11. Show that

$$f(z) = \int_0^1 \frac{\sin zt}{t}\, dt$$

is an entire function.
a. by applying Morera's Theorem,
b. by obtaining a power series expansion for $f$.

12. With $f$ as in (11) show that

$$f'(z) = \int_0^1 \cos zt\, dt$$

a. by writing

$$f(z) = \int_0^1 \int_0^z \cos zt\, dz\, dt$$
$$= \int_0^z \left( \int_0^1 \cos zt\, dt \right) dz, \quad \text{etc.,}$$

b. by using the power series for $f$.

13. Show that $g(z) = z_0 + e^{i\theta} z$, $\theta = \operatorname{Arg}(z_1 - z_0)$, maps the real axis onto the line $L$ through $z_0$ and $z_1$.

14. Suppose $f$ is bounded and analytic in $\operatorname{Im} z \geq 0$ and real on the real axis. Prove that $f$ is constant.

15. Given an entire function which is real on the real axis and imaginary on the imaginary axis, prove that it is an odd function: i.e., $f(z) = -f(-z)$.

16. Suppose $f$ is analytic in the semi-disc: $|z| \leq 1$, $\operatorname{Im} z > 0$ and real on the semi-circle $|z| = 1$, $\operatorname{Im} z > 0$. Show that if we set

$$g(z) = \begin{cases} f(z) & |z| \leq 1, \quad \operatorname{Im} z > 0 \\ \overline{f\left(\frac{1}{\bar{z}}\right)} & |z| > 1, \quad \operatorname{Im} z > 0 \end{cases}$$

then $g$ is analytic in the upper half-plane.

17. Show that there is no non-constant analytic function in the unit disc which is real-valued on the unit circle.

18. Suppose $f$ is analytic in the upper semi-disc: $|z| \leq 1$, $\operatorname{Im} z > 0$ and is continuous to the boundary. Explain why it is not possible that $f(x) = |x|$ for all real values of $x$.