# Derivadas

$$\frac{df(t_0)}{dt} = \lim_{t \to t_0} \frac{f(t) - f(t_0)}{t - t_0} = \lim_{\Delta t \to 0} \frac{f(t_0 + \Delta t) - f(t_0)}{\Delta t}$$

## Calculemos:

$\rightarrow f(t) = c \qquad (c = cte)$

$$\Rightarrow \frac{df(t_0)}{dt} = \lim_{t \to t_0} \frac{f(t) - f(t_0)}{t - t_0} = \lim_{t \to t_0} \frac{c - c}{t - t_0} = \lim_{t \to t_0} \frac{0}{t - t_0} = 0.$$

$\rightarrow f(t) = a \cdot t^n \qquad (a \in \mathbb{R}) \qquad (n \in \mathbb{R})$

$$\frac{df(t)}{dt} = \lim_{\Delta t \to 0} \frac{f(t + \Delta t) - f(t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{a(t + \Delta t)^n - a \cdot t^n}{\Delta t}$$

$(1 + \Delta)^n \approx (1 + n\Delta) \qquad |\Delta| \ll 1$

Ej:

$(1 + 0{,}001)^3 = 1{,}003003001$

$(1 + 3 \cdot 0{,}001) = 1{,}003$

$$= \lim_{\Delta t \to 0} \frac{a \cdot t^n \left(1 + \frac{\Delta t}{t}\right)^n - a \cdot t^n}{\Delta t}$$

$$\approx \lim_{\Delta t \to 0} \frac{\cancel{a \cdot t^n}\left(1 + \frac{n \Delta t}{t}\right) - \cancel{a t^n}}{\Delta t}$$

$$\approx \lim_{\Delta t \to 0} \frac{a \cdot n \cdot t^{n-1} \cdot \cancel{\Delta t}}{\cancel{\Delta t}}$$

$$\Rightarrow \frac{df(t)}{dt} = a \cdot n \cdot t^{n-1}$$

$\rightarrow f(t) = b \cdot \cos(dt) \qquad d, t \in \mathbb{R}$

$$\frac{df(t)}{dt} = \frac{d}{dt}(b \cdot \cos(dt)) = \lim_{\Delta t \to 0} \frac{b \cdot \cos(dt + d\Delta t) - b\cos(d \cdot t)}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{b\left[\cos(dt)\underbrace{\cos(d\Delta t)}_{1} - \operatorname{sen}(dt)\underbrace{\operatorname{sen}(d\Delta t)}_{d\Delta t} - b\cos(d \cdot t)\right]}{\Delta t}$$

$$\Rightarrow \frac{d}{dt}(b\cdot\cos(d\cdot t)) = \lim_{\Delta t \to 0} \frac{-b\cdot d\cdot \mathrm{sen}(d\cdot t)\cancel{\Delta t}}{\cancel{\Delta t}} = -b\cdot d\cdot \mathrm{sen}(d\cdot t)$$

$$\Rightarrow \frac{d}{dt}(b\cdot\cos(d\cdot t)) = -b\cdot d\cdot \mathrm{sen}(d\cdot t)$$

<u>Tabla</u>

$\rightarrow f(t) = cte \Rightarrow \frac{df(t)}{dt} = 0$

$\rightarrow f(t) = a\cdot t^n \Rightarrow \frac{df(t)}{dt} = a\cdot n\cdot t^{n-1}$

$\rightarrow f(t) = \cos(t) \Rightarrow \frac{df(t)}{dt} = -\mathrm{sen}(t)$

$\rightarrow f(t) = \mathrm{sen}(t) \Rightarrow \frac{df(t)}{dt} = \cos(t)$

$\rightarrow f(t) = g(t) + h(t) \Rightarrow \frac{df(t)}{dt} = \frac{dg(t)}{dt} + \frac{dh(t)}{dt}$

$\rightarrow f(t) = cte\cdot g(t) \Rightarrow \frac{df(t)}{dt} = cte\cdot\frac{dg(t)}{dt}$

$\rightarrow f(t) = g(t)\cdot h(t) \Rightarrow \frac{df(t)}{dt} = \frac{dg(t)}{dt}\cdot h(t) + g(t)\cdot\frac{dh(t)}{dt}$

$\rightarrow f(t) = \frac{g(t)}{h(t)} \Rightarrow \frac{df(t)}{dt} = \frac{\frac{dg(t)}{dt}\cdot h(t) - g(t)\cdot\frac{dh(t)}{dt}}{h(t)^2}$

$\rightarrow f(g(t)) \Rightarrow \frac{df(g(t))}{dt} = \frac{df(g(t))}{dt}\cdot\frac{dg(t)}{dt}$

Ej: sea $f(g(t)) = \cos(bt)$, $g(t) = b\cdot t$ $\Rightarrow \boxed{\frac{df(g(t))}{dt} = -\mathrm{sen}(bt)\cdot b}$