

Fórmulas Control 1
MA26B Matemáticas Aplicadas
Semestre 2004-2

Curvas

s : parámetro de longitud de arco, T : vector tangente unitario, N : vector normal unitario, B : vector binormal, κ : curvatura, τ : torsión

Se tiene

$$\begin{aligned}\frac{dN}{ds} &= -\kappa T + \tau B \\ \frac{dB}{ds} &= -\tau N.\end{aligned}$$

Operadores diferenciales

Si F y G son campos C^1 , f es C^1 escalar

$$\begin{aligned}\nabla \cdot (fF) &= \nabla f \cdot F + f \nabla \cdot F \\ \nabla \times (fF) &= \nabla f \times F + f \nabla \times F \\ \nabla \cdot (F \times G) &= G \cdot (\nabla \times F) - F \cdot (\nabla \times G)\end{aligned}$$

Divergencia y rotor en algunas coordenadas

Divergencia en cilíndricas:

$$\nabla \cdot F = \frac{1}{r} \frac{\partial}{\partial r}(rF_r) + \frac{1}{r} \frac{\partial F_\theta}{\partial \theta} + \frac{\partial F_z}{\partial z}$$

Divergencia en esféricas:

$$\nabla \cdot F = \frac{1}{\rho^2} \frac{\partial}{\partial \rho}(\rho^2 F_\rho) + \frac{1}{\rho \sin \theta} \frac{\partial F_\varphi}{\partial \varphi} + \frac{1}{\rho \sin \theta} \frac{\partial}{\partial \theta}(F_\theta \sin \theta)$$

Rotor en cilíndricas:

$$\nabla \times F = \frac{1}{r} \left(\frac{\partial F_z}{\partial \theta} - r \frac{\partial F_\theta}{\partial z} \right) \hat{r} + \left(\frac{\partial F_r}{\partial z} - \frac{\partial F_z}{\partial r} \right) \hat{\theta} + \frac{1}{r} \left(\frac{\partial}{\partial r}(rF_\theta) - \frac{\partial F_r}{\partial \theta} \right) \hat{z}$$

Rotor en esféricas:

$$\begin{aligned}\nabla \times F &= \frac{1}{\rho^2 \sin \theta} \left(\frac{\partial}{\partial \theta}(F_\varphi \rho \sin \theta) - \frac{\partial}{\partial \varphi}(F_\theta \rho) \right) \hat{\rho} + \frac{1}{\rho \sin \theta} \left(\frac{\partial F_\rho}{\partial \varphi} - \frac{\partial}{\partial \rho}(F_\varphi \rho \sin \theta) \right) \hat{\theta} \\ &\quad + \frac{1}{\rho} \left(\frac{\partial}{\partial \rho}(F_\theta \rho) - \frac{\partial F_\rho}{\partial \theta} \right) \hat{\varphi}\end{aligned}$$