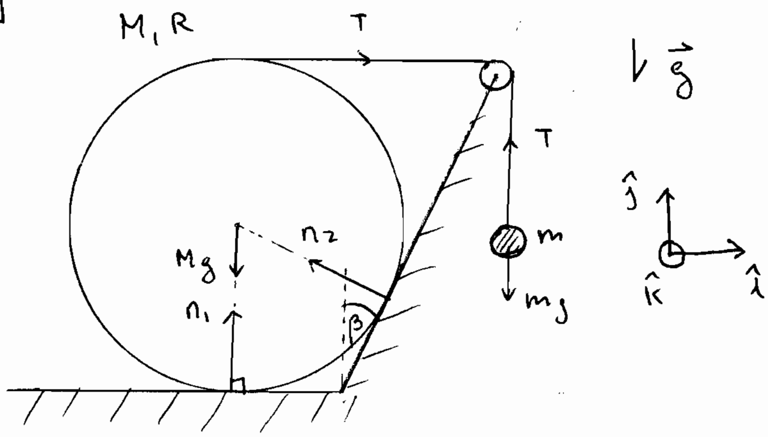
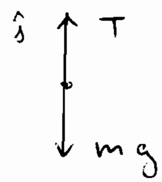
P1]



Datos: M, R

¿m?

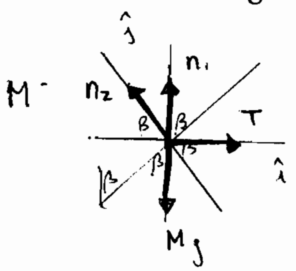
m:



$$\vec{T} + m\vec{g} = m\,\vec{a}_m$$

$$\hat{j})\quad T - mg = m\,a_m \quad (1)$$

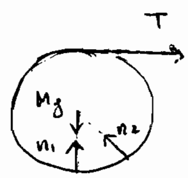
M:



$$\vec{T} + \vec{n}_1 + \vec{n}_2 + M\vec{g} = M\,\underbrace{\cancel{\vec{a}_{CM\,M}}}_{\to 0 \text{ por enunciado}}$$

$$\hat{i})\quad T + 0 - n_2\cos\beta + 0 = 0 \quad (2)$$

$$\hat{j})\quad 0 + n_1 + n_2\sin\beta - Mg = 0 \quad (3)$$



$$\vec{\tau}_{CM}(\vec{T}) + \underbrace{\cancel{\vec{\tau}_{CM}(M\vec{g})}}_{0} + \underbrace{\cancel{\vec{\tau}_{CM}(\vec{n}_1)}}_{0} + \underbrace{\cancel{\vec{\tau}_{CM}(\vec{n}_2)}}_{0} = J_{CM}\,\vec{\alpha}$$

$$\hat{k})\quad -RT = \frac{1}{2}MR^2\alpha \qquad \text{pero} \qquad a_m = R\cdot\alpha$$

$$\Rightarrow \quad -\cancel{R}T = \frac{1}{2}M\cancel{R^2}\frac{a_m}{\cancel{R}}$$

$$\Rightarrow \quad a_m = -\frac{2T}{M} \quad (4)$$

Condición $n_1 = 0$

$$\left.\begin{array}{l}(3) \to \ n_2\sin\beta = Mg \\ (2) \to \ n_2\cos\beta = T\end{array}\right\} \ (3)/(2) \Rightarrow \quad \tan\beta = \frac{Mg}{T}$$

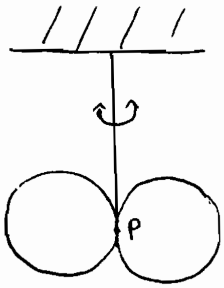
$$\Rightarrow \quad T = \frac{Mg}{\tan\beta} \quad (5)$$

$$(4) \text{ en } (1) \Rightarrow \quad T - mg = -\frac{2m}{M}T$$

$$\Rightarrow \quad T = \frac{m}{M+2m}\,Mg \quad (6)$$

$$(5) = (6) \Rightarrow \quad \frac{\cancel{Mg}}{\tan\beta} = \frac{m}{M+2m}\cancel{Mg} \quad \Rightarrow \quad \boxed{m = \frac{M}{\tan\beta - 2}}$$

P2]



Datos: M, R, $\tau = -\lambda\theta$

(a) $I_{\substack{esf \\ \%\, CM}} = \frac{2}{5} M R^2$

Steiner $\quad I_p = I_{cm} + M D^2$

$$I_{\substack{1\,esf \\ \%\, P}} = \frac{2}{5} MR^2 + MR^2 = \frac{7}{5} MR^2$$

$$\Rightarrow \boxed{I_{\substack{2\,esf \\ \%\, P}} = \frac{14}{5} M R^2}$$

(b) $\tau_p = I_p \ddot{\theta}$

$$\Rightarrow -\lambda\theta = \frac{14}{5} M R^2 \ddot{\theta}$$

$$\Rightarrow \ddot{\theta} + \underbrace{\frac{5\lambda}{14 M R^2}}_{\omega^2} \theta = 0$$

$$\Rightarrow \boxed{T = 2\pi \sqrt{\frac{14 M R^2}{5\lambda}}}$$

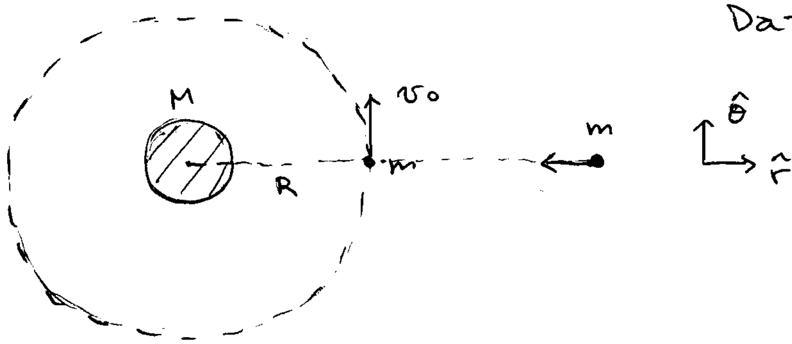
(c) $\theta(t) = A \cos\left(\sqrt{\frac{5\lambda}{14 M R^2}}\, t + \phi\right)$

$$CI: \left.\begin{array}{l} \theta(0) = \theta_0 \\ \dot{\theta}(0) = 0 \end{array}\right\} \Rightarrow \begin{array}{l} A = \theta_0 \\ \phi = 0 \end{array}$$

$$\Rightarrow \boxed{\theta(t) = \theta_0 \cos\left(\sqrt{\frac{5\lambda}{14 M R^2}}\, t\right)}$$

P3]

Datos: M, m, R, v_1



(a) $\vec{F} = m\vec{a}$

$$\Rightarrow -\frac{GMm}{R^2}\hat{r} = -m\frac{v_0^2}{R}\hat{r}$$

$$\Rightarrow \boxed{v_0 = \sqrt{\frac{GM}{R}}}$$

(b) $\vec{P}_f - \vec{P}_i = \vec{F}\cdot\Delta t$

$\hat{\theta}$) $(2m)V_\theta - m v_0 = \underbrace{F_\theta}_{0}\cdot\Delta t$

$$\Rightarrow V_\theta = \frac{v_0}{2} = \frac{1}{2}\sqrt{\frac{GM}{R}}$$

$\hat{r}$) $(2m)V_r - (-m v_1) = F_r \cdot \underbrace{\Delta t}_{0}$ se asume impacto instantáneo

$$\Rightarrow V_r = -\frac{v_1}{2}$$

Entonces $\vec{V} = -\frac{v_1}{2}\hat{r} + \frac{1}{2}\sqrt{\frac{GM}{R}}\,\hat{\theta}$

(c) Energía después del choque

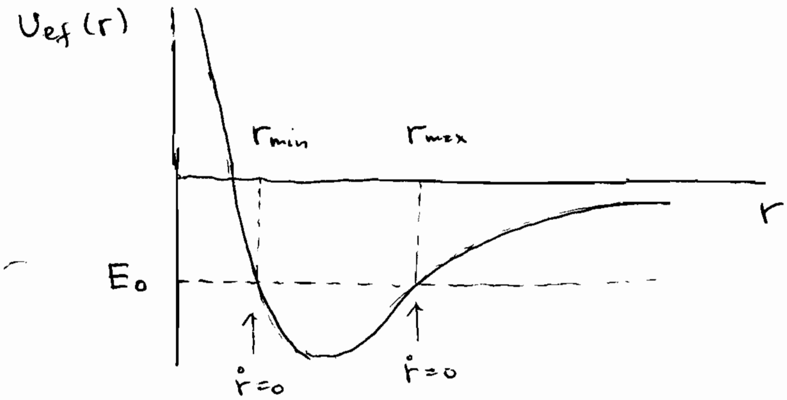
$$E_0 = \frac{1}{2}(2m)\vec{V}^2 - \frac{GM(2m)}{R}$$

$$= \frac{1}{2}(2m)\left[\frac{v_1^2}{4}\cdot\underbrace{\hat{r}\cdot\hat{r}}_{1} + -\left(\frac{v_1}{2}\right)\left(\frac{1}{2}\sqrt{\frac{GM}{R}}\right)\underbrace{\hat{r}\cdot\hat{\theta}}_{0} + \frac{1}{4}\frac{GM}{R}\underbrace{\hat{\theta}\cdot\hat{\theta}}_{1}\right] - \frac{2GMm}{R}$$

$$= \frac{1}{4}m v_1^2 - \frac{7}{4}\frac{GMm}{R} \quad (1)$$

La energía mecánica se puede expresar en función de un potencial efectivo:

$$E = \frac{1}{2} m \dot{r}^2 + \underbrace{\frac{L_0^2}{2mr^2} - \frac{GMm}{r}}_{U_{ef}(r)} \qquad (2)$$



$$(1) = (2) \Rightarrow \quad E_0 = \underbrace{\frac{1}{2} m \dot{r}^2}_{0} + \frac{L_0^2}{2m\, r^2} - \frac{GMm}{r}$$

0 (se impone para encontrar r_{min} y r_{max})

$\Rightarrow$ Se resuelve la ecuación cuadrática y se obtiene r_{min} (perigeo) y r_{max} (apogeo)

(d) Para que el cuerpo proyectil-satélite escape del campo gravitacional del planeta $\Rightarrow E_0 = 0$

(trayectoria parabólica)

de (1) $\Rightarrow \frac{1}{4} \cancel{m} v_1^2 - \frac{7}{4} \frac{GM\cancel{m}}{R} = 0$

$$\Rightarrow \boxed{v_1 = \sqrt{\frac{7GM}{R}}}$$