$$* \int_{I_1} \varphi(z)\,dz \xrightarrow[\varepsilon \downarrow 0]{} \int_0^R \varphi(x)\,dx \xrightarrow[R\to\infty]{} \int_0^\infty \varphi(x)\,dx .$$

$$* \int_{I_2} \varphi(z)\,dz \xrightarrow[\varepsilon\downarrow 0]{} -\int_0^R \left[\log(x)+2\pi i\right]^2 \frac{P(x)}{Q(x)}\,dx \xrightarrow[R\to\infty]{} -\int_0^\infty \varphi(x)\,dx - \int_0^\infty 4\pi i \operatorname{Log} x \frac{P(x)}{Q(x)}\,dx + 4\pi^2 \int_0^\infty \frac{P(x)}{Q(x)}\,dx$$

Así,

$$2\pi i \sum_{Q(w)=0}' \operatorname{Res}(\varphi(z),w) = 0 + 0 + \int_0^\infty \varphi(x)\,dx - \int_0^\infty \varphi(x)\,dx - 4\pi i \int_0^\infty \operatorname{Log} x \frac{P(x)}{Q(x)}\,dx + 4\pi^2 \int_0^\infty \frac{P(x)}{Q(x)}\,dx$$

y

$$\int_0^\infty \operatorname{Log}(x) \frac{P(x)}{Q(x)}\,dx = \frac{1}{4\pi i}\left[4\pi^2 \int_0^\infty \frac{P(x)}{Q(x)}\,dx - 2\pi i \sum{}' \operatorname{Res}(\varphi(z),w)\right]$$

$$= -\pi i \int_0^\infty \frac{P(x)}{Q(x)}\,dx - \frac{1}{2}\sum{}' \operatorname{Res}(\varphi(z),w) .$$

(b) $$\int_0^\infty \frac{\operatorname{Log}(x)}{1+x^3}\,dx = -\pi i \int_0^\infty \frac{dx}{1+x^3} - \frac{1}{2}\sum_{j=1}^{3} \operatorname{Res}\left(\frac{[\operatorname{Log}(z)]^2}{z^3+1} ; w_j\right) \quad \begin{cases} w_1 = e^{i\pi/3} \\ w_2 = -1 \\ w_3 = e^{i5\pi/3} \end{cases}$$

* En clase auxiliar vimos que $\int_0^\infty \frac{dx}{1+x^3} =$

$$* \operatorname{Res}(\varphi(z), w_1) = \frac{(i\pi/3)^2}{(e^{i\pi/3}+1)(e^{i\pi/3}-e^{5\pi i/3})} = \frac{-2\pi^2/9}{(3+\sqrt{3}i)(\sqrt{3}i)} = R_1$$

$$\operatorname{Res}(\varphi(z), w_2) = \frac{(i\pi)^2}{(-1-e^{i\pi/3})(-1-e^{5i\pi/3})} = \frac{+4\pi^2}{(3-\sqrt{3}i)(3+\sqrt{3}i)} = R_2$$

$$\operatorname{Res}(\varphi(z), w_2) = \frac{(5i\pi/3)^2}{(e^{5\pi i/3}+1)(e^{5\pi i/3}-e^{i\pi/3})} = \frac{50\pi^2/9}{(3+\sqrt{3}i)(\sqrt{3}i)} = R_3 .$$

$$\text{Así,} \quad \int_0^\infty \frac{\operatorname{Log}(x)}{1+x^3}\,dx = -\pi i\left(\qquad \right) - \frac{1}{2}(R_1+R_2+R_3)$$