Pauta P1, EJ. 6 ME33A

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} \Rightarrow v_r = \frac{1}{r}\frac{\partial}{\partial \theta}\left(2r^2 sen(2\theta)\right) = \frac{1}{r}\cdot 2r^2 \frac{\partial\, sen(2\theta)}{\partial \theta} = 2r\cdot \cos(2\theta)\cdot 2 = 4r\cos(2\theta)$$

$$v_\theta = -\frac{\partial \psi}{\partial r} \Rightarrow v_\theta = -\frac{\partial}{\partial r}\left(2r^2 sen(2\theta)\right) = -2\cdot 2r\, sen(2\theta) = -4r\, sen(2\theta)$$

$$\Rightarrow v_r = 4r\cos(2\theta)$$

$$v_\theta = -4r\, sen(2\theta)$$

Además, $v_r = \frac{\partial \phi}{\partial r} = 4r\cos(2\theta) \Rightarrow \partial\phi = 4r\cos(2\theta)\, dr \,\Big/\!\int \Rightarrow \phi = 4\cos(2\theta)\cdot\frac{r^2}{2} \Rightarrow \boxed{\phi = 2r^2\cos(2\theta)}$

Aplicando Bernoulli:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g z_1 = P_2 + \frac{1}{2}\rho v_2^2$$

$$v^2 = v_r^2 + v_\theta^2$$

$$\Rightarrow v_1^2 = \left(4\cdot 0{,}5\cdot\cos(2\cdot\tfrac{\pi}{2})\right)^2 + \left(-4\cdot 0{,}5\cdot sen(2\cdot\pi/2)\right)^2$$
$$\Rightarrow v_1^2 = (2\cdot\cos(\pi))^2 + (-2\cdot sen(\pi))^2 = (2\cdot -1)^2 + (-2\cdot 0)^2 = 4$$
$$\Rightarrow v_1^2 = 4\ m^2/s^2$$

$$v_2^2 = (4\cdot 1\cdot\cos(2\cdot 0))^2 + (-4\cdot 1\cdot sen(2\cdot 0))^2 = (4\cdot 1)^2 + (-4\cdot 0)^2$$
$$\Rightarrow v_2^2 = 16\ \frac{m^2}{s^2}$$

$$\Rightarrow P_1 + \frac{1}{2}\rho\cdot 4 + \rho g\cdot 0{,}5 = P_2 + \frac{1}{2}\rho\cdot 16$$

$$\Rightarrow 30KPa + \frac{1}{2}\cdot 1000\frac{kg}{m^3}\, 4\frac{m^2}{s^2} + 1000\frac{kg}{m^3}\cdot 9{,}8\frac{m}{s^2}\cdot 0{,}5m = P_2 + \frac{1}{2}\cdot 1000\frac{kg}{m^3}\cdot 16\frac{m^2}{s^2}$$

$$\Rightarrow 30KPa + 2\cdot 1000\frac{kg}{ms^2} + 4{,}9\cdot 1000\frac{kg}{ms^2} - 8\cdot 1000\frac{kg}{ms^2} = P_2$$

$$\Rightarrow P_2 = 30KPa + 2KPa + 4{,}9KPa - 8KPa$$

$$\Rightarrow \boxed{P_2 = 28{,}9KPa}$$

PAUTA P2, EJ. 6 MC33A

$$\phi = \frac{\Gamma}{2\pi}\theta$$

$$\psi = -\frac{\Gamma}{2\pi}\ln(r)$$

$$v_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta} \Rightarrow v_\theta = \frac{1}{r}\frac{\Gamma}{2\pi}$$

$$v_r = 0$$

$$v^2 = v_\theta^2 = \left(\frac{\Gamma}{2\pi r}\right)^2$$

APLICANDO Bernoulli: $\cancel{P_1} + \rho g \cancel{z_1}^{0} + \cancel{\frac{v_1^2}{2}}^{0}\rho = \cancel{P_2} + \rho g z_2 + \frac{v_2^2}{2}\rho$

$$z_1 = 0$$
$$v_1 = 0$$
$$P_1 = P_2$$

$$\Rightarrow \cancel{\rho} g z_2 + \frac{v_2^2}{2}\cancel{\rho} = 0 \Rightarrow z_2 = \frac{-v_2^2}{2g}$$

$$v_2^2 = \left(\frac{\Gamma}{2\pi r}\right)^2 \Rightarrow z_2 = -\frac{\Gamma^2}{4\pi^2 r^2 2\cdot g} \Rightarrow \boxed{z_2 = -\frac{\Gamma^2}{8\pi^2 r^2 g}}$$