UNIVERSIDAD DE CHILE

CÁLCULO EN VARIAS VARIABLES
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Ejercicio # 3

1. a) Considere la función $F(x_1, x_2, y) = y \cdot arctg(1 - y^2) + 3x_1 + 5y - 8x_2^3 = 0$ y el punto $(x_1, x_2, y) = (1, 1, 1)$.
   Pruebe que se satisfacen las condiciones del Teorema de la función implícita y calcule $\frac{\partial y}{\partial x_1}(1, 1) \qquad \frac{\partial y}{\partial x_2}(1, 1)$.

   b) Sea $f : \mathbb{R}^5 \to \mathbb{R}^2$ talque

   $$f(u, v, w, x, y) = \begin{pmatrix} uvw + x + y + 2 \\ ux - vy + w^2 \end{pmatrix}$$

   Muestre que se puede despejar (x, y) en términos de (u, v, w) en torno a $(u_0, v_0, w_0) = (1, 2, 3)$. Calcule $\frac{\partial x}{\partial v}(1, 2, 3) \quad \frac{\partial y}{\partial w}(1, 2, 3)$

2. Usando multiplicadores de Lagrange, encuentre los máximos y mínimos de la función f(x,y,z)=y bajo las restricciones $z = x + y \qquad 2x^2 + y^2 + 2z^2 = 8$

Ejercicio #3

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a) $F(x_1, x_2, y) = y \operatorname{arctg}(1-y^2) + 3x_1 + 5y - 8x_2^3 = 0$ en $(x_1, x_2, y) = (1,1,1)$

$F(1,1,1) = 1 \cdot \operatorname{arctg}(0) + 3 + 5 - 8 = 0$

$$\frac{\partial F}{\partial y} = \operatorname{arctg}(1-y^2) + \frac{y}{1+(1-y^2)^2} \cdot -2y + 5$$

$$= \operatorname{arctg}(1-y^2) - \frac{2y^2}{1+(1-y^2)^2} + 5$$

$$\frac{\partial F}{\partial y}(1,1,1) = 0 - \frac{2 \cdot 1^2}{1+(1-1^2)^2} + 5 = 3 \neq 0 \Rightarrow \frac{\partial F}{\partial y} \text{ es invertible}$$

$\therefore$ y se puede despejar en términos de x_1, x_2 en torno a $(1,1,1)$

$$\frac{\partial F}{\partial x} = \begin{pmatrix} 3 & -24x_2^2 \end{pmatrix} \Rightarrow \frac{\partial F}{\partial x}(1,1,1) = \begin{pmatrix} 3 & -24 \end{pmatrix}$$

$$\therefore \begin{pmatrix} \frac{\partial y}{\partial x_1} & \frac{\partial y}{\partial x_2} \end{pmatrix} = -\frac{1}{3}\begin{pmatrix} 3 & -24 \end{pmatrix} = \begin{pmatrix} -3 & 8 \end{pmatrix}$$

$$\Rightarrow \frac{\partial y}{\partial x_1}(1,1) = -3 \qquad \frac{\partial y}{\partial x_2}(1,1) = 8$$

Ejercicio #3

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b) $f(u,v,w,x,y) = \begin{pmatrix} u\cdot v\cdot w + x + y + 2 \\ u\cdot x - v\cdot y + w^2 \end{pmatrix}$

$$1\cdot 2\cdot 3 + x + y + 2 = 0 \implies x + y = -8$$
$$1\cdot x - 2y + 3^2 = 0 \implies x - 2y = -9$$

$$\implies -3y = -9 - (-8) = -1 \implies \boxed{y = \tfrac{1}{3}} \implies x = -y - 8 = -\tfrac{1}{3} - 8 = \boxed{-\tfrac{25}{3} = x}$$

$$(x_0, y_0, u_0, v_0, w_0) = \left(-\frac{25}{3}, \frac{1}{3}, 1, 2, 3\right)$$

$$\begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ u & -v \end{bmatrix} \implies \begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix} \text{ invertible}$$

$\therefore$ x e y se pueden despejar en términos de (u,v,w)

$$\begin{bmatrix} \frac{\partial f_1}{\partial u} & \frac{\partial f_1}{\partial v} & \frac{\partial f_1}{\partial w} \\ \frac{\partial f_2}{\partial u} & \frac{\partial f_2}{\partial v} & \frac{\partial f_2}{\partial w} \end{bmatrix} = \begin{bmatrix} vw & uw & u\cdot v \\ x & -y & 2w \end{bmatrix} \implies \begin{bmatrix} 6 & 3 & 2 \\ -\frac{25}{3} & -\frac{1}{3} & 6 \end{bmatrix}$$

Por lo tanto

$$\begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \end{bmatrix} = -\frac{1}{3}\begin{bmatrix} -2 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 6 & 3 & 2 \\ -\frac{25}{3} & -\frac{1}{3} & 6 \end{bmatrix} = -\frac{1}{3}\begin{bmatrix} -\frac{11}{3} & -\frac{17}{3} & -10 \\ -\frac{43}{3} & -\frac{10}{3} & 4 \end{bmatrix}$$

$$\frac{\partial x}{\partial v}(1,2,3) = \frac{17}{9}$$

$$\frac{\partial y}{\partial w}(1,2,3) = -\frac{4}{3}$$

Ejercicio 3

2- $L = y - \lambda(x+y-z) - \beta(2x^2+y^2+2z^2-8)$

$$\left.\begin{aligned}
\frac{\partial L}{\partial x} &= -\lambda - 4\beta x = 0 \\
\frac{\partial L}{\partial y} &= 1 - \lambda - 2\beta y = 0 \\
\frac{\partial L}{\partial z} &= \lambda - 4\beta z = 0 \\
\frac{\partial L}{\partial \lambda} &\Rightarrow x+y=z \\
\frac{\partial L}{\partial \beta} &\Rightarrow 2x^2+y^2+2z^2=8
\end{aligned}\right\} \begin{aligned} &(x,y,z) = \pm(-1,\ 2,\ 1) \\ &P_1 = (-1,\ 2,\ 1) \quad P_2 = (1,-2,-1) \end{aligned}$$

Evaluando se tiene

$f(-1,2,1) = 2$ máximo

$f(1,-2,-1) = -2$ mínimo.