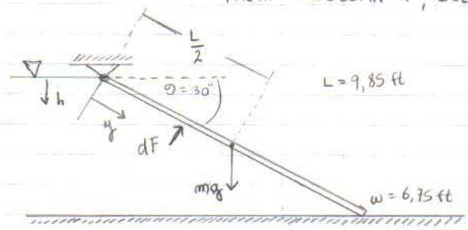
PAUTA PROBLEMA 1, EJERCICIO 2



$$F = \int P\, dA \quad ; \quad \frac{dP}{dh} = \rho \cdot g$$

$$\sum M_o = 0 \quad ; \quad M = \int y \cdot dF$$

SUPONEMOS QUE EL FLUIDO ES ESTÁTICO, QUE ρ ES CONSTANTE Y PATM ACTÚA SOBRE LA SUPERFICIE DEL AGUA Y SOBRE LA PUERTA.

$$P = \rho g h = \rho g \,\mathrm{sen}(\theta)$$

$$\sum M_o = 0 \Rightarrow \int y\, dF - mg \frac{L}{2}\cos(\theta) = 0$$

$$\Rightarrow mg = \frac{2}{L\cos(\theta)} \int y\, dF = \frac{2}{L\cos(\theta)} \int y\, P dA = \frac{2}{L\cos(\theta)} \int y \cdot \rho \cdot g \cdot h\, dA$$

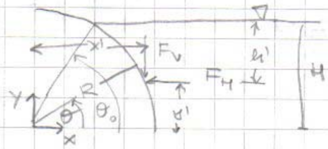
$$\Rightarrow mg = \frac{2}{L\cos(\theta)} \int_0^w \int_0^L y \cdot \rho \cdot g \cdot y \cdot \mathrm{sen}(\theta)\, dy\, dz = \frac{2w}{L\cos(\theta)} \cdot \rho \cdot g \cdot \mathrm{sen}(\theta) \int_0^L y^2\, dy$$

$$\Rightarrow mg = \frac{2w\rho g}{\cancel{L}} \,\mathrm{tg}(\theta) \cdot \frac{L^3}{3} = \frac{2w\rho g L^2}{3} \,\mathrm{tg}(\theta) = \frac{2 \cdot 6{,}75\,\mathrm{ft} \cdot 1{,}94\, \mathrm{slug}/\mathrm{ft}^3 \cdot 32{,}2\, \mathrm{ft}/\mathrm{s}^2 \cdot 9{,}85^2\, \mathrm{ft}^2}{3} \cdot \mathrm{tg}(30°)$$

$$\Rightarrow mg = 15746{,}42 \ \frac{\mathrm{slug} \cdot \mathrm{ft}}{\mathrm{s}^2} \Rightarrow \boxed{mg = 15746{,}42\,[\mathrm{lbf}]}$$ $\rightarrow$ EL PESO MÍNIMO DE LA COMPUERTA ES $P_{min} \sim 15800\,\mathrm{lbf}$

$$\Rightarrow \boxed{m = 489{,}02 \ \mathrm{slug}}$$

PAUTA PROBLEMA 2, EJERCICIO 2



Encontrar F_V, F_H, x', y'

Solución.

Ecuaciones básicas (a) $\dfrac{dP}{dh} = \rho g$, (b) $F_V = \int P dA_y$, (c) $x' F_V = \int x dF_V$

(d) $F_H = -P_c A$ (e) $h' = h_c + \dfrac{I_{xx}}{h_c A}$

Suponemos: (1) fluido estático
(2) ρ = constante
(3) Patm sobre la superficie libre del fluido

Integrando (a) tenemos $P = \rho g h$

De la geometría del problema se tiene:

$h = H - R\sin(\theta)$ $y = R\sin(\theta)$, $x = R\cos(\theta)$

$\theta_0 = \sin^{-1}\left(\frac{H}{R}\right)$ $dA = W R d\theta$

• Cálculo de F_V:

$$F_V = \int P dA_y = \int \rho g h\, dA \sin\theta = \int_0^{\theta_0} \rho g (H - R\sin(\theta))\sin(\theta) W R d\theta$$

$$= \rho g W R \int_0^{\theta_0} (H\sin(\theta) - R\sin^2(\theta))\, d\theta$$

$$= \rho g W R \left[-H\cos(\theta) - R\left(\frac{\theta}{2} - \frac{\sin(2\theta)}{4}\right)\right]\Bigg|_0^{\theta_0}$$

$$= \rho g W R \left[H(1-\cos(\theta_0)) - R\left(\frac{\theta_0}{2} - \frac{\sin(2\theta_0)}{4}\right)\right]$$

Evaluando para $\theta_0 = \sin^{-1}\left(\frac{0,65}{0,75}\right) = 60^\circ$ ó $\frac{\pi}{3}$

$$\boxed{F_V = 2,47\ [kN]}$$

$$x' F_V = \rho g W R \int_0^{\theta_0} R\cos(\theta)\left(H\sin(\theta) - R\sin^2(\theta)\right) d\theta$$

$$= \rho g W R^2 \int_0^{\theta_0} \left(H\sin(\theta)\cos(\theta) - R\sin^2(\theta)\cos(\theta)\right) d\theta$$

$$= \rho g W R^2 \left[H \frac{\sin^2(\theta)}{2} - R\frac{\sin^3(\theta)}{3}\right]_0^{\theta_0}$$

$$\Rightarrow \quad x' = \frac{\rho g W R^2}{F_V}\left[\frac{H\sin^2(\theta_0)}{2} - \frac{R}{3}\sin^3(\theta_0)\right]$$

$$\boxed{x' = 0,645\ [m]}$$

- Calculo de F_H

$$F_H = P_c \cdot A = \rho g h_c H W = \rho g \frac{H}{2} H W = \frac{\rho g H^2 W}{2}$$

$$\boxed{F_H = 7,35\ [kN]}$$

ó tambien

$$F_H = \rho g R W \int_0^{\theta_0} (H - R\sin\theta)\cos\theta\, d\theta = \rho g R W \left[H\sin(\theta_0) - R\frac{\sin^2(\theta_0)}{2}\right]$$

$$= 7,35\ [kN]$$

$$h' = h_c + \frac{I_{xx}}{h_c A} = h_c + \frac{1}{12}\frac{WH^3}{h_c A} = \frac{H}{2} + \frac{1}{12}\frac{WH^3}{\frac{H}{2}HW} = \frac{2}{3}H$$

$$y' = H - h' = H - \frac{2}{3}H = \frac{1}{3}H$$

$$\boxed{y' = 0{,}21\ [m]}$$

Tambien se podía sacar de la siguiente forma

$$tg(\theta^*) = \frac{F_V}{F_H} \Rightarrow \theta^* = 18{,}6°$$

$$x' = R\cos(\theta^*)$$
$$y' = R\,sen(\theta^*)$$

- Las ecuaciones para dibujar son:

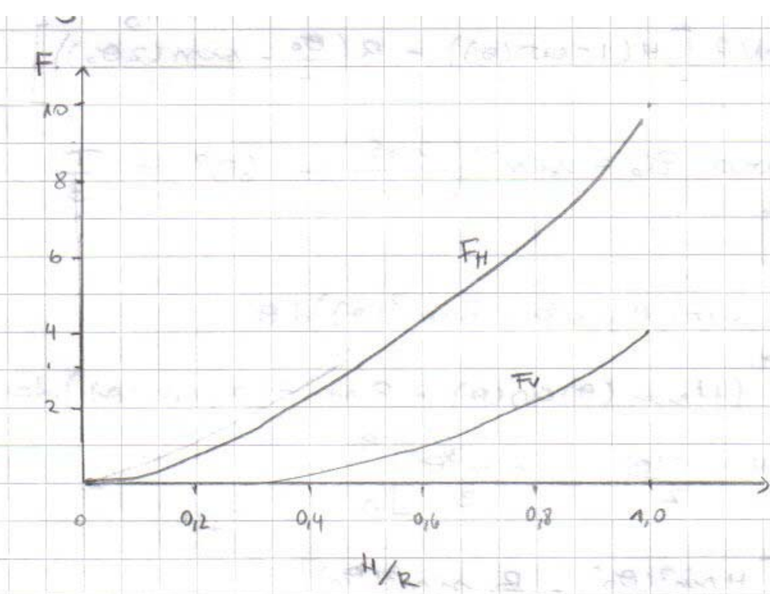
$$\theta_0 = sen\left(\frac{H}{R}\right)$$

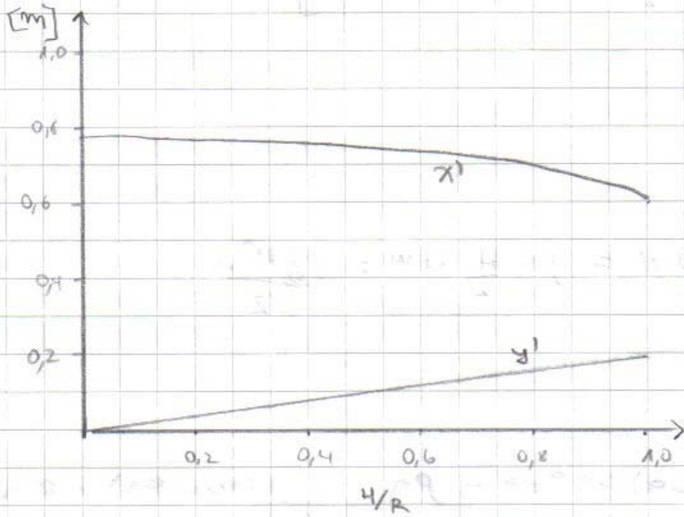
$$F_V = \rho g W R^2 \left[\frac{H}{R}(1-\cos(\theta_0)) - \frac{\theta_0}{2} + \frac{sen(2\theta_0)}{4}\right]$$

$$x' = \frac{\rho g W R^3 sin^2(\theta_0)}{F_V}\left[\frac{1}{2}\frac{H}{R} - \frac{1}{3}sen(\theta_0)\right]$$

$$F_H = \frac{\rho g H^2 W}{2}$$

$$y' = \frac{H}{3}$$


F
10
8
6
4
2
0
0,2
0,4
0,6
0,8
1,0
H/R
F_H
F_V


[m]
1,0
0,8
0,6
0,4
0,2
0,2
0,4
0,6
0,8
1,0
H/R
x'
y'