Caso 5. $(a,b)=(2,1)$. En este caso $I=\{1,2\}, g_1(2,2)=g_2(2,1)=0$
$\nabla f(a,b)=(-2,-2), \nabla g_1(a,b)=(4,2), \nabla g_2(a,b)=(1,2)$
$F_0=\{\mathbf{d}:(-2,-2)\cdot\mathbf{d}<0\}, G_0=\{\mathbf{d}:(4,2)\cdot\mathbf{d}<0,(1,2)\cdot\mathbf{d}<0\} \Rightarrow F_0\cap G_0=\phi$
En este caso $I=\{1,2\}$ ya que $g_1(2,1)=0, g_2(2,1)=0, g_3(2,1)\neq 0$ y $g_4(2,1)\neq 0$
$\mathbf{d}\in F_0$ ssi $(-2,-2)\cdot\mathbf{d}<0 \Rightarrow d_1+d_2>0$.

Afirmación: $\mathbf{d}\notin G_0$. Supongamos lo contrario, i.e.,$(4,2)\cdot\mathbf{d}<0$ y $(1,2)\cdot\mathbf{d}<0 \Rightarrow 4d_1+d_2<0$ y $d_1+2d_2<0$, pero $d_1+d_2>0$, y por lo tanto,

$$\left.\begin{aligned} 4d_1+d_2<0 \Leftrightarrow 3d_1+(d_1+d_2)<0 \Leftrightarrow d_1<0 \\ d_1+2d_2<0 \Leftrightarrow (d_1+d_2)+d_2<0 \Rightarrow d_2<0 \end{aligned}\right\} \begin{array}{l} \Rightarrow \mathbf{d_1}+\mathbf{d_2}<\mathbf{0}, \\ \text{contradicción} \\ \text{con } d_1+d_2>0 \end{array}$$

Caso 6. $(a,b)=(\sqrt{5},0)$. En este caso $I=\{1,3\}, g_1(a,b)=g_3(a,b)=0$
$\nabla f(a,b)=(2(\sqrt{5}-3),-4), \nabla g_1(a,b)=(2\sqrt{5},0), \nabla g_3(a,b)=(-1,0)$
$F_0(a,b)=\{\mathbf{d}:(2(\sqrt{5}-3),-4)\cdot\mathbf{d}\}; G_0=\{\mathbf{d}:(2\sqrt{5},0)\cdot\mathbf{d}<0,(-1,0)\cdot\mathbf{d}<0\}=\phi$
$\Rightarrow F_0\cap G_0\neq\phi$

Caso 7. $(a,b)=(0,2)$. En este caso $I=\{2,4\}, g_2(0,2)=g_4(0,2)=0$
$\nabla f(a,b)=(-6,0)\nabla g_2(a,b)=(1,2), \nabla g_4(a,b)=(0,-1)$
$F_0=\{\mathbf{d}:(-6,0)\cdot\mathbf{d}<0\}, G_0=\{\mathbf{d}:(1,2)\mathbf{d}<0,(0,-1)\cdot\mathbf{d}<0\} \Rightarrow F_0\cap G_0\neq\phi$

Caso 8. $(a,b)=(0,0)$. En este caso $I=\{3,4\}, g_3(0,0)=g_4(0,0)=0$
$\nabla f(a,b)=(-6,-4), \nabla g_3(a,b)=(-1,0), g_4(a,b)=(0,-1)$
$F_0=\{\mathbf{d}:(-6,-4)\cdot\mathbf{d}<0\}, G_0=\{\mathbf{d}:(-1,0)\cdot\mathbf{d}<0,(0,-1)\cdot\mathbf{d}<0\}$
$\Rightarrow F_0\cap G_0\neq\phi$

Graficamente.

Condiciones necesarias de Fritz-John
Se debe determinar si existen escalares $\alpha_0, \alpha_i, i\in I$ no-negativos (**no todos nulos**) tales que

$$\alpha_0\nabla f(a,b)+\sum_{i\in I}\alpha_i\nabla g_i(a,b)=(0,0)(\alpha_i g_i(a,b)=0)$$