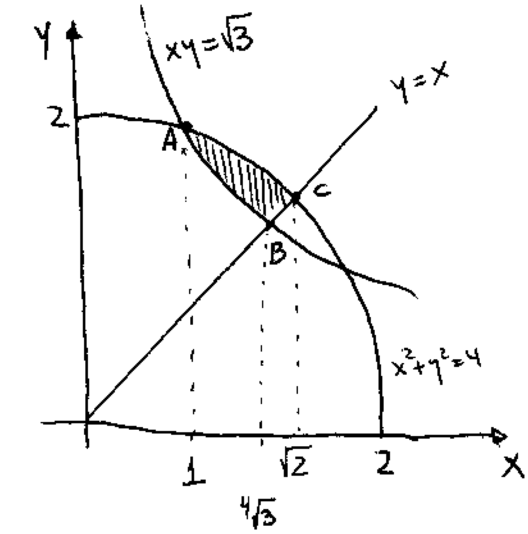
Ejercicio #4

a)



$$U = xy$$
$$V = x^2+y^2$$

Notemos que el trazo $\overline{AB} \Rightarrow U=\sqrt{3}$

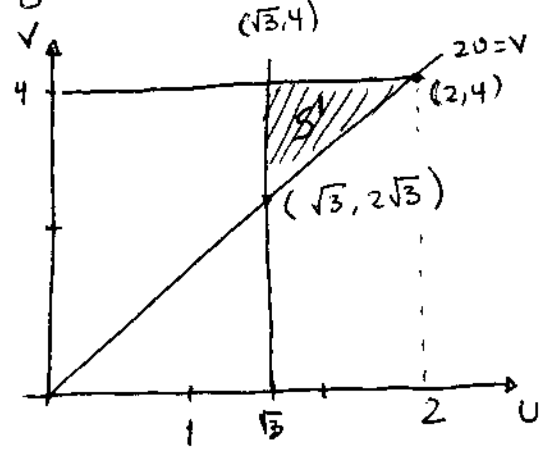
$\overline{AC} \Rightarrow V = 4$

Además $y=x \Rightarrow (y-x)^2=0$

$\Rightarrow y^2+x^2 = 2xy$

$V = 2U$

Luego



$$dudv = \begin{vmatrix} y & x \\ 2x & 2y \end{vmatrix} dxdy$$

$$= (2y^2+2x^2)\,dxdy = 2(y^2-x^2)dxdy$$

b)

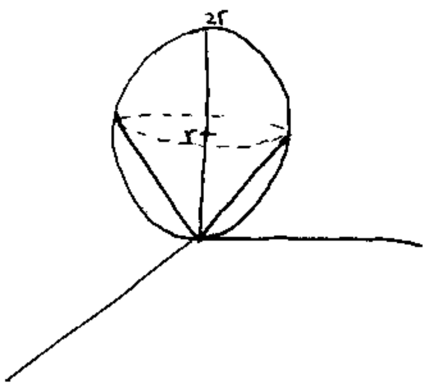
Por lo tanto

$$K = \iint_S xy(y^2-x^2)\,dxdy = \iint_{S'} U\cdot\frac{1}{2}\,dUdV$$

$$= \frac{1}{2}\int_{\sqrt{3}}^{2}\int_{2U}^{4} U\,dV\,dU = \frac{1}{2}\int_{\sqrt{3}}^{2} U(4-2U)\,dU = \sqrt{3}-\frac{5}{3}$$

Ejercicio #5

$$x^2+y^2+z^2=2rz \Rightarrow x^2+y^2+(z-r)^2=r^2$$



En cilíndricas la región queda definida por

$$\rho \le z \le r+\sqrt{r^2-\rho^2}$$
$$0 \le \rho \le r$$
$$0 \le \theta \le 2\pi$$

a) $$V = \int_0^{2\pi}\int_0^{r}\int_{\rho}^{r+\sqrt{r^2-\rho^2}} \rho \cdot dz\, d\rho\, d\theta$$

b) En esféricas la región está definida por:

$$\rho^2 = 2r\rho\cos\theta \Rightarrow \rho = 2r\cos\theta$$

$$\Rightarrow \quad 0 \le \rho \le 2r\cos\theta$$
$$0 \le \theta \le \frac{\pi}{4}$$
$$0 \le \phi \le 2\pi$$

$$V = \int_0^{2\pi}\int_0^{\pi/4}\int_0^{2r\cos\theta} \rho^2 \operatorname{sen}\theta\, d\rho\, d\theta\, d\phi = \frac{2\pi}{3}\int_0^{\pi/4} 2^3 r^3 \cos^3\theta \operatorname{sen}\theta\, d\theta$$

$$= \frac{16}{3}\pi r^3 \int_0^{\pi/4} \cos^3\theta \operatorname{sen}\theta\, d\theta = \frac{16}{3}\pi r^3 \cdot \left(-\frac{\cos^4\theta}{4}\right)\Bigg|_0^{\pi/4} = \frac{16}{3\cdot 4}\pi r^3 \underbrace{\left[-\frac{1}{4}+1\right]}_{\frac{3}{4}}$$

$$V = \frac{\cancel{16}\,\pi \cdot r^3 \cdot \cancel{3}}{\cancel{3}\cdot\cancel{4}\cdot\cancel{4}} = \pi r^3$$