

IDENTIDADES VECTORIALES

$$\begin{aligned}
 \vec{A} \cdot (\vec{B} \times \vec{C}) &= \vec{B} \cdot \vec{C} \times \vec{A} = \vec{C} \cdot \vec{A} \times \vec{B} \\
 \vec{A} \times (\vec{B} \times \vec{C}) &= (\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C} \\
 \vec{A} \times (\vec{B} \times \vec{C}) + \vec{B} \times (\vec{C} \times \vec{A}) + \vec{C} \times (\vec{A} \times \vec{B}) &= \vec{0} \\
 (\vec{A} \times \vec{B}) \cdot (\vec{C} \times \vec{D}) &= \vec{A} \cdot [\vec{B} \times (\vec{C} \times \vec{D})] \\
 (\vec{A} \times \vec{B}) \cdot (\vec{C} \times \vec{D}) &= (\vec{A} \cdot \vec{C})(\vec{B} \cdot \vec{D}) - (\vec{A} \cdot \vec{D})(\vec{B} \cdot \vec{C}) \\
 (\vec{A} \times \vec{B}) \times (\vec{C} \times \vec{D}) &= [(\vec{A} \times \vec{B}) \cdot \vec{D}]\vec{C} - [(\vec{A} \times \vec{B}) \cdot \vec{C}]\vec{D}
 \end{aligned}$$

IDENTIDADES CON OPERADORES DIFERENCIALES

$$\begin{aligned}
 \nabla(\phi\psi) &= \phi\nabla\psi + \psi\nabla\phi \\
 \vec{\nabla} \cdot (\phi\vec{A}) &= \vec{A} \cdot \nabla\phi + \phi\vec{\nabla} \cdot \vec{A} \\
 \vec{\nabla} \times (\phi\vec{A}) &= \phi\vec{\nabla} \times \vec{A} - \vec{A} \times \nabla\phi \\
 \vec{\nabla} \cdot (\vec{A} \times \vec{B}) &= \vec{B} \cdot (\vec{\nabla} \times \vec{A}) - \vec{A} \cdot (\vec{\nabla} \times \vec{B}) \\
 \vec{\nabla} \times (\vec{A} \times \vec{B}) &= \vec{A}(\vec{\nabla} \cdot \vec{B}) - \vec{B}(\vec{\nabla} \cdot \vec{A}) + (\vec{B} \cdot \nabla)\vec{A} - (\vec{A} \cdot \nabla)\vec{B} \\
 \nabla(\vec{A} \cdot \vec{B}) &= \vec{A} \times (\nabla \times \vec{B}) + \vec{B} \times (\nabla \times \vec{A}) + (\vec{B} \cdot \nabla)\vec{A} + (\vec{A} \cdot \nabla)\vec{B} \\
 \nabla \times \nabla\phi &= \vec{0} \\
 \nabla \cdot (\nabla \times \vec{A}) &= \vec{0}
 \end{aligned}$$

Casos particulares de interés

Sea \vec{r} el vector posición y ∇ el operador gradiente correspondiente. Entonces:

$$\begin{aligned}
 \vec{\nabla} \cdot \vec{r} &= 3 \\
 \vec{\nabla} \cdot \hat{r} &= \frac{2}{r} \\
 (\vec{a} \cdot \nabla)\hat{r} &= \frac{1}{r}[\vec{a} - \hat{r}(\vec{a} \cdot \hat{r})] \equiv \frac{\vec{a}_\perp}{r} \\
 \vec{\nabla} \times \vec{r} &= \vec{0} \\
 \vec{\nabla} \times \hat{r} &= \vec{0}
 \end{aligned}$$

SISTEMAS DE COORDENADAS

CARTESIANAS: $(x, y, z) \rightarrow (\hat{x}, \hat{y}, \hat{z})$

$$\begin{aligned}\vec{\nabla} \psi &= \left(\hat{x} \frac{\partial}{\partial x} + \hat{y} \frac{\partial}{\partial y} + \hat{z} \frac{\partial}{\partial z} \right) \psi \\ \nabla^2 \psi &= \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \psi \\ \vec{\nabla} \cdot \vec{A} &= \frac{\partial}{\partial x} A_x + \frac{\partial}{\partial y} A_y + \frac{\partial}{\partial z} A_z \\ \vec{\nabla} \times \vec{A} &= \hat{x} \left(\frac{\partial}{\partial y} A_z - \frac{\partial}{\partial z} A_y \right) + \hat{y} \left(\frac{\partial}{\partial z} A_x - \frac{\partial}{\partial x} A_z \right) + \hat{z} \left(\frac{\partial}{\partial x} A_y - \frac{\partial}{\partial y} A_x \right)\end{aligned}$$

CILINDRICAS: $(\rho, \phi, z) \rightarrow (\hat{\rho}, \hat{\phi}, \hat{z})$

$$\begin{aligned}\vec{\nabla} \psi &= \left(\hat{\rho} \frac{\partial}{\partial \rho} + \hat{\phi} \frac{1}{\rho} \frac{\partial}{\partial \phi} + \hat{z} \frac{\partial}{\partial z} \right) \psi \\ \nabla^2 \psi &= \left(\frac{1}{\rho} \frac{\partial}{\partial \rho} \rho \frac{\partial}{\partial \rho} + \frac{1}{\rho} \frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2} \right) \psi \\ \vec{\nabla} \cdot \vec{A} &= \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho A_\rho) + \frac{1}{\rho} \frac{\partial}{\partial \phi} (A_\phi) + \frac{\partial}{\partial z} A_z \\ \vec{\nabla} \times \vec{A} &= \hat{\rho} \left(\frac{1}{\rho} \frac{\partial}{\partial \phi} A_z - \frac{\partial}{\partial z} A_\phi \right) + \hat{\phi} \left(\frac{\partial}{\partial z} A_\rho - \frac{\partial}{\partial \rho} A_z \right) + \hat{z} \frac{1}{\rho} \left(\frac{\partial}{\partial \rho} (\rho A_\phi) - \frac{\partial}{\partial \phi} A_\rho \right) \\ &= \left(\hat{\rho} \frac{1}{\rho} \frac{\partial}{\partial \phi} - \hat{\phi} \frac{\partial}{\partial \rho} \right) A_z + \frac{1}{\rho} \left(\hat{z} \frac{\partial}{\partial \rho} - \hat{\rho} \frac{\partial}{\partial z} \right) (\rho A_\phi) + \left(\hat{\phi} \frac{\partial}{\partial z} - \hat{z} \frac{1}{\rho} \frac{\partial}{\partial \phi} \right) A_\rho\end{aligned}$$

ESFERICAS: $(r, \theta, \phi) \rightarrow (\hat{r}, \hat{\theta}, \hat{\phi})$

$$\begin{aligned}\vec{\nabla} \psi &= \left(\hat{r} \frac{\partial}{\partial r} + \hat{\theta} \frac{1}{r} \frac{\partial}{\partial \theta} + \hat{\phi} \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} \right) \psi \\ \nabla^2 \psi &= \left(\frac{1}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \sin \theta \frac{\partial}{\partial \theta} + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \phi^2} \right) \psi \\ \vec{\nabla} \cdot \vec{A} &= \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 A_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta A_\theta) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} (A_\phi) \\ \vec{\nabla} \times \vec{A} &= \hat{r} \frac{1}{r \sin \theta} \left(\frac{\partial}{\partial \theta} (\sin \theta A_\phi) - \frac{\partial}{\partial \phi} A_\theta \right) + \hat{\theta} \frac{1}{r} \left(\frac{1}{\sin \theta} \frac{\partial}{\partial \phi} A_r - \frac{\partial}{\partial r} (r A_\phi) \right) \\ &\quad + \hat{\phi} \frac{1}{r} \left(\frac{\partial}{\partial r} (r A_\theta) - \frac{\partial}{\partial \theta} A_r \right)\end{aligned}$$