HEAT BALANCE IN LAKES AND RESERVOIRS

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1 Heat exchange with the atmosphere

A lake can exchange heat with the atmosphere, inflows, outflows, and the bed sediments. These processes are represented schematically in Fig. 1, for a lake of volume V and mean temperature T, where H_n denotes the net heat flux from the atmosphere to the water body per unit surface area, A_s; H_{sed} denotes the net heat flux from the water column to the bed sediments per unit bed area, A_{sed}; H_i denotes the net heat flux input from the inflows; and H_o denotes the net heat flux extracted by the outflows. The bulk balance of heat within the water body is given by:

$$C_p\ \rho\ V\ \frac{dT}{dt} = A_s\ H_n - A_{sed}\ H_{sed} + H_i - H_o \tag{1}$$

where ρ and C_p denote water density and specific heat, respectively.

Considering, for the sake of simplicity, only the exchange with the atmosphere, then the variation in time of the bulk water temperature is governed by:

$$C_p\ \rho\ V\ \frac{dT}{dt} = A_s\ H_n \tag{2}$$

This is a relatively simple equation stating that the bulk temperature of the water body is driven by variations in time of the net heat influx from the atmosphere. In order to solve for $T(t)$, $H_n(t)$ must be estimated first. This can be done by taking into account the different processes that determine the heat exchange between the atmosphere and the water column:

$$H_n = H_{sw} + H_H - H_B - H_L - H_s \tag{3}$$

where H_{sw} denotes the heat flux due to short wave radiation from the sun; H_H denotes the heat flux from clouds and atmosphere in general, which absorbe part of the solar radiation and then reflect it back to the earth in the form of long wave radiation; H_B is the heat flux due to black-body radiation from the free surface; H_L denotes heat losses due to evaporation; and H_s corresponds to heat losses due to sensible heat transfer, associated to heat conduction and advection resulting from wind, waves and currents.

These fluxes can be estimated from the following empirical relationships:

- Short wave radiation

$$H_{sw} = H_0\ a_t\ (1 - R_s)\ C_a \tag{4}$$

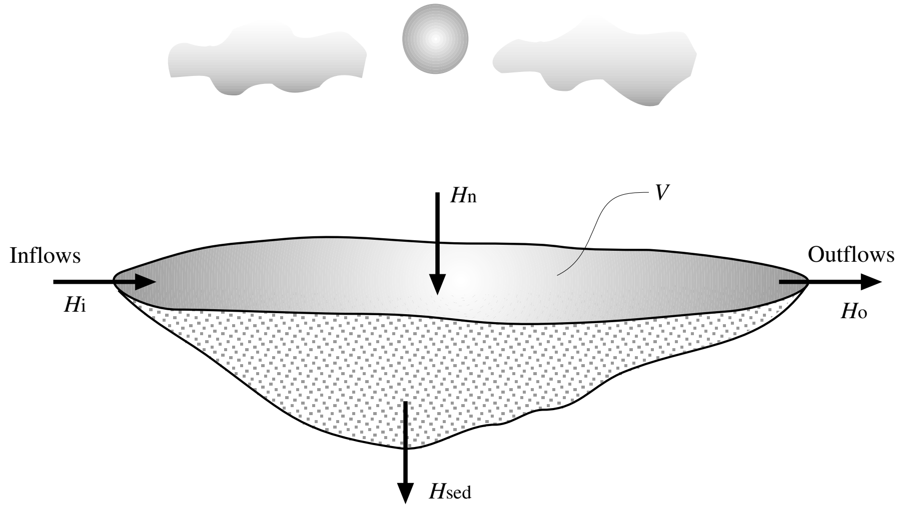

Figure 1: Heat exchange in lakes and reservoirs.

where H_0 represents short wave solar radiation arriving at the outer atmosphere, a_t denotes an atmospheric transmission coefficient, R_s is the water surface albedo or reflection coefficient and C_a denotes the fraction of solar radiation not absorbed by clouds.

- Long wave radiation

$$H_H = 0.97\, \epsilon_a\, \sigma\, (T_a + 273.16)^4 \tag{5}$$

This represents Stefan-Boltzman's law for the black-body long wave radiation of the atmosphere, where ϵ_a is the emissivity of air, which depends on a number of factors, such as the fraction of the sky covered by clouds, vapour pressure, etc.; σ is Stefan-Boltzman constant; and T_a denotes dry bulb air temperature in ^{o}C. This equation assumes that 3 % of the incoming long wave radiation is reflected back to the atmosphere.

- Back radiation from free surface

$$H_B = \epsilon_w\, \sigma\, (T_s + 273.16)^4 \tag{6}$$

This equation is also Stefan-Boltzman's law, but now applied to the black-body long wave radiation emited by the water surface. Here, ϵ_w denotes the emissivity of water and T_s denotes the water surface temperature in ^{o}C.

- Heat losses due to evaporation

$$H_L = \rho\, L_w\, E \tag{7}$$

where L_w denotes latent heat, that is, the amount of heat required to evaporate water per unit mass, and E is the evaporation rate, which is proportional to the water vapor pressure gradient between the water and atmosphere and depends on the wind speed near the water surface. E can be estimated as:

$$E = (a + b\, u_w)\,(p_{vs} - p_{va}) \tag{8}$$

where a and b are coefficients, u_w denotes wind speed, p_{vs} denotes saturated vapor pressure at the water surface temperature and p_{va} denotes the vapor pressure at the air temperature.

- Heat losses due to conduction and convection

$$H_s = H_L\, R_B \tag{9}$$

where R_B denotes *Bowen's Ratio*, such that:

$$R_B = C_B\, \frac{p_a}{p_{asl}}\, \frac{T_s - T_a}{p_{vs} - p_{va}} \tag{10}$$

where C_B represents a coefficient, p_a denotes local atmospheric pressure and p_{asl} denotes atmospheric pressure at sea level.

To analyze vertical heat transport in the water body it is important to consider the penetration of short wave solar radiation through the water column. The remaining radiation at a depth z measured downwards from the water surface is given by *Beer's Law*:

$$H_{sw}(z) = H_{sw}\ \exp(-k_e\, z) \tag{11}$$

where $H_{sw}(z)$ represents the heat flux at a depth z, H_{sw} corresponds to the heat flux across the water surface and k_e denotes the light extinction coefficient. This coefficient depends on turbidity and color of the water column and can be measured with a Secchi disk or a photometer.

2 Equilibrium Temperature Method

A simplified methodology to estimate the net heat flux exchange between the atmosphere and the water column is that known as *Equilibrium Temperature Method*. This method is based on the observation that, given a set of meteorological conditions that remain fixed for a sufficiently long time, a water body tends to reach a constant equilibrium temperature, which remains constant as long as those meteorological conditions prevail. From this observation, the net heat flux H_n can be estimated as:

$$H_n = C_e\,(T_e - T_s) \tag{12}$$

where C_e represents a coefficient of surface heat exchange, T_e denotes equilibrium temperature and T_s denotes water surface temperature.

Although this expression is rather simple, estimating C_e and T_e is as complex as estimating all the terms in the complete heat balance equation discussed in the previous section.

To estimate T_e it is convenient to rewrite equation (3) as follows:

$$H_n = H_1 - H_2 \tag{13}$$

with:

$$H_1 = H_{sw} + H_H \tag{14}$$

and:

$$H_2 = H_B + H_L + H_s \tag{15}$$

Note that H_1 does not depend on water temperature, while H_2 does. Indeed, expanding the latter term yields:

$$H_2 = \epsilon_w\ \sigma\ (T_s + 273.16)^4 + \rho\ L_w\ (a + b\ u_w)\ (p_{vs} - p_{va})\ (1 + C_B\ \frac{p_a}{p_{asl}}\ \frac{T_s - T_a}{p_{vs} - p_{va}}) \tag{16}$$

Introducing approximations in both terms, it is possible to simplify this equation to:

$$H_2 \approx \epsilon_w\ \sigma\ (T_K^4 + 4\ T_K^3\ T_s + 6\ T_K^2\ T_s^2) - \rho\ L_w\ (a + b\ u_w)\ \{(C_B\ \frac{p_a}{p_{asl}}\ T_a + \beta\ T_d) - $$
$$-(C_B\ \frac{p_a}{p_{asl}} + \beta)\ T_s\} \tag{17}$$

where $T_K = 273.16\ ^o\mathrm{C}$ and β represents the local slope of the function that relates saturated vapor pressure and temperature, such that:

$$p_{vs} - p_{va} \approx \beta\ (T_s - T_d) \tag{18}$$

where T_d represents the dew point temperature corresponding to p_{va}.

Considering that when $T_s = T_e$ the net heat flux from the atmosphere vanishes, that is $H_n = 0$, then in such case $H_1 = H_2$, which gives the following equation to determine T_e for the prevailing atmospheric conditions:

$$H_1 - \epsilon_w\ \sigma\ (T_K^4 + 4T_K^3\ T_e + 6T_K^2\ T_e^2) + \rho\ L_w\ (a + b\ u_w)\ \{(C_B\ \frac{p_a}{p_{asl}}\ T_a + \beta\ T_d) - $$
$$-(C_B\ \frac{p_a}{p_{asl}} + \beta)\ T_e\} = 0 \tag{19}$$

To estimate C_e it is convenient to take the derivative of H_n with respect to T_s. Since:

$$H_n = C_e\ (T_e - T_s) = H_1 - H_2(T_s) \tag{20}$$

then:

$$\frac{\partial H_n}{\partial T_s} = -C_e = -\frac{\partial H_2}{\partial T_s} \tag{21}$$

Thus, taking the derivative of (17) yields:

$$C_e \approx \epsilon_w\ \sigma\ (4\ T_K^3 + 12\ T_K^2\ T_e) + \rho\ L_w\ (a + b\ u_w)\ (C_B\ \frac{p_a}{p_{asl}} + \beta) \tag{22}$$

References:

- Hydrodynamics and transport for water quality modelling. Martin and McCutcheon (1999). Lewis Publishers.
- Applied Hydrology. Chow, Maidment and Mays (1988). McGraw-Hill