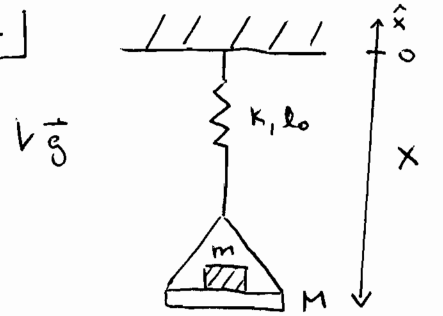
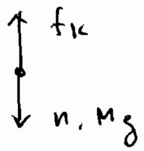
P1]



Datos: M, m, k

M:



$$f_k - Mg - n = M\ddot{x}$$

donde $f_k = -k\,(x - \underbrace{(-l_0)}_{})\; = -k\,(x + l_0)$

posición del largo natural c/r al sistema de referencia

$$\Rightarrow\; -k(x + l_0) - Mg - n = M\ddot{x} \quad (1)$$

m:

$$n - mg = m\ddot{x} \quad (2)$$

$$(1)+(2) \Rightarrow -k\,(x + l_0) - (M+m)\,g = (M+m)\,\ddot{x}$$

$$\Rightarrow \ddot{x} + \frac{k}{M+m}(x + l_0) + g = 0$$

$$\Rightarrow \ddot{x} + \frac{k}{M+m}\left(x + l_0 + \frac{M+m}{k}\cdot g\right) = 0$$

$$\Rightarrow x(t) = A\cos\left(\sqrt{\frac{k}{M+m}}\,t + \phi\right) + l_0 + \frac{M+m}{k}\cdot g \quad (3)$$

$$(3) \text{ en } (2) \Rightarrow n = mg - m\cdot A\,\frac{k}{M+m}\cos(\;) \geq 0$$

↑ se impone

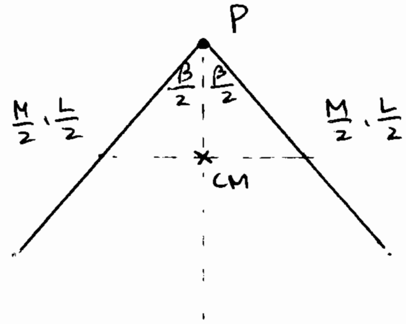
$$\Rightarrow A\cos(\;) \leq \frac{M+m}{k}\cdot g$$

Casos extremos: • si $\cos(\;) = 1 \Rightarrow A \leq \frac{M+m}{k}g$

• si $\cos(\;) = -1 \Rightarrow -A \leq \frac{M+m}{k}g \iff A \geq -\frac{M+m}{k}g$

$$\Rightarrow \boxed{|A| \leq \frac{M+m}{k}\cdot g}$$

P2]

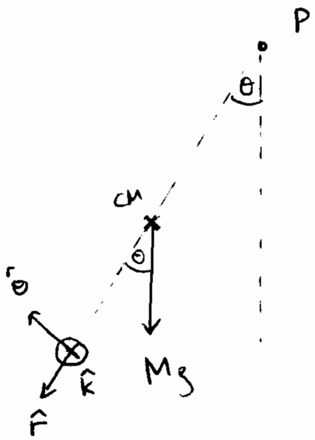


$\forall \vec{g}$ Datos: M, L, β

¿ T ?

distancia del CM a P: $\frac{L}{4} \cos \frac{\beta}{2}$

$$I_p = \frac{1}{3}\left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2 + \frac{1}{3}\left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2 = \frac{1}{12} M L^2$$



$$\tau_p = I_p \ddot{\theta}$$

$$-\frac{\cancel{L}}{\cancel{4}} \cos\frac{\beta}{2} \cdot \cancel{M} g \sin\theta = \frac{1}{\cancel{12}_{\,3}} \cancel{M} L^{\cancel{2}} \ddot{\theta}$$

$$\Rightarrow \ddot{\theta} + \frac{3 g \cos \beta/2}{L} \sin\theta = 0$$

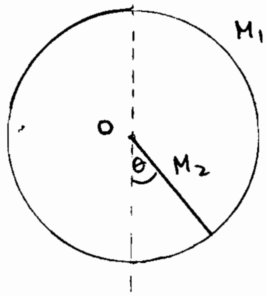
pequeñas oscilaciones $\sin\theta \approx \theta$

$$\Rightarrow \ddot{\theta} + \underbrace{\frac{3 g \cos \beta/2}{L}}_{\omega^2} \theta = 0$$

$$\Rightarrow \omega = \sqrt{\frac{3 g \cos \beta/2}{L}} \quad ; \quad \omega = \frac{2\pi}{T}$$

$$\Rightarrow \boxed{T = 2\pi \sqrt{\frac{L}{3 g \cos \beta/2}}}$$

P3]

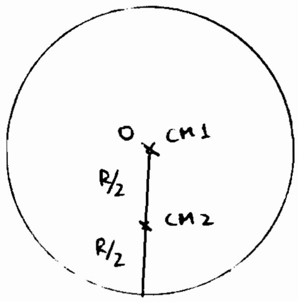


$\forall \vec{g}$ Datos: M, R

Llamemos M_1: masa del aro

M_2: masa de la barra

$\Rightarrow$ densidad lineal de masa $\lambda = \dfrac{M}{2\pi R + R} = \dfrac{1}{2\pi + 1}\dfrac{M}{R}$



distancia del CM al pto O

$= \dfrac{M_1 \cdot 0 + M_2 \cdot R/2}{M}$ donde $M_1 = \lambda \cdot 2\pi R$, $M_2 = \lambda \cdot R$

$= \dfrac{\lambda R^2}{2M}$

$= R/2(2\pi+1)$

$$I_o = M_1 R^2 + \frac{1}{3} M_2 R^2$$

$$= \frac{2\pi + 1/3}{2\pi + 1} M R^2$$

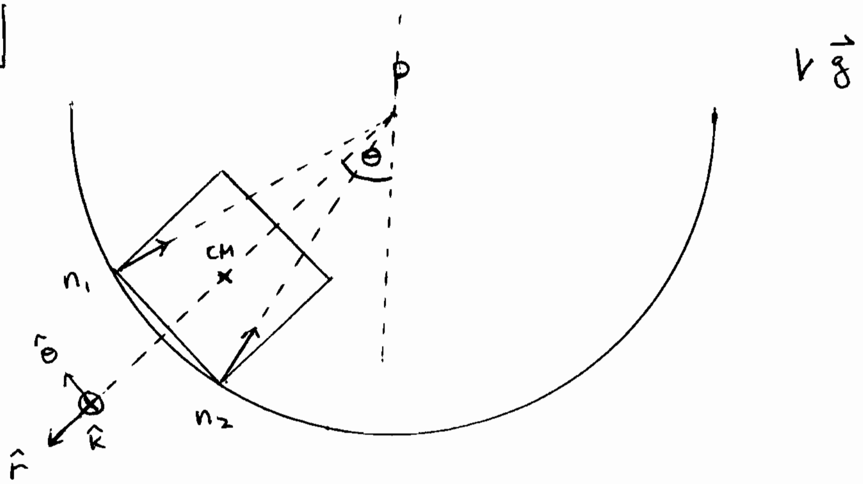
$$\tau_o = I_o \ddot{\theta} \Rightarrow -\frac{\cancel{R}}{2\cancel{(2\pi+1)}} \cancel{M} g \sin\theta = \frac{2\pi + 1/3}{2\pi+1} \cancel{M} R^{\cancel{2}} \ddot{\theta}$$

osc. pequeñas $\Rightarrow \ddot{\theta} + \underbrace{\dfrac{g/R}{2(2\pi + 1/3)}}_{\omega^2} \theta = 0$

$$\Rightarrow \boxed{T = 2\pi \sqrt{2(2\pi + 1/3) \cdot \frac{R}{g}}}$$

P4

Datos: b, R.



Recuerdo: $\begin{cases} \tau_{CM} = I_{CM}\, \ddot{\theta} \\ \tau_0 = I_0\, \ddot{\theta} \end{cases}$, donde O es un punto fijo en torno al cual rota el sólido.

En este caso aplicamos $\tau_P = I_P\, \ddot{\theta}$

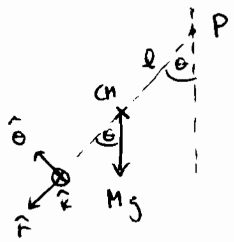
Llamemos a la distancia $\overline{P\,CM} \equiv \ell$

$$I_{\substack{\text{cuadrado} \\ \text{c/r CM}}} = \frac{1}{3} M b^2 \quad \text{(propuesto)}$$

Por Steiner, $I_{\substack{\text{cuadrado} \\ \text{c/r P}}} = \frac{1}{3} M b^2 + M \ell^2$

$$\tau_P = I_P\, \ddot{\theta} \Rightarrow -\ell M g \sin\theta = \left(\frac{1}{3} M b^2 + M \ell^2\right) \ddot{\theta}$$

$$\Rightarrow \ddot{\theta} + \frac{g/\ell}{\frac{1}{3}\left(\frac{b}{\ell}\right)^2 + 1} \cdot \sin\theta = 0$$

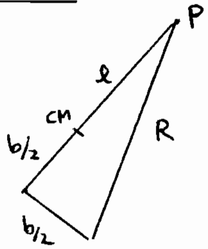


Pequeñas oscilaciones $\Rightarrow$ ($\theta \ll 1 \Rightarrow \sin\theta \simeq \theta$)

$$\ddot{\theta} + \underbrace{\frac{g/\ell}{\frac{1}{3}\left(\frac{b}{\ell}\right)^2 + 1}}_{\omega^2} \cdot \theta = 0$$

$$\Rightarrow \boxed{f = \frac{1}{2\pi} \sqrt{\frac{g/\ell}{\frac{1}{3}\left(\frac{b}{\ell}\right)^2 + 1}}}$$

Falta calcular ℓ:



$$R^2 = (\ell + b/2)^2 + (b/2)^2$$

$$\Rightarrow \ell = \sqrt{R^2 - (b/2)^2} - b/2$$

Si $b \ll R \Rightarrow \ell \simeq R$ y $b/\ell \ll 1 \Rightarrow f \simeq \frac{1}{2\pi}\sqrt{\frac{g}{\ell}}$ resultado conocido