutually exclusive.

B. Equilibrium in the Assets Markets

The three nonmoney assets are assumed to be perfect substitutes. Hence arbitrage between them implies that they have the same expected short-term rate of return.³ Their common expected rate of return must in turn be such that agents are satisfied with the proportion of money in their portfolios.

Portfolio balance is characterized by a conventional *LM* relation in inverse form:

$$(2) \quad i = cy - h(m - p) \quad c > 0; h > 0]$$

where i denotes the short-term nominal rate, y denotes income, and m and p denote the logarithms of nominal money and the price level. The short-term real rate is defined as

$$(3) \quad r^* \equiv i - \dot{p}^*$$

An asterisk denotes an expectation; \dot{p}^* is the expected rate of inflation.

1. Arbitrage between Short- and Long-Term Bonds

The long-term bonds are consols with yield I and price $1/I$. The expected short-term nominal rate of return from holding consols is therefore

$$I \left(1 + \frac{d}{dt} \left(\frac{1}{I} \right) \right) = I - \dot{I}^*/I$$

It is the sum of the yield and the expected nominal capital gain. Arbitrage between

short and long bonds implies⁴

$$(4) \quad I - \dot{I}^*/I = i$$

or equivalently $\dot{I}^*/I = I - i$. If the long-term rate I is above the short-term rate i , agents must be expecting a capital loss on consols, that is, an increase in the long-term rate. Let R be the long-term real rate. Then by an equation similar to (4), we can define it implicitly by

$$(5) \quad r^* = R - \dot{R}^*/R$$

2. Arbitrage between Short-Term Bonds and Shares

As q is the real value of the stock market, the expected real rate of return on holding shares is $\dot{q}^*/q + \pi/q$, where π denotes real profit. Real profit is in turn assumed to be an increasing function of output:

$$\pi = \alpha_0 + \alpha_1 y; \quad \alpha_1 \geq 0$$

Arbitrage between short-term bonds and shares therefore implies

$$(6) \quad \frac{\dot{q}^*}{q} + \frac{\alpha_0 + \alpha_1 y}{q} = r^*$$

Equations (1)–(6) characterize output, the stock market and interest rates as functions of policy variables m and g , expectations \dot{q}^* and \dot{p}^* , and the price level p . The system is recursive: long rates are determined by equations (4) and (5) but do not in turn determine other variables. Following Tobin, the only link between assets and goods markets is the value of the stock market, q . To close the model, assume that expectations are formed rationally. This leaves us with the need for only one equation, the equation describing the behavior of the price level.

II. Steady State and Dynamics with Fixed Prices

We start with the assumption that prices are fixed. Hence, there is no actual and no expected inflation; nominal and real rates

²A more satisfactory—and more complex—formulation would allow for inventories to be rebuilt later during the adjustment process.

³I use “short term” instead of “instantaneous” which is more proper but also more cumbersome.

⁴The implicit assumption is that agents hold their expectations with subjective certainty. If this was not the case, the arbitrage equations would have to pay attention to Jensen's inequality.

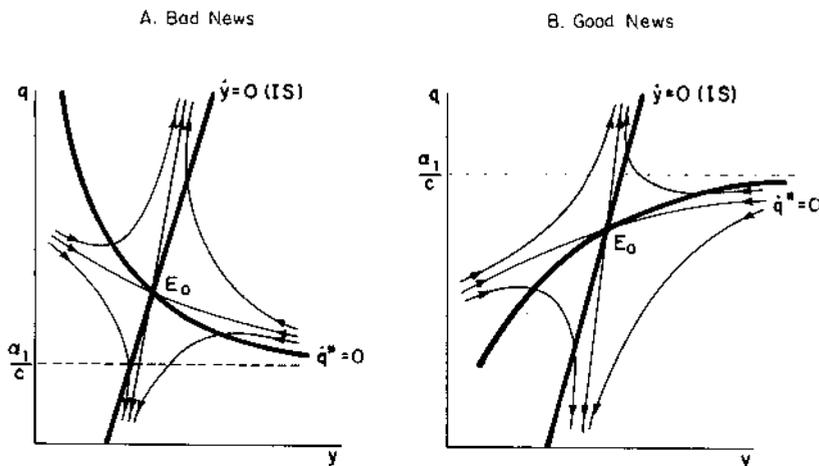


FIGURE 1

are identical and the system simplifies to

$$(1) \quad \dot{y} = \sigma(aq - by + g)$$

$$(2') \quad r = cy - h(m - p)$$

$$(6') \quad \frac{\dot{q}^*}{q} + \frac{\alpha_0 + \alpha_1 y}{q} = r$$

The real interest rate replaces the nominal rate in (2). It is no longer an expected rate and is now denoted by r rather than r^* . The term-structure relation is given by (5).

A. Steady State

In steady state $\dot{y} = 0$, output equals spending which is given by

$$y = \frac{a}{b}q + \frac{1}{b}g$$

Output depends on the stock market and fiscal policy. (Recall that we do not allow prices to adjust and that, in this "steady state," output is demand determined, an assumption that we shall want to relax later.) From (2') and (6'), if $\dot{q} = \dot{q}^* = 0$:

$$q = \frac{\pi}{r} = \frac{\alpha_0 + \alpha_1 y}{cy - h(m - p)}$$

The stock market is the ratio of steady-state profit to the steady-state interest rate. Both profit and interest are increasing functions

of output: output increases profit directly; it also increases the transaction demand for money and the interest rate. The effect of output on the stock market is therefore ambiguous and two cases have to be considered:

Let \bar{q} denote the steady-state value of q . Then if $(c\bar{q} - \alpha_1) > 0$, then the interest rate effect will dominate and an increase in output will have the net effect of decreasing the stock market. For lack of a better term, this case will be called the *bad news* case. The other will be called the *good news* case.

The loci $\dot{y} = 0$ and $\dot{q}^* = 0$ are drawn in Figure 1; the steady state is characterized graphically in each of the two cases.⁵

B. Dynamics

In the absence of changes in current or future policies, the assumption of rational expectations implies that $\dot{q}^* = \dot{q}$. Thus the dynamic behavior of the economy is characterized by two differential equations in q

⁵In the good news case, the existence of an equilibrium where the ($\dot{q} = 0$) locus intersects the IS from above follows from

$$\lim_{y \rightarrow \infty} \frac{dq}{dy} \Big|_{\dot{q}=0} = 0$$

There might however be two equilibria. The other one however has both undesirable comparative statics and dynamic properties.

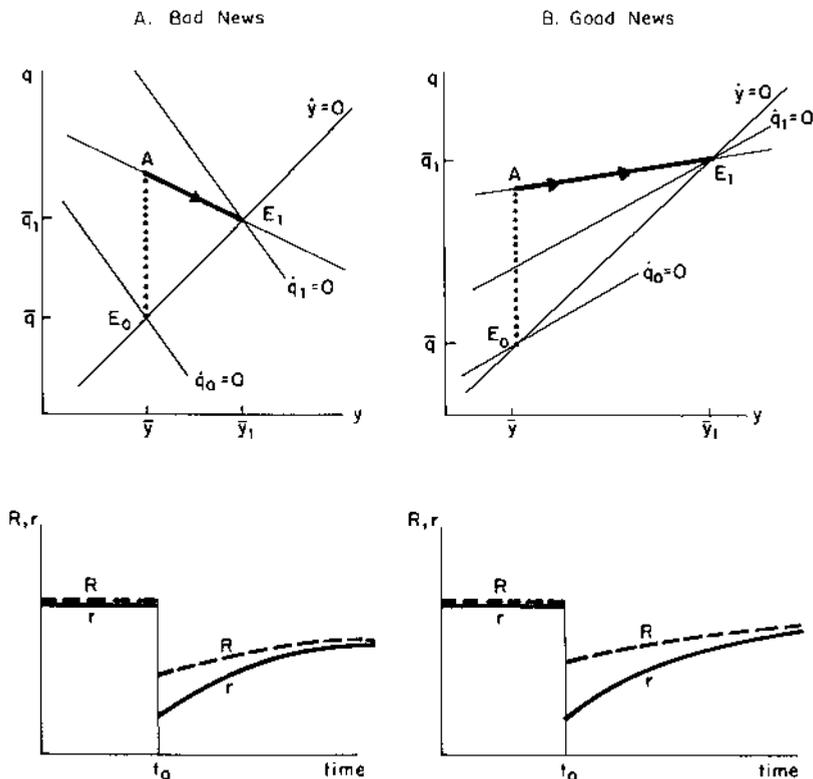


FIGURE 2. THE EFFECTS OF AN UNANTICIPATED MONETARY EXPANSION

and y . Trajectories corresponding to these equations of motion are drawn in Figure 1. In each case, the steady state is a saddle-point equilibrium. (Algebraic proofs and derivations are given in Appendix A.) Given the value of y which is given at any moment of time, there is a unique value of q which is such that the economy converges to its steady state. Following a standard if not entirely convincing practice,⁶ I shall assume that q always adjusts so as to leave the economy on the stable path to equilibrium.

III. A Monetary Expansion under Fixed Prices

What are the effects of an increase in money? The "steady-state" effects are clear: output and the stock market are higher in the new equilibrium. The higher money stock lowers the real interest rate and thus the cost of capital. This lower cost leads to a

higher stock market value, higher spending, higher output and profit. These comparative statics are very similar to the usual *IS-LM*. The dynamic adjustment is of more interest. To characterize it we need to distinguish between the case where the monetary expansion is unanticipated (i.e., announced and implemented simultaneously) and the case where it is anticipated (i.e., known for some time before its implementation).

A. An Unanticipated Monetary Expansion

The dynamic adjustment path is characterized in Figure 2 (for the system linearized around its initial steady state). It is drawn under the assumption that the economy is initially in steady state E_0 . The stock market jumps to A and the economy converges to E_1 over time. The behavior of interest rates, which cannot be read off the phase diagram, is plotted below.

⁶See my 1979 paper for further discussion.

When the increase in money takes place, output is given and it is the short-term rate which falls to maintain portfolio balance. To understand why the stock market jumps, we can integrate forward the arbitrage equation (6) (taking into account the transversality condition imposed by the requirement that the economy converges). This gives q as the present discounted value of profits:

$$q_t = \int_t^{\infty} \pi^*(s) e^{-\int_t^s r^*(v) dv} ds$$

The jump is then easily understood by looking forward at the adjustment path: interest rates are anticipated to be lower than before and profits are anticipated to be higher. What happens to the long rate? Although the short-term rate initially falls, the expectation of increasing output leads to an expected increase in the demand for money and thus to an expected increase in the short rate: the long-term rate falls, but by less than the short; the term structure slopes upwards after the increase in money. Over time, production increases in response to spending. What happens to the stock market? As time passes, the initial low discount rates and low profits disappear from the integral and are "replaced" by higher discount rates and profits. In the bad news case, the discount rate effect dominates: the stock market initially overshoots its steady-state value and decreases thereafter. In this case consol prices and shares have a similar qualitative behavior. The opposite holds in the good news case: after their initial increase consol prices and shares move in opposite directions.

Note that the initial jump in the stock market is unanticipated, but that its movement afterwards is anticipated. This movement is in no way inconsistent either with rational expectations or the assumption of no excess return on shares. In this particular case, rational expectations for the stock market turn out to be regressive (see Appendix A). If \bar{q}_1 denotes the new steady-state value of q , we get

$$\dot{q}^* = \dot{q} = \mu(q - \bar{q}_1) \quad \mu < 0$$

This result is however not very general and will not hold below.

B. An Anticipated Monetary Expansion

Suppose that the monetary expansion is announced at time t_0 to take place at time $t_1 > t_0$. The steady-state effects are the same as before, but the dynamic adjustment is different. It is characterized in Figure 3.

Technically, the adjustment path is uniquely determined by the following requirements. At time t_1 , the economy, if it is to converge, must be somewhere on the stable arm of the postmonetary expansion system SS' . At time t_0 , output \bar{y} is given. Between t_0 and t_1 the system must satisfy the equations of motion on the premonetary expansion system. There is a unique trajectory which satisfies all these requirements; which one it is depends on the length of the period between t_0 and t_1 . (Charles Wilson gives a more detailed explanation and the associated algebra in a model with a similar structure.) The curve $A'B'$ corresponds to a given period $(t_1 - t_0)$, $A''B''$ to a longer period.

The announcement of the monetary expansion is itself expansionary. The stock market jumps at time t_0 in anticipation of lower interest rates and higher profits after time t_1 . This increases spending and output over time.

Between the announcement and the implementation, output increases. As the money stock is still constant, the short-term rate also increases. Because of anticipated lower short rates after t_1 , however, the long-term rate declines. Thus the term structure "twists"; long and short rates moving in opposite directions. As the period of lower rates comes closer, the stock market increases. Whether or not it overshoots its steady state depends again on whether we are in the bad news or good news case.

At the time of the implementation, the short-term rate falls to maintain portfolio balance. *Little else happens*; the long-term rate and the stock market do not jump. If they did, this would imply anticipated infinite rates of capital gain or loss. After the implementation, the behavior of the economy is qualitatively similar to the case of an unanticipated increase.

Between t_0 and t_1 , the movements in output, stock market, and interest rates happen

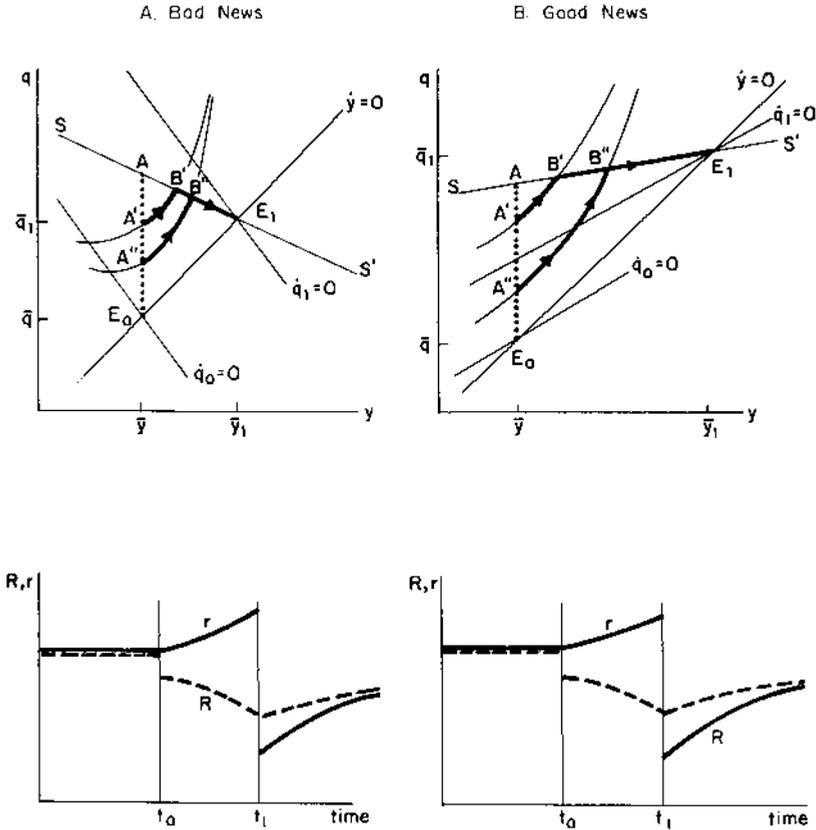


FIGURE 3. THE EFFECTS OF AN ANTICIPATED MONETARY EXPANSION

without apparent changes in policy. An outside observer, unaware of the announcement, may indeed conclude from an examination of the behavior of q and y over time, that an initial "speculative" boom in the stock market is the "cause" of the increase in output, and that the subsequent increase in money takes place to reduce the pressure on interest rates.

IV. A Fiscal Expansion under Fixed Prices

What are the effects of a fiscal expansion? The steady-state effects are again familiar: the fiscal expansion increases output, profit, and the interest rate. Hence, the effect on the stock market is ambiguous; the stock market decreases in the bad news case, increases in the other.

Turning to the dynamics, I shall directly consider the effect of an anticipated fiscal

expansion, say, announced at t_0 to be implemented at time t_1 . It is indeed probably the case that most changes in fiscal policy are known before they are implemented. (The case of an unanticipated change is just the limit of this case as t_1 tends to t_0 .) The adjustment is characterized in Figure 4.

There are now important differences between the bad and good news cases. In the bad news case, the policy change is bad news for the stock market which falls at the time of announcement. The reason lies in the anticipated increase in the sequence of short-term rates after the policy is implemented. In this case, it more than compensates the anticipated increase in the sequence of profits. Between the announcement and the implementation, fiscal policy has a perverse effect on output: because of the decrease in the stock market, private spending decreases and public spending is

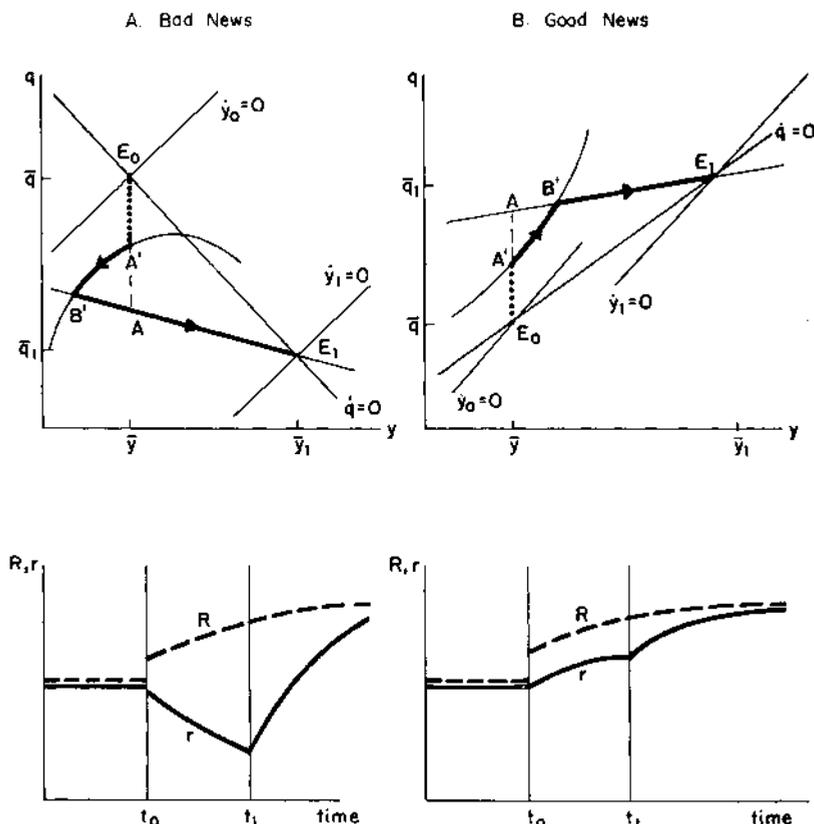


FIGURE 4. THE EFFECTS OF AN ANTICIPATED FISCAL EXPANSION

unchanged. Output therefore decreases until time t_1 , and so does the short-term rate. The long-term rate, however, increases in anticipation of higher short rates in the future. At the announcement, the term structure is upward sloping and twists during the period between announcement and implementation.

At and after the implementation, public spending increases, and so does output over time. The short-term rate increases with output; the term structure remains positively sloped, its slope decreasing as the economy reaches its new steady state. The stock market and consol prices have, in this case, the same qualitative behavior.

There is no perverse effect of the announcement of a change in policy in the good news case. The anticipation of higher profits more than offsets the anticipation of higher rates and the stock market increases. This jump and subsequent increase in the

stock market leads to an increase in output, and the long-term rate jumps in anticipation of higher short rates; the term structure is upward sloping. At the time of the implementation, the only noticeable effect is a larger rate of increase of output. An outside observer might again conclude that the policy change was not necessary, as the economy was already expanding. Note finally that, in this case, consol prices and the stock market move in opposite directions.

V. A Monetary Expansion under Flexible Prices

There is an uneasy feeling in characterizing the effects of nominal money over time assuming prices constant. I shall relax this assumption, at the cost of some additional complexity, and show how this affects the results obtained above. The discussion will be limited to characterizations of monetary policy.

If prices were perfectly flexible, changes in the level of money would be neutral, leaving output and the stock market unaffected. The dynamic adjustment of nominal variables could be characterized using this model (this would extend the analysis of Thomas Sargent and Neil Wallace) but would not be of considerable interest. We want instead to allow for movements in output, at least temporarily. The simplest price adjustment is probably

$$(7) \quad \dot{p} = \dot{p}^* = \theta(\bar{p} - p) \quad \theta > 0$$

where \bar{p} is the price level associated with full-employment output \bar{y} and the level of nominal money \bar{m} . This adjustment process implies that money is neutral in the long run as prices adjust to their equilibrium value over time; it also assumes that the reason for the slow adjustment of prices is not irrational expectations, as $\dot{p} = \dot{p}^*$, but inertia or the existence of predetermined nominal contracts. The extension of equation (7) to include an unemployment term such as $(y - \bar{y})$ would make (7) look more like a Phillips curve. But it would make the analysis more cumbersome and not affect results substantially.

A. Steady State and Dynamics

From equations (1)–(7), steady-state output, interest rates, and the stock market are invariant to nominal money. Nominal money simply affects prices proportionately.

To understand how (7) affects the dynamics, consider an (unanticipated) increase in nominal money. When this expansion takes place, real balances are higher as prices cannot instantaneously adjust, decreasing the nominal interest rate. Prices are, however, now expected to increase and the expected rate of inflation decreases the real rate of interest given the nominal rate; this effect is usually referred to as the "Mundell effect." Both effects work in the same direction, decreasing the real rate. Over time, real balances decrease and the expected inflation becomes smaller; both effects work again in the same direction, now increasing the real rate. Algebraically, using (2), (3) and (7)

gives

$$\begin{aligned} r^* &= i - \dot{p}^* = cy - h(m - p) + \theta(p - \bar{p}) \\ &= (\theta + h)p + \psi \end{aligned}$$

where ψ does not depend directly on p . A higher value of p leads to a higher r^* through the real balance effect (h) and the Mundell effect (θ).

Linearizing the system around its steady state gives

$$(8) \quad \begin{bmatrix} \dot{q}^* \\ \dot{y} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} \bar{r} & c\bar{q} - \alpha_1 & (h + \theta)\bar{q} \\ \sigma a & -\sigma b & 0 \\ 0 & 0 & -\theta \end{bmatrix} \begin{bmatrix} q - \bar{q} \\ y - \bar{y} \\ p - \bar{p} \end{bmatrix}$$

This system has three roots. Because of its recursive structure, two of the roots are the same as in the fixed-price case, $\mu < 0$ and $\xi > 0$. The third is simply $-\theta$, the (negative of) the speed of adjustment of prices.

One root, ξ , is positive. Two variables y and p cannot "jump." Thus the initial conditions for (y, p) together with the requirement that the system converges to steady state determines a unique trajectory and, as in Sections II–IV, a unique value for q .

B. An Unanticipated Monetary Expansion

Consider an unanticipated monetary expansion dm at time $t_0 = 0$. If we assume the economy to be in steady state before the change, the behavior of output, prices and the stock market is given by (see Appendix B)

$$(9) \quad q - \bar{q} = -\frac{\theta + h}{(\theta + \mu)(\theta + \xi)} \times [(\sigma b - \theta)e^{-\theta t} - (\sigma b + \mu)e^{\mu t}] \bar{q} dm$$

$$(10) \quad y - \bar{y} = -\frac{\theta + h}{(\theta + \mu)(\theta + \xi)} \times [e^{-\theta t} - e^{\mu t}] \sigma a \bar{q} dm$$

$$(11) \quad p - \bar{p} = -e^{-\theta t} dm$$

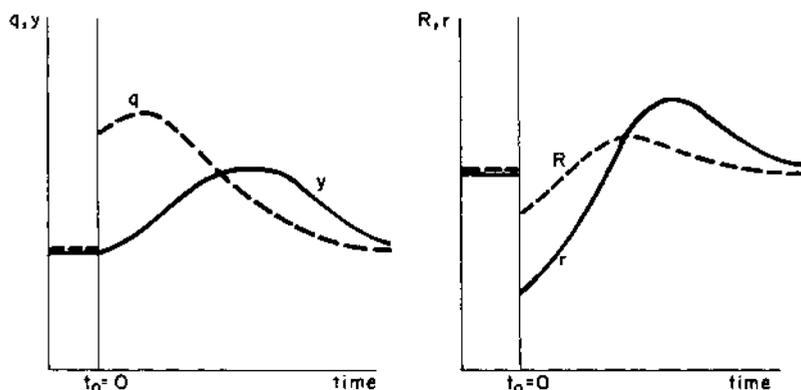


FIGURE 5. THE EFFECTS OF AN UNANTICIPATED MONETARY EXPANSION UNDER FLEXIBLE PRICES

(These relations hold only if $\theta \neq -\mu$. The Appendix gives the relations for $\theta = -\mu$.) If there was no Mundell effect (i.e., if, for example, money balances paid the expected rate of inflation), the relations defining q and y would have h rather than $(\theta + h)$ in the numerator. The roots θ , μ , ξ would be unaffected.

The first question is whether the analysis of Sections II–IV is the limit of the flexible price case when prices adjust extremely slowly, that is, as θ tends to zero. As shown in the Appendix, this is indeed the case. Thus the dynamics of Sections II–IV are “approximately correct” if prices adjust slowly.

The second question is how price flexibility affects the impact effect of monetary policy. Intuition suggests the elements of the answer: Assume that there is no Mundell effect; money balances pay the expected rate of inflation. Then the more flexible prices are, the faster the real money stock will return to its previous level. This in turn suggests a faster return of profits and real interest rates to their previous levels, thus a smaller initial jump in the stock market. This in turn implies a lower initial increase in spending and a lower rate of increase of output.

The countervailing effect of more flexible prices is through the Mundell effect. The more flexible prices are, the higher the initial rate of inflation and, *ceteris paribus*, the lower the real rate of interest. This lower

initial sequence of real interest rates tends to increase the initial jump in the stock market, leading to a higher initial rate of increase in output.

Evaluating (9) at $t=0$ gives

$$(q - \bar{q})|_{t=0} = \frac{\theta + h}{\theta + \xi} \bar{q} dm > 0$$

As θ , h , and ξ are positive, a monetary expansion always increases the stock market and spending. The effect of an increase in θ is indeed ambiguous. With no Mundell effect, the numerator would simply be h : in this case more flexible prices lead to a smaller impact effect. The Mundell effect works in the opposite direction; the net effect depends on $(h - \xi)$. The positive root ξ of the system does not depend on h . Thus there are no restrictions on the sign of $(h - \xi)$.

What can we deduce about the adjustment path from equations (9)–(11)? The monetary expansion leads to an expansion of output over time followed by a return to steady state; output remains above its equilibrium value during the period of adjustment. The real interest rate increases rapidly after its initial decline, overshooting its equilibrium value. This is due partly to lower real money balances, partly to higher transaction demand for money, and partly to lower expected inflation. As a result, the long-term real rate initially declines by less than the short rate; the term structure is

initially upward sloping, flattening over time to become downward sloping. The sequences of initially increasing profits and interest rates followed by decreasing profits and interest rates have a complex effect on the stock market which may keep increasing after its initial jump and which may also decrease below its steady-state value during the adjustment process. These results are derived in Appendix B and summarized graphically in Figure 5.

VI. Summary and Extensions

This paper has shown the interaction between output and the stock market. The effect of a change either in current or anticipated policy is a discrete change in the stock market due to the change in the anticipated sequence of profits and real interest rates. This, in turn, together with the change in policy, affects spending and output over time, validating the initial anticipations of profits and interest rates. The stock market is not the "cause" of the increase in output, no more than the increase in output is the cause of the initial stock market change. They are both the results of changes in policy.

Whether policies are anticipated or not is important; the announcement itself will usually lead to a change in anticipated profits and discount rates, leading to a change in the stock market. Although in this case the change in the stock market and the resulting increase in output will precede the change in policy, they are still caused by it; the implementation of the policy may have little apparent effect. Under plausible assumptions, the announcement of an expansionary fiscal policy may have a perverse effect, decreasing output before the actual implementation of the policy.

The effect of more flexible prices is to decrease the overall effect of changes in nominal money. The effect on the initial impact of changes in money is, however, ambiguous: the smaller overall effect leads to a smaller change in the stock market, but the faster initial inflation may lead to lower real interest rates initially, and to a larger initial change in the stock market.

There are at least two logical extensions of this model. The first is a more detailed treatment of aggregate demand, allowing in particular a more detailed treatment of fiscal policy. This is done in a medium-sized econometric model (see my 1980 paper) and parallels a similar attempt by Ray Fair. The second is a more detailed treatment of aggregate supply: the implicit assumption of this paper is that the economy is in Keynesian unemployment, with no effects of the real wage. Allowing for an effect of the real wage and taking explicit account of capital accumulation are the next items on the agenda.

APPENDIX A: THE FIXED-PRICE CASE

The system composed of (1), (2'), and (6') is linearized around its steady state:

$$\begin{bmatrix} \dot{q}^* \\ \dot{y} \end{bmatrix} = \begin{bmatrix} \bar{r} & c\bar{q} - \alpha_1 \\ \sigma a & -\sigma b \end{bmatrix} \begin{bmatrix} q - \bar{q} \\ y - \bar{y} \end{bmatrix}$$

The assumption that the *IS* intersects the *LM* from below implies that the determinant of the above Jacobian is negative. The system has two roots of opposite sign, $\mu < 0$ and $\xi > 0$. The characteristic vector associated with μ , (x_1, x_2) is such that

$$(A1) \quad \frac{x_1}{x_2} = \frac{c\bar{q} - \alpha_1}{\mu - \bar{r}} = \frac{\sigma b + \mu}{\sigma a}$$

The equations of motion along the stable arm are thus given by

$$(A2) \quad \begin{aligned} q - \bar{q} &= c_1(\sigma b + \mu)e^{\mu t} \\ y - \bar{y} &= c_1\sigma a e^{\mu t} \end{aligned}$$

where c_1 depends on the initial conditions for y . Thus, along the stable arm:

$$(A3) \quad \frac{dq}{dy} \geq 0 \Leftrightarrow (\sigma b + \mu) \geq 0 \Leftrightarrow (c\bar{q} - \alpha_1) \leq 0$$

An increase in nominal money, dm , increases steady state q and y . Denoting their

new steady-state values by \bar{q}_1 and \bar{y}_1 , we get

$$(A4) \quad (\bar{q}_1 - \bar{q}) = -\frac{\sigma b \bar{q} h}{\mu \xi} dm$$

$$(\bar{y}_1 - \bar{y}) = -\frac{\sigma a \bar{q} h}{\mu \xi} dm$$

At the time of the increase, $t=0$, y is fixed:

$$(y - \bar{y}_1)|_{t=0} = \bar{y} - \bar{y}_1 \Rightarrow c_1 = \frac{\bar{q} h}{\mu \xi} dm$$

This gives, replacing c_1 in (A2)

$$(q - \bar{q}_1) = \frac{\bar{q} h}{\mu \xi} (\sigma b + \mu) e^{\mu t} dm$$

Or, using (A4)

$$(A5) \quad (q - \bar{q}) = \frac{\bar{q} h}{\mu \xi} ((\sigma b + \mu) e^{\mu t} - \sigma b) dm$$

APPENDIX B: THE FLEXIBLE PRICE CASE

The characteristic polynomial of the matrix in (8) has three roots. Two are the same as in the fixed-price case, $\mu < 0$ and $\xi > 0$. The last is $-\theta < 0$.

The vectors associated with the negative roots μ and $(-\theta)$ are, respectively (up to a factor of proportionality), if $\mu \neq -\theta$

$$\begin{bmatrix} \sigma b + \mu & \sigma a & 0 \end{bmatrix}$$

$$\text{and } \begin{bmatrix} \sigma b - \theta & \sigma a & \frac{(\theta + \xi)(\theta + \mu)}{(h + \theta)\bar{q}} \end{bmatrix}$$

The stable trajectory therefore satisfies

$$(A6) \quad q - \bar{q} = c_1(\sigma b - \theta)e^{-\theta t} + c_2(\sigma b + \mu)e^{\mu t}$$

$$(A7) \quad y - \bar{y} = c_1 \sigma a e^{-\theta t} + c_2 \sigma a e^{\mu t}$$

(A8)

$$p - \bar{p} = c_1(\theta + \xi)(\theta + \mu)(h + \theta)^{-1} \bar{q}^{-1} e^{-\theta t}$$

The variables c_1, c_2 are determined by initial conditions for y and p . An increase in dm leaves \bar{y}, \bar{q} unchanged and increases \bar{p} by dm . So after an unanticipated increase dm at $t_0=0$, $p - \bar{p} = -dm$ and $y - \bar{y} = 0$. This determines uniquely c_1 and c_2 using (A7) and (A8) at $t_0=0$. Replacing these values of c_1 and c_2 in (A6) to (A8) gives equations (9)–(11) in the text. Note that if $\theta=0$, (9) reduces to equation (A5).

For the case where $\mu = -\theta$, L'Hospital's rule can be used on (9) and (10) to derive $(y - \bar{y})$ and $(q - \bar{q})$. This gives

$$q - \bar{q} = \bar{q}(\mu - h)(\mu - \xi)^{-1} \\ \times (1 - t(\sigma b + \mu))e^{\mu t} dm$$

$$y - \bar{y} = \bar{q} \sigma a (\mu - h)(\mu - \xi)^{-1} t e^{\mu t} dm$$

We may now characterize the dynamic behavior of q . The initial jump in q at $t=0$ is given by

$$(q - \bar{q})|_{t=0} = \frac{\theta + h}{\theta + \xi} \bar{q} dm > 0$$

As $(q - \bar{q})$ is the sum of two declining exponentials, it has for $t \geq 0$ at most one interior maximum or minimum. If such an extremum exists, it happens at t^* given by

$$e^{-(\theta + \mu)t^*} = -\frac{\mu}{\theta} \frac{\sigma b + \mu}{\sigma b - \theta} \quad \text{for } \theta \neq -\mu$$

$$t^* = -\sigma b \mu^{-1} (\sigma b + \mu)^{-1} \quad \text{for } \theta = -\mu$$

The initial rate of change of q is given by

$$\dot{q}|_{t=0} = -\frac{(\theta + h)}{(\theta + \mu)(\theta + \xi)} \\ \times [-\theta(\sigma b - \theta) - \mu(\sigma b + \mu)] \bar{q} dm \\ = \frac{\theta + h}{\theta + \xi} [\sigma b + \mu - \theta] \bar{q} dm$$

Thus, for $\dot{q}|_{t=0} > 0$, a necessary condition is that $\sigma b + \mu > 0$, or equivalently $c\bar{q} - \alpha_1 < 0$: the good news condition is necessary but not sufficient anymore.

The same analysis is easily done for y . The variable y increases initially, reaches its maximum for:

$$t^{**} = \frac{-\ln(-\mu) + \ln(\theta)}{\theta + \mu} \text{ for } \theta \neq -\mu$$

$$1/\theta \text{ for } \theta = \mu$$

It decreases to steady state after that. The behavior of the short-term real rate is directly obtained from

$$(r - \bar{r}) = c(y - \bar{y}) + (h = \theta)(p - \bar{p})$$

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