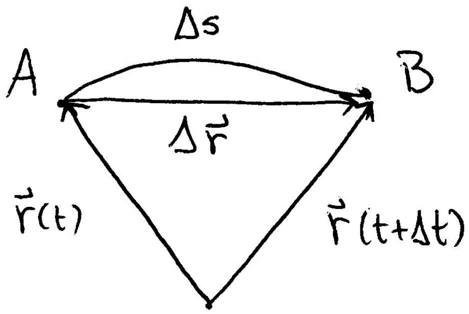
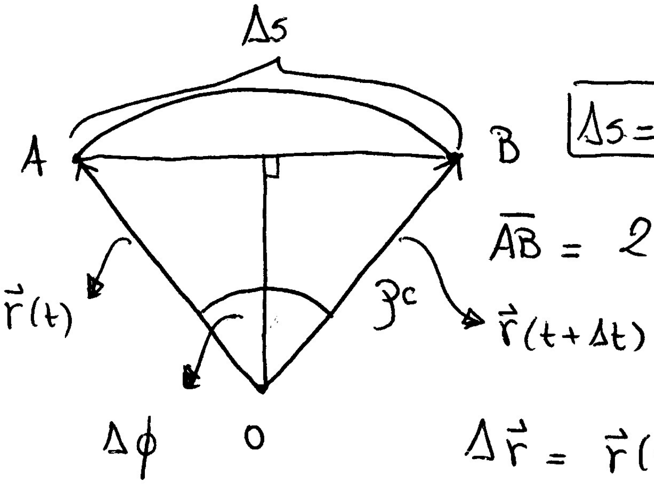
# Velocidad y Rapidez




$$\vec{v} = \frac{\vec{r}(t+\Delta t) - \vec{r}(t)}{\Delta t}$$

$$= \frac{\Delta \vec{r}(t)}{\Delta t} \underset{\Delta t \to 0}{\longrightarrow} \frac{d\vec{r}}{dt}$$

$$v = \frac{\Delta s}{\Delta t} \underset{\Delta t \to 0}{\longrightarrow} \frac{ds}{dt}$$




$$\boxed{\Delta s = \rho_c \cdot \Delta\phi} \underset{\Delta t \to 0}{\longrightarrow} \left\{ \frac{ds}{d\phi} = \rho_c \right\}$$

$$\overline{AB} = 2\,\rho_c \cdot \operatorname{sen}\left(\frac{\Delta\phi}{2}\right)$$

$$\Delta\vec{r} = \vec{r}(t+\Delta t) - \vec{r}(t)$$

$$\approx \frac{\Delta\vec{r}}{\Delta t} \cdot \Delta t$$

$$\approx \vec{v}(t) \cdot \Delta t$$

$$\underset{\Delta t \to 0}{\Rightarrow} \quad d\vec{r} = \vec{v}(t) \cdot dt$$

$$\|\Delta\vec{r}\| = |\overline{AB}| = 2\,\rho_c \cdot \operatorname{sen}\left(\frac{\Delta\phi}{2}\right)$$

$$\|\vec{v}\| = \lim_{\Delta t \to 0} \frac{\|\Delta\vec{r}\|}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = |v|$$

$\|\Delta\vec{r}\| \approx \Delta s$

ya que $\Delta\phi << 1$
"chico"

$\Rightarrow \|\vec{v}\| = |v|$

se def $\hat{t}$ tq $\|\hat{t}\| = 1$ y

$$\boxed{\vec{v} = v \cdot \hat{t}}$$

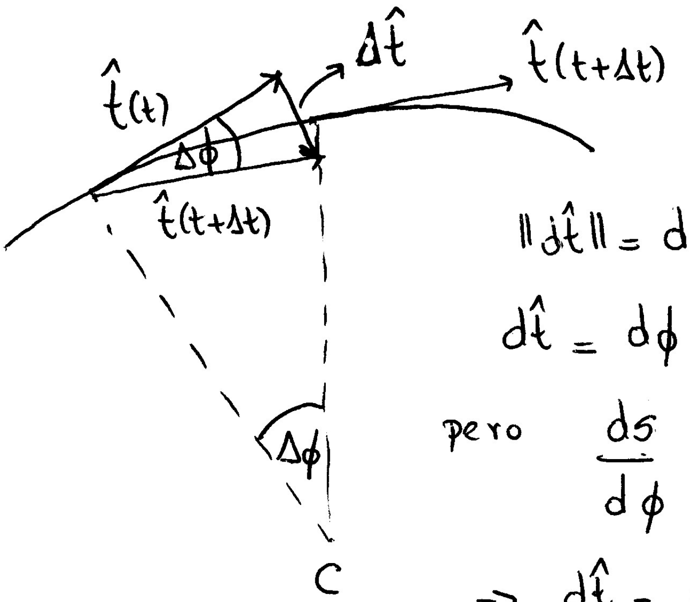
mo $\|\hat{t}\| = 1$

$$\Rightarrow \hat{t} \cdot \hat{t} = 1$$

$$\Rightarrow \frac{d}{dt}(\hat{t} \cdot \hat{t}) = 0$$

$$\Rightarrow \hat{t} \cdot \frac{d\hat{t}}{dt} = 0 \Rightarrow \boxed{\hat{t} \perp \frac{d\hat{t}}{dt}}$$

Pero.




cuando $\Delta\phi \approx 0$ "chico", $\Delta\hat{t}$ apunta hacia el "centro"

$\|d\hat{t}\| = d\phi$ "cuando son chicos"

$$d\hat{t} = d\phi \cdot \hat{n}$$

pero $\frac{ds}{d\phi} = \rho_c \Rightarrow d\phi = \frac{ds}{\rho_c}$

$$\Rightarrow d\hat{t} = \frac{ds}{\rho_c} \cdot \hat{n}$$

$$\Rightarrow \boxed{\frac{d\hat{t}}{ds} = \frac{1}{\rho_c} \cdot \hat{n}}$$

Aceleración

$$\vec{a}(t) = \frac{d\vec{v}(t)}{dt} = \frac{d(v(t) \cdot \hat{t})}{dt}$$

$$= v(t)\frac{d\hat{t}}{dt} + \frac{dv}{dt} \cdot \hat{t}$$

$$\frac{d\hat{t}}{dt} = \frac{d\hat{t}}{ds}\,\frac{ds}{dt} = \frac{1}{\rho_c}\cdot\hat{n}\cdot v$$

$$\Rightarrow \quad \vec{a}(t) = \frac{v^2}{\rho_c}\cdot\hat{n} + \frac{dv}{dt}\cdot\hat{t}$$

$$= \vec{a}_c + \vec{a}_t$$

$$\vec{a}\times\hat{t} = \frac{v^2}{\rho_c}(\hat{n}\times\hat{t}) + \frac{dv}{dt}\cdot\cancel{(\hat{t}\times\hat{t})}^{\,0}$$

$$\|\vec{a}\times\hat{t}\| = \left|\frac{v^2}{\rho_c}\right|\cdot\cancel{\|\hat{n}\|}^{\,1}\cdot\cancel{\|\hat{t}\|}^{\,1}\,\cancel{\operatorname{sen}\frac{\pi}{2}}^{\,1}$$

$$\Rightarrow \boxed{\frac{v^2}{\|\vec{a}\times\hat{t}\|} = \rho_c}$$