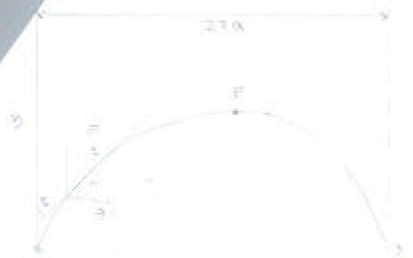
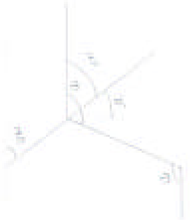


$$X = a(\theta - \sin\theta)$$
$$Y = a(1 - \cos\theta)$$



$z = 100 - \alpha$

$tg\,\alpha = m$ (pendiente)

$$m = \frac{dY}{dX} = \frac{(dY/d\theta)}{(dX/d\theta)} = \frac{\sin\theta}{1-\cos\theta} = ctg\frac{\theta}{2}$$

$\boxed{m = ctg\frac{\theta}{2}}$ $\quad tg\,\alpha = ctg\frac{\theta}{2}$

$\boxed{\alpha = arctg\left(ctg\frac{\theta}{2}\right)}$

también

$\boxed{ctg\frac{\theta}{2} = tg(100 - z)}$

Arco L: $ds = \sqrt{dx^2 + dY^2}$

$$ds^2 = a^2\left((1-\cos\theta)^2 + \sin^2\theta\right)d\theta^2$$
$$= 2a^2(1-\cos\theta)\,d\theta^2 = 4a^2\sin^2\tfrac{1}{2}\theta\,d\theta^2$$
$$ds = 2a\sin\frac{\theta}{2}\,d\theta$$

$$L = \int_0^\theta 2a\sin\frac{\theta}{2}\,d\theta$$

$\boxed{L = 4a\left(1 - \cos\frac{\theta}{2}\right)}$

pero $ctg\frac{\theta}{2} = tg(100 - z)$

$$\cos\frac{\theta}{2} = \sin\frac{\theta}{2}\,tg(100 - z)$$
$$\cos\frac{\theta}{2} = \sqrt{1 - \cos^2\frac{\theta}{2}}\;ctg\frac{\theta}{2}$$
$$\cos^2\frac{\theta}{2} = \left(1 - \cos^2\frac{\theta}{2}\right)ctg^2\frac{\theta}{2}$$
$$= \frac{ctg\frac{\theta}{2}}{\sqrt{1 + ctg^2\frac{\theta}{2}}} = \frac{tg(100-z)}{\sec(100-z)} = \sin(100-z)$$

$\boxed{\cos\frac{\theta}{2} = \sin(100 - z)}$

$\boxed{L = 4a(1 - \sin(100 - z))}$

$R = 4a\sin\frac{\theta}{2}$ ó $\boxed{R = 4a\cos(100 - z)}$

$R(z)\,L(z) = 16a^2\cos(100-z)(1 - \sin(100-z))$ $\quad (z = 100)$

$R \cdot L = A^2 = 16a^2$ $\quad \boxed{a = \frac{A}{4}}$

$$\text{Área} = \int_0^{2\pi a} y\,dx = \int_0^{2\pi a} y\,\frac{dx}{d\theta}\,d\theta = \int_0^{2\pi a} a^2(1-\cos\theta)^2\,d\theta$$

$$\text{Área} = 3\pi a^2$$

consideración económica

$$\left(2\pi a B - 3\pi a^2\right) P_{pasto} + 8a\, P_{cerca} = 550000 \text{ U\$}$$

$$a = \frac{A}{4} = \frac{100}{4} = 25 \qquad \underline{a = 25}$$

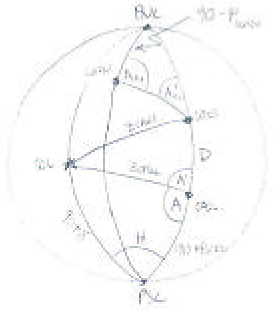
$P_{cer} = 1558{,}46$ U\$/m

$P_{pas} = 10{,}72$ U\$/m^2

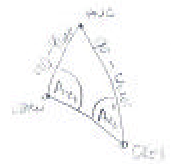
$$\boxed{B = \frac{500\,000 - 8a\,P_{cer} + 3\pi a^2\,P_{pas}}{2\pi a\,P_{ast}}}$$

$$\boxed{B = 180{,}73 \text{ m}}$$

Pauta P3 Control 3, Otoño 2005



En el triángulo formado por Cornelia - Cressida - PNC se tiene:



$$\frac{\overbrace{\text{sen}(90-\varphi_{CORN})}^{P_{CORN}}}{\text{sen } A_1} = \frac{\text{sen}(90-\varphi_{CRES})}{\text{sen}(A_1')}$$

$$\text{sen}(90-\varphi_{CRES}) = \frac{0.867 \times 0.975}{0.155} = 0.984$$

$$\varphi' = 79.679^\circ$$

$$\Rightarrow \varphi_{CRES} = 90 - 79.679^\circ = 10.32^\circ$$

- Como Cressida está 4350 km al norte de Ofelia:

$$D = \frac{4460}{6371} = 0.7^{RAD} = 40.11^\circ$$

$$\varphi_{OFELIA} = \varphi_{CRES} - D = 10.32^\circ - 40.11^\circ = -29.79^\circ \quad (\text{Latitud Sur})$$

En el triángulo Ofelia - Cressida - Sol se tiene:

$$Z_{OFEL} = 42.066^{RAD} = 37.9594^\circ$$

$$Z_{CRES} = \text{tg}^{-1}(1.21) = 50.43^\circ$$

$$D = 40.11^\circ$$

- $$\cos A' = \frac{\cos Z_{CRES} - \cos Z_{OFEL} \cos D}{\text{sen } Z_{OFEL} \text{ sen } D} = \frac{0.637 - 0.790 \times 0.765}{0.614 \times 0.644} = 0.084$$

$$A' = 85.18^\circ$$

$$\Rightarrow A = 180 - A' = 94.82^\circ$$

Luego en el triángulo de posición de Ofelia:

$$\cos(90+\delta) = \cos Z_{OFEL} \cos(90+\varphi_{OFEL}) + \text{sen } Z_{OFEL} \text{ sen}(90+\varphi_{OFEL}) \cos A$$
$$= 0.790 \cdot 0.497 + 0.614 \times 0.868 \times (-0.084) = 0.347$$

$$\boxed{\delta = -20.33^\circ}$$ (Negativo pues está en el hemisferio sur → verano en Ofelia ✓ OK)

AHORA SE PUEDE CALCULAR EL ÁNGULO HORARIO DEL SOL PARA EL MISMO TRIÁNGULO DE POSICIÓN.

$$\cos H = \frac{\cos z_{obs} - \cos(90+\delta)\cos(90+\varphi_{obs})}{\operatorname{sen}(90+\delta)\operatorname{sen}(90+\varphi_{obs})} = \frac{0.990 - 0.347 \times 0.497}{0.938 \times 0.938} = 0.758$$

$$\therefore H = 40.7^{\circ} \longrightarrow 2.714^{h}$$

$$e = T_v - T_m$$

$$T_v = 2.714^{h}$$

$$\left.\begin{array}{l} \lambda = TU - TC \Rightarrow TC = TU - \lambda \\ TC = H_m + 12^{h} \end{array}\right\} \Rightarrow H_m = TU - \lambda - 12^{h}$$

$$\left.\begin{array}{l} \lambda_u = -4^{h} \Rightarrow TU = 15^{h}19^{m} + 4^{h} = 19^{h}19^{m} = 19.3167^{h} \\ \lambda = 71^{\circ}24' = 71.4^{\circ} = 4.76^{h} \end{array}\right\} \Rightarrow H_m = 19.3167^{h} - 4.76^{h} - 12^{h} = 2.557^{h}$$

$$e = 2.714 - 2.557 = 0.157^{h}$$ (Tv > Tm ✓OK PUES EL VERANO ES [illegible])

$$\therefore E = 12 + e = 12.157^{h} = 12^{h}9^{m}25.2^{s}$$

EN RESUMEN

$$\boxed{\begin{array}{l} \delta = -20.33^{\circ} = -20^{\circ}19'48'' \\ E = 12.157^{h} = 12^{h}9^{m}25.2^{s} \end{array}}$$