14/04/2005 C1

2)

1. Proceso adiabático : $Q=0$ (no cuasiestático)

1ª Ley : $\Delta U = \cancel{Q}^{=0} - W$ (1)

$$U_f = n_f u_f + (n_i - n_f) u'$$

$$U_i = n_i u_i$$

$$\therefore \Delta U = n_f u_f + (n_i - n_f) u' - n_i u_i \quad (2)$$

Además : $W = P' \cdot (n_i - n_f) v' > 0$ (3)

Reemplazando (2) y (3) en (1) :

$$n_f u_f + (n_i - n_f) u' - n_i u_i = -(n_i - n_f) P' v'$$

o $$n_f u_f + (n_i - n_f) [\underbrace{u' + P'v'}_{h'}] = n_i u_i$$

$$\therefore \frac{n_f}{n_i} u_f + h' - \frac{n_f}{n_i} h' = u_i$$

$$\therefore \frac{n_f}{n_i}(h' - u_f) = h' - u_i \Longrightarrow \boxed{\frac{n_f}{n_i} = \frac{h' - u_i}{h' - u_f}}$$

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3) $$\left(\frac{\partial V}{\partial T}\right)_P = \frac{nR}{P} + \frac{na}{T^2} = M(T,P)$$

$$\wedge \quad \left(\frac{\partial V}{\partial P}\right)_T = -nT f(P) = N(T,P)$$

De $V(T,P) \to dV = \underbrace{\left(\frac{\partial V}{\partial T}\right)_P}_{M} dT + \underbrace{\left(\frac{\partial V}{\partial P}\right)_T}_{N} dP$

De la condición de exactitud (integrabilidad),

$$\left(\frac{\partial M}{\partial P}\right)_T = \left(\frac{\partial N}{\partial T}\right)_P \longrightarrow -\frac{nR}{P^2} = -n f(P)$$

$$\therefore \boxed{f(P) = \frac{R}{P^2}}$$

Con ésto, $$\left.\begin{aligned} \left(\frac{\partial V}{\partial T}\right)_P &= \frac{nR}{P} + \frac{na}{T^2} \quad (1) \\ \left(\frac{\partial V}{\partial P}\right)_T &= -\frac{nRT}{P^2} \quad (2) \end{aligned}\right\}$$

De (1) : $dV = \frac{nR}{P} dT + \frac{na}{T^2} dT$ ($P=$cte)

Integrando a $P=$cte:

$$V = nR\frac{T}{P} - \frac{na}{T} + F(P) \quad (3)$$

cte. de integración en su forma más general.

Calculando de (3) $\left(\frac{\partial V}{\partial P}\right)_T$:

$$\left(\frac{\partial V}{\partial P}\right)_T = -\frac{nRT}{P^2} + F'(P) \quad (4)$$

Igualando los 2º miembros de (4) y (2)

$$-\cancel{\frac{nRT}{P^2}} + F'(P) = -\cancel{\frac{nRT}{P^2}} \Rightarrow F'(P) = 0$$

Luego $F(P) = \text{cte} \equiv b$ (digamos) (5)

Finalmente, reempl. en (3) :

$$\boxed{PV = nRT + \left(b - \frac{na}{T}\right)P} \quad //$$