Duopoly in exhaustible resource exploration and extraction

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Abstract. Strategic considerations of exploration and extraction are investigated in a two-player, two-period, two-stage perfect equilibrium framework. Relative to two 'plant' monopoly, the duopolists explore more and extract more period by period. A mixed game in which there is co-operation 'upstream' in exploration and Cournot competition 'downstream' in quantitites extracted is investigated. We also note that increasing returns to scale in exploration can introduce an unstable solution with a corner solution the presumed stable equilibrium.

Duopole dans l'exploration et l'extraction dans un secteur de ressource épuisable. Les auteurs examinent certaines considérations stratégiques dans les décisions d'exploration et d'extraction dans un univers d'équilibre parfait en deux étapes quand il y a deux joueurs et deux périodes. Par rapport aux résultats dans le cas d'un monopole avec deux unités d'opération, les résultats, dans le cas de duopole, montrent que les duopoleurs explorent davantage et extraient davantage période par période. Les auteurs analysent un jeu mixte dans lequel il y a coopération en amont dans l'exploration et concurrence à la Cournot en aval dans les quantités de ressource extraites. On montre que des rendements croissants à l'échelle dans l'exploration peuvent entraîner une solution intérieure instable et une solution présumée d'équilibre dans un coin.

I. INTRODUCTION

Exploration can affect current output price when discoveries are relatively large. Large discoveries are associated with finds of new oil fields or major ore bodies. The phenomenon is inherently stochastic, regular, steady search with many small or negative (dry wells) hits and the occasional large finds.[1] One discoverer's large

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1 Arrow and Chang (1982) consider an agent exploring in a stochastic framework in which only

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hit inflicts capital losses on owners of known but unsold stock and this introduces strategic considerations into exploration. Successful explorers can also ultimately increase their market share of, say, oil. We first investigate this strategic rivalry in a non-stochastic framework. Rivals anticipate the effects of competitor's exploration in a perfect equilibrium.[2] We deal with duopolist explorer-extractors in a two-stage game. Each player explores in anticipation of playing a Cournot game in quantities extracted in a second stage. We conjecture over-exploration relative to pure price-taking rivalry and observe over-exploration relative to a 'two-plant monopoly.' We present an illustrative numerical example. The interesting dimension of resource exploration rivalry as distinct from say R&D rivalry is how success by a discoverer not only leads to encroachment on a rival's current and subsequent market but inflicts a capital loss on the rival's current known reserves.[3]

This paper is organized as follows. The second section introduces the basic analytic model. The third section presents some numerical comparative static results for the case of a linear market demand with quadratic extraction costs. In section IV we discuss some extensions of our model. In particular, we show how our results on strategic exploration can be extended to include stochastic exploration in a three-stage game where nature makes the first move. Section V concludes the paper. Strategic uncertainty is discussed in the appendix.

II. TWO-PERIOD, TWO-PLAYER EXPLORATION AND EXTRACTION RIVALRY

Each player opens with some stock or known reserves, knows its extraction costs and both deterministic exploration technology and market demand (either prices or demand schedules, depending on the competitive mode below). We will set out the competitive (price-taking) case and two plant monopoly cases very briefly first in order to provide a comparison with the price-setting duopoly case of particular interest. Firm i's profits are

$$\pi^i = Q^i p(Q^1 + Q^2) - c^i(Q^i) - w^i(x^i) + \beta \Bigg\{ (S^i + S^i(x^i) - Q^i) \cdot p\left(\sum_{i=1}^{2}(S^i + S^i(x^i) - Q^i)\right) - c^i(S^i + S^i(x^i) - Q^i)\Bigg\} \qquad (i = 1,\ 2), \qquad (1)$$

where Q^i = firm i's output or exploration in period 1. $Q^i \leqq S^i$.
S^i = firm i's reserves or known stock in period i.

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small finds can occur. See also Pindyck (1980). Devarajan and Fisher (1982) and Lasserre (1985) make exploration costs rise with cumulative discoveries.

2 Non-rivalrous exploration in a non-stochastic framework is analysed in Pindyck (1978).

3 A parallel in R&D rivalry would be success by player i forcing player j to write off part of his or her fixed capital in addition to having part of his or her market share reduced. In Spencer and Brander (1983), for example, the duopoly rivalry in R&D affects only current variable costs and market shares, not fixed costs. See the Concluding Remarks below.

$p(\cdot)$ = the stationary demand schedule for output extracted by the two firms.
$c^i(Q^i)$ = the total extraction cost for firm i.
x^i = physical resources devoted to exploration by firm i in period 1.
$w^i(\cdot)$ = the cost of exploration.
$S^i(x^i)$ = the discovery of stock made in period 1 to be extracted in period 2. $S^i(x^i)$ is assumed positive, increasing, and concave in x^i with $S^i(0) = 0$. $w^i(x^i)$ is assumed to be positive and increasing in x^i. We shall assume that $w^i(0) = 0$ and $d^2w^i/dS^{i2} \geqq 0$ in the normal case but shall comment on the case of increasing returns to exploration effort; that is, the case of $d^2w^i/dS^{i2} < 0$.
β = the discount factor (equal to $1/(1+r)$) where r is the discount rate, say, equal to the rate of interest.

Simplification is achieved by making the horizon exogenous. All stock is extracted over two periods. We focus on the exploration-extraction rivalry per se, not on attrition and withdrawal of a rival over an extended horizon, endogenously determined.

1. Price-taking behaviour

With respect to quantity, firm i 'follows' the r per cent rule in rent on the marginal ton

$$[p_1 - mc_1^i] = \left(\frac{1}{1+r}\right)[p_2 - mc_2^i] \qquad (i = 1,\ 2), \tag{2}$$

where p_1 is the exogenously determined price in period 1 and p_2 is the exogenously determined price of output in period 2 and mc_k^i is the firm i's marginal cost of extraction in period k. With respect to exploration activity, we have

$$\frac{dw^i}{dx^i} = \left(\frac{1}{1+r}\right)[p_2 - mc_2^i]\,\frac{dS^i}{dx^i} \qquad (i = 1,\ 2). \tag{3}$$

This is simply the rule that the cost of the marginal unit of exploration should equal the value of its marginal product, suitably discounted and valued at 'net price,' $p_2 - mc_2^i$.

2. Two 'plant' monopoly

A decision-maker maximizes the sum $\pi^1 + \pi^2$ by choice of Q^1, Q^2, x^1, and x^2. With respect to quantity, in plant i we have

$$[mr_1^i - mc_1^i] = \left(\frac{1}{1+r}\right)[mr_2^i - mc_2^i] - Q^i\,\frac{\partial p_1 \partial \sum_1}{\partial \sum_1 \partial Q^i}$$

$$-\left(\frac{1}{1+r}\right)\frac{\partial p_2 \partial \sum_2}{\partial \sum_2 \partial Q^i}(S^j + S^j(x^j) - Q^j)$$

$$(i = 1,\ 2) \qquad (j = 1,\ 2;\ i \neq j) \tag{4}$$

where mr_k^i is the marginal revenue in period k for plant i and $\sum_1 = Q^1 + Q^2$. With respect to exploration, we have

$$\frac{dw^i}{dx^i} = \left(\frac{1}{1+r}\right)[p_2 - mc_2^i]\frac{dS^i}{dx^i} + \left(\frac{1}{1+r}\right)\left(\frac{\partial p_2}{\partial\left(\sum_2\right)}\cdot\frac{\partial \sum_2}{\partial S^i}\cdot\frac{dS^i}{dx^i}\right)$$

$$\cdot(S^i + S^i(x^i) - Q^i) + \left(\frac{1}{1+r}\right)\left(\frac{\partial p_2}{\partial\left(\sum_2\right)}\cdot\frac{\partial \sum_2}{\partial S^i}\cdot\frac{dS^i}{dx^i}\right)$$

$$\cdot(S^j + S^j(x^j) - Q^j) \qquad (i = 1,\ 2;\ i \neq j). \tag{5}$$

In comparison with the price taking case, the monopolist considers the effect of his or her discoveries on the second period price at the margin and the effect of this price decline on his or her own quantity extracted in plant i plus the effect of this price decline on the output in period 2 of plant j. $\sum_2$ is an abbreviation for $S^1 + S^1(x^1) - Q^1 + S^2 + S^2(x^2) - Q^2$. The monopoly solution involves the simultaneous choice of Q^1, Q^2, x^1, and x^2.

3. Non-co-operative subgame perfect duopoly

In this case each player knows and is committed to play a Cournot game in quantities extracted once exploration levels are selected. Given a Cournot game in quantities extracted, one can solve backwards to a game pure in exploration levels. If this game is Cournot in exploration levels, we label it non-co-operative. (Below we consider the exploration levels selected co-operatively or as in the two plant monopoly.) In the non-co-operative game, the players satisfy in the Cournot output game:

$$mr_1^i - mc_1^i = \left(\frac{1}{1+r}\right)[mr_2^i - mc_2^i] \qquad \begin{cases} i = 1,\ 2 \\ j = 1,\ 2; \quad i \neq j \end{cases}. \tag{6}$$

(6) can be solved (perhaps implicitly) to yield $Q^1 = f^1(x^1,\ x^2)$ and $Q^2 = f^2(x^1,\ x^2)$. If these equations are placed in (1) for Q^1 and Q^2, we have a pair

of profit functions in exploration levels (x^1, x^2) alone. The non-co-operative game in exploration levels is defined as the solution x^1 and x^2 to the pair $\partial\pi^1/\partial x^1 = 0$ and $\partial\pi^2/\partial x^2 = 0$. Given our specification of demand and costs, these two equations are

$$\frac{dw^i}{dx^i} = \left(\frac{\partial p_1}{\partial Q^j}\cdot\frac{dQ^j}{dx^i}\cdot Q^i\right) + \left(\frac{1}{1+r}\right)[p_2 - mc_2^i]\,\frac{dS^i}{dx^i}$$
$$+\left(\frac{1}{1+r}\right)\left(\frac{\partial p_2}{\partial\left(\sum_2\right)}\cdot\frac{\partial\sum_2}{\partial S^i}\cdot\frac{dS^i}{dx^i}\right)\cdot(S^i + S^i(x^i) - Q^i)$$
$$+\left(\frac{1}{1+r}\right)\left(\frac{\partial p_2}{\partial\left(\sum_2\right)}\cdot\frac{\partial\sum_2}{\partial Q^j}\cdot\frac{dQ^j}{dx^i}\right)\cdot(S^i + S^i(x^i) - Q^i)$$
$$(i = 1,\ 2;\ i \neq j), \qquad (7)$$

where dQ^1/dx^2 and dQ^2/dx^1 are defined from the reaction functions from the Cournot game in quantities (or from the basic implicit equations, $Q^1 = f^1(x^1, x^2)$ and $Q^2 = f^2(x^1, x^2)$. The own effects dQ^i/dx^i vanish above because $d\pi^i/dQ^i$ are zero from (6). For our specification, one totally differentiates the pairs of equations in (6) to obtain

$$\partial\left(\frac{mr_1^1 - mc_1^1 - \beta(mr_2^1 - mc_2^1)}{\partial Q^1}\right)dQ^1 + \partial\left(\frac{mr_1^1 - mc_1^1 - \beta(mr_2^1 - mc_2^1)}{\partial Q^2}\right)dQ^2$$
$$= -\partial\left(\frac{mr_1^1 - mc_1^1 - \beta(mr_2^1 - mc_2^1)}{\partial x^1}\right)dx^1 - \partial\left(\frac{mr_1^1 - mc_1^1 - \beta(mr_2^1 - mc_2^1)}{\partial x^2}\right)$$
$$\cdot\, dx^2 \qquad (8)$$

$$\partial\left(\frac{mr_1^2 - mc_1^2 - \beta(mr_2^2 - mc_2^2)}{\partial Q^1}\right)dQ^1 + \partial\left(\frac{mr_1^2 - mc_1^2 - \beta(mr_2^2 - mc_2^2)}{\partial Q^2}\right)dQ^2$$
$$= -\partial\left(\frac{mr_1^2 - mc_1^2 - \beta(mr_2^2 - mc_2^2)}{\partial x^1}\right)dx^1 - \partial\left(\frac{mr_1^2 - mc_1^2 - \beta(mr_2^2 - mc_2^2)}{\partial x^2}\right)$$
$$\cdot\, dx^2 \qquad (9)$$

and solves for dQ^1/dx^1, dQ^2/dx^1, dQ^1/dx^2, and dQ^2/dx^2. In (7), we make use of dQ^1/dx^2 and dQ^2/dx^1, to solve the Cournot game (non-co-operative game) in exploration levels.

Since (4) and (5) define the 'two plant monopoly' case, and (6) and (7) define the non-co-operative duopoly case, we can readily compare the two games. Equations in (5) and (7) differ only in the first and last terms on the right-hand sides. In the special case of a linear 'industry,' examined in detail below, we have $dQ^1/dx^2 = dQ^2/dx^1 = 0$ and the first and last terms in the pair of equations in (7) vanish, suggesting that the monopolist takes account of the spillover of his or her exploration activity from one 'plant' on the other whereas the duopolists completely ignore the spillovers. This is an extreme case illustrating our argument duopolists 'overexplore' because they ignore the 'damage' their discoveries have on rival's market share and capital losses associated with known reserves of stock. Though formally, discovery can only drive down future prices in our first-order conditions for exploration effort, future prices are tied to present prices through the first-order conditions on quantities extracted. This is as it should be – namely, discoveries to be mined in the future drive down current output price. These effects lead to the capital losses mentioned. Note in (7) that $dS^i/dx^i > 0$ and $\partial \sum_2 /\partial S^i = -\partial \sum_2 /\partial Q^i$. The sign of dQ^j/dx^i turns on how the demand schedule is specified. The monopolist takes account of 'plant' i's discovery on total profit, whereas the non-co-operating duopolist takes account of his or her discovery on his or her market share (industry price) and on the effect of his or her discovery on his or her profit alone via the effect on his or her rivals extraction. This would suggest intuitively that the duopolist will be dealing with a smaller 'cross effect' of marginal discovery, since his or her profits will be a fraction of total profits, and thus the duopolist will cut back exploration relative to the monopoly less given internalization of spillovers from exploration. Certainly in the example with a linear demand schedule, the monopolist explores in total less than the two duopolists combined.[4]

4. *Co-operative exploration and Cournot output strategy*[5,6]

In this case, given $Q^1 = f^1(x^1, x^2)$ and $Q^2 = f^2(x^1, x^2)$ implicitly defined from the Cournot reaction function in (4), the duopolists maximize the sum of their

4 Comparing the duopoly case with the two-plant monopoly case is straightforward, since both market structures can be considered to face the same downward sloping demand curve and in each case the equilibrium output price is determined endogenously. A direct comparison with the competitive case is not so easy, since price is exogenous to the competitive firm. We do in fact conjecture that the duopolists overexplore relative to *both* the two-plant monopoly and the competitive case. The fact that duopolists over explore relative to the competitive case has been shown in a simple, open-loop model by Sadorsky (1988).

5 The case of co-operation downstream and competition in exploration upstream is another case. We have not reported on it, since it seems less empirically relevant.

6 Co-operative exploration and Cournot output strategy is certainly realistic. As a result of the growing demand for crude oil and dwindling supplies of existing reserves, firms are forced to undertake more costly exploration to enlarge the reserve base. At the exploration stage the ben-

profits by simultaneously choosing exploration levels x^1 and x^2. Thus the Cournot game in quantities is still defined by the Q^1 and Q^2 which solve (6) given x^1 and x^2. Equations (8) and (9) define dQ^1/dx^1, dQ^2/dx^1, dQ^1/dx^2, and dQ^2/dx^2. Maximizing the sum of profits, π^1 and π^2 from (1) where Q^1 and Q^2 are implicit functions of x^1, and x^2 yields the same basic pair of equations in (7) plus new cross-effect terms indicated by braces. That is,

$$\frac{dw^1}{dx^1} = \beta(p_2 - mc_2^1)\frac{dS^1}{dx^1} + (S^1 + S^1(x^1) - Q^1)\beta\frac{\partial p_2}{\partial\sum_2}\left[\frac{\partial\sum_2}{\partial S^1}\cdot\frac{dS^1}{dx^1} + \frac{\partial\sum_2}{\partial Q^2}\cdot\frac{dQ^2}{dx^1}\right]$$

$$+\left\{(S^2 + S^2(x^2) - Q^2)\beta\frac{\partial p_2}{\partial\sum_2}\cdot\frac{\partial\sum_2}{\partial S^1}\cdot\frac{dS^1}{dx^1} + Q^2\frac{\partial p_1}{\partial\sum_1}\cdot\frac{\partial\sum_1}{\partial Q^1}\cdot\frac{dQ^1}{dx^1}\right\} \tag{10}$$

$$\frac{dw^2}{dx^2} = \beta(p_2 - mc_2^2)\frac{dS^2}{dx^2} + (S^2 + S^2(x^2) - Q^2)\beta\frac{\partial p_2}{\partial\sum_2}\left[\frac{\partial\sum_2}{\partial S^2}\cdot\frac{dS^2}{dx^2} + \frac{\partial\sum_2}{\partial Q^1}\cdot\frac{dQ^1}{dx^2}\right]$$

$$+\left\{(S^1 + S^1(x^1) - Q^1)\beta\frac{\partial p_2}{\partial\sum_2}\cdot\frac{\partial\sum_2}{\partial S^1}\cdot\frac{dS^2}{dx^2} + Q^1\frac{\partial p_1}{\partial\sum_1}\cdot\frac{\partial\sum_1}{\partial Q^2}\cdot\frac{dQ^2}{dx^2}\right\}. \tag{11}$$

This new game is defined by the Q^1, Q^2, x^1, x^2 which solve the two equations in (6), and (10), and (11), where dQ^1/dx^1, dQ^2/dx^1, dQ^1/dx^2, and dQ^2/dx^2 are defined in (8) and (9). One conjectures that less exploration will be done in this game relative to the non-co-operative duopoly game, because here each internalizes his or her spillover from undertaking exploration in the exploration phase of competition for profits. The sign of the first 'new term' in (10) is negative, since $\partial p_2/\partial\sum_2 < 0$, which should induce less exploration; and the sign of the second 'new term' turns on the sign of dQ^1/dx^1, which for the case of a linear demand schedule is positive, making the sign of this 'new term' for a linear demand schedule positive.

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efits outweigh the costs if firms can join together and agree to share the costs of these high-cost exploration projects. In Canada the Hibernia and Lloydminster mega projects are prime examples. In the output market, however, these firms can still behave non-co-operatively and compete for market shares. We point out that competition at the exploration stage and co-operation at the output stage is not really an interesting possibility, since anti-trust laws limit the degree of collusion in output markets.

III. LINEAR DEMAND AND QUADRATIC EXTRACTION COSTS

This example with discovery concave in effort yields mostly expected results. Duopolists extract period by period more than the corresponding two-plant monopolist, and duopolists explore (and discover) more than the corresponding two-plant monopolist. The mixed case of 'downstream' duopoly and 'upstream' co-operation in exploration yields the striking results that exploration levels are the same as those for the two-plant monopoly. Downstream quantities are larger for the mixed case. In other words, fully anticipated rivalry downstream has no effect upstream on the levels of exploration under co-operation (two plant monopoly).

The profit functions for the two players (firms or plants) are

$$\pi^1 = [A - B(Q^1 + Q^2)]Q^1 - c^1(Q^1)^2 - w^1x^1 + \beta[A - B(S^1 + S^1(x^1) - Q^1 + S^2 + S^2(x^2) - Q^2]\{S^1 + S^1(x^1) - Q^1\} - \beta c^1 \cdot [S^1 + S^1(x^1) - Q^1]^2 \tag{12}$$

$$\pi^2 = [A - B(Q^1 + Q^2)]Q^2 - c^2(Q^2)^2 - w^2x^2 + \beta[A - B(S^1 + S^1(x^1) - Q^1 + S^2 + S^2(x^2) - Q^2]\{S^2 + S^2(x^2) - Q^2\} - \beta c^2 \cdot [S^2 + S^2(x^2) - Q^2]^2. \tag{13}$$

1. Two-plant monopoly

In this case we solve $\partial\pi/\partial Q^1 = \partial\pi/\partial Q^2 = \partial\pi/\partial x^1 = \partial\pi/\partial x^2 = 0$ where $\pi = \pi^1 + \pi^2$. We can solve the first two equations for Q^1 and Q^2 in terms of x^1 and x^2 and parameters. These are then

$$Q^1 = \left(\frac{\beta}{1+\beta}\right) Z^1 + \left[\frac{c^2}{2Bc^2 + 2Bc^1 + 2c^1c^2}\right] \left(\frac{1-\beta}{1+\beta}\right) A \tag{14}$$

$$Q^2 = \left(\frac{\beta}{1+\beta}\right) Z^2 + \left[\frac{c^1}{2Bc^2 + 2Bc^1 + 2c^1c^2}\right] \left(\frac{1-\beta}{1+\beta}\right) A, \tag{15}$$

where $Z^i = S^i + S^i(x^i)$. If Q^1 and Q^2 from (14) and (15) are substituted into $\partial\pi/\partial x^1 = \partial\pi/\partial x^2 = 0$, we obtain

$$-w^1 \frac{dx^1}{dS^1} + \frac{2\beta A}{(1+\beta)} - \left(\frac{2\beta}{1+\beta}\right)(B + c^1)Z^1 - \left(\frac{2\beta}{1+\beta}\right) BZ^2 = 0 \tag{16}$$

$$-w^2 \frac{dx^2}{dS^2} + \frac{2\beta A}{(1+\beta)} - \left(\frac{2\beta}{1+\beta}\right)(B + c^2)Z^2 - \left(\frac{2\beta}{1+\beta}\right) BZ^1 = 0. \tag{17}$$

This pair of non-linear equations defines the x^1 and x^2 for the pure monopoly case. Given x^1 and x^2, we obtain the solution values for Q^1 and Q^2 above. For the

TABLE 1

inverse demand	$p = 10\text{-}(\cdot)$	discovery function $E^i(x^i)^{0.5} E^1 = E^2 = 1.0$	
extraction cost	$c^i \cdot (\cdot)^2, c^1 = c^2 = 0.1$	$\beta = 1/1 + r = 0.9$	
exploration cost	$w^i x^i, w^1 = w^2 = 0.55$	T for two-plant monopoly	
reserves	$S^1 = S^2 = 2.4$	D for duopoly	
		M for the mixed case	

		Q^1	Q^2	x^1	x^2
Base case	T	1.982607	1.982607	2.313299	2.313300
	D	2.358098	2.358098	4.977300	4.977296
	M	2.021767	2.021768	2.313298[a]	2.313300
$\beta = 0.85$	T	1.986952	1.986950	2.263144	2.263128
	D	2.367954	2.367954	4.850182	4.850183
	M	2.047278	2.047277	2.263135	2.263128
$S^2 = 2.3$	T	1.995980	1.952369	2.399977	2.424612
	D	2.363753	2.332523[b]	5.030712	5.184702
	M	2.035141	1.991528	2.399983	2.424603
$c^2 = 0.11$	T	1.992768	1.966214	2.342229	2.248544
	D	2.361190	2.346082	4.998472	4.882116
	M	2.027105	2.009748	2.342230	2.248545
$w^2 = 0.56$	T	1.986263	1.974340	2.336840	2.260519
	D	2.360370	2.347825	4.988724	4.880999
	M	2.025422	2.013501	2.336833	2.260524
$E^2 = 0.99$	T	1.986684	1.973388	2.339557	2.300254
	D	2.360632	2.346638	5.001197	4.968818
	M	2.025845	2.012547	2.339559	2.300251

a Values for x^1 and x^2 are the same for the T (monopoly) and M (mixed) cases (see text).
b This is not economically feasible, since Q^2 exceeds the initial reserves of size 2.3 in the first period.

case $S^i(x^i) \equiv E^i x^{0.5}$ we solve some examples. Results are reported in table 1. We discuss them below.

2. Duopoly

In this case the 'downstream' Cournot game is played in Q^1 and Q^2, and the 'upstream' game is played in x^1 and x^2 given that each player knows and is committed to play a downstream game in Q^1 and Q^2. The first-order conditions or reaction functions $\partial\pi^1/\partial Q^1 = 0$ and $\partial\pi^2/\partial Q^2 = 0$ (corresponding to the equations in (6)) are

$$Q^1 = \frac{A}{2B + 2c^1}\left(\frac{1-\beta}{1+\beta}\right) + \left(\frac{\beta}{1+\beta}\right)Z^1 + \left(\frac{\beta}{1+\beta}\right)\left(\frac{B}{2B+2c^1}\right)BZ^2 - \left(\frac{B}{2B+2c^1}\right)Q^2 \qquad (18)$$

$$Q^2 = \frac{A}{2B + 2c^2}\left(\frac{1-\beta}{1+\beta}\right) + \left(\frac{\beta}{1+\beta}\right)Z^2 + \left(\frac{\beta}{1+\beta}\right)\left(\frac{B}{2B+2c^2}\right)BZ^1 - \left(\frac{B}{2B+2c^2}\right)Q^1. \tag{19}$$

Observe that $B/(2B + 2c^1) < 1$, implying that the reaction functions cross in a way compatible with stability of convergences to the solution Q^1 and Q^2. Also, $d^2\pi^i/d(Q^i)^2 < 0$ or the second-order conditions are satisfied. The solution to (18) and (19) is

$$Q^1 = \left(\frac{\beta}{1+\beta}\right)Z^1 + \left[\frac{B + 2c^2}{3B^2 + 4c^1c^2 + 4Bc^1 + 4Bc^2}\right]\left(\frac{1-\beta}{1+\beta}\right)A \tag{20}$$

$$Q^2 = \left(\frac{\beta}{1+\beta}\right)Z^2 + \left[\frac{B + 2c^1}{3B^2 + 4c^1c^2 + 4Bc^1 + 4Bc^2}\right]\left(\frac{1-\beta}{1+\beta}\right)A. \tag{21}$$

These equations imply

$$\frac{dQ^1}{dx^1} = \left(\frac{\beta}{1+\beta}\right)\frac{dS^1(x^1)}{dx^1} \quad \text{and} \quad \frac{dQ^2}{dx^2} = \left(\frac{\beta}{1+\beta}\right)\frac{dS^2(x^2)}{dx^2} \tag{22}$$

and $dQ^1/dx^2 = dQ^2/dx^1 = 0$. If one substitutes (20) and (21) in the profits functions in (12) and (13), one has two non-linear equations in x^1 and x^2. The first-order conditions are

$$-w^1\frac{dx^1}{dS^1} + \left(\frac{2\beta}{1+\beta}\right)A - \left(\frac{2\beta}{1+\beta}\right)(B + c^1)Z^1 - \left(\frac{\beta}{1+\beta}\right)BZ^2 = 0 \tag{23}$$

$$-w^2\frac{dx^2}{dS^2} + \left(\frac{2\beta}{1+\beta}\right)A - \left(\frac{2\beta}{1+\beta}\right)(B + c^2)Z^2 - \left(\frac{\beta}{1+\beta}\right)BZ^1 = 0. \tag{24}$$

Comparison of (16) and (17), the monopoly case, with (23) and (24), the duopoly case, reveals that only the 'off-diagonal' terms are larger for the monopoly case. (23) and (24) can be viewed as the reduced-form reaction functions for the duopoly game. We solved for Q^1, Q^2, x^1, and x^2, for $S^i(x^i) \equiv E^i \cdot (x^i)^{0.5}$. The results are reported in table 1. Note that the slope of the reaction function in (23), for example, is

$$\frac{dx^1}{dx^2} = \frac{\left(\frac{\beta}{1+\beta}\right)B\frac{dS^2}{dx^2}}{w^1\left(\frac{dS^1}{dx^1}\right)^{-2}\frac{d^2S^1}{d(x^1)^2} - \left(\frac{2\beta}{1+\beta}\right)(B + c^1)\frac{dS^1}{dx^1}}.$$

Stability of reactions requires that dx^1/dx^2 above, corresponding to player 1's reaction function, be less than dx^1/dx^2, corresponding to player 2's reaction function, that is, in the neighbourhood of the solution (x^1, x^2):

$$\frac{\dfrac{dS^2}{dx^2}}{w^1\left(\dfrac{dS^1}{dx^1}\right)^{-2}\dfrac{d^2S^1}{d(x^1)^2}-\left(\dfrac{2\beta}{1+\beta}\right)(B+c^1)\dfrac{dS^1}{dx^1}} < \frac{\dfrac{dS^1}{dx^1}}{w^2\left(\dfrac{dS^2}{dx^2}\right)^{-2}\dfrac{d^2S^2}{d(x^2)^2}-\left(\dfrac{2\beta}{1+\beta}\right)(B+c^2)\dfrac{dS^2}{dx^2}}.$$

For the symmetric case and $d^2S^1/d(x^1)^2 < 0$, stability obtains. Clearly, however, for $d^2S^1/d(x^1)^2 > 0$ it is possible to have the above inequality reversed and observe unstable reactions in the neighbourhood of the solution. In other words, increasing returns to scale in exploration can bring about instability of 'the equilibrium.' Ultimately, a stable solution may be obtained at a boundary with one player's not exploring, a 'natural monopoly.'[7]

3. Co-operation in exploration and downstream duopoly

This case displays the striking property that *the same level of exploration is undertaken as with pure monopoly, though downstream quantities extracted differ.* Equations (20) and (21) solve the downstream game in quantities given x^1 and x^2. One inserts for Q^1 and Q^2 in $\pi = \pi^1 + \pi^2$ from (20) and (21) and solves for $\partial\pi/\partial x^1 = 0$ and $\partial\pi/\partial x^2 = 0$. The resulting equations turn out to be (16) and (17). Hence our result on equal exploration efforts in the two different games. In table 1 we report on some numerical solutions.

A comparison of the outcomes under three different strategic modes is contained in table 1. T is the two-plant monopoly case, D the two-stage Cournot duopoly case, and M the mixed case of downstream duopoly in quantities and upstream monopoly or co-operation in extraction. The base case is symmetric, marginal extraction cost is increasing in quantity, and discoveries are concave in effort. Demand is linear in price. For the base case we observe that each of the duopolists delivers more in period 1 than for T and M and explore almost twice as much as under T and M. Competition thus promotes greater output and exploration (and discovery, since the link is non-stochastic). We see our result numerically established earlier; namely,

7 A natural monopoly may result when one firm is substantially larger than the other. In this case the largest firm has presumably more money to spend on research and development projects relating to exploration technology. The increased spending on R&D may ultimately lead the larger firm to acquire a better exploration technology with increasing returns to scale in the exploration production function.

the T and M modes explore at the same intensity, though quantities delivered in period 1 differ, being larger for the M mode.

Consider the comparative static results in table 1. A rise in the interest rate (the discount factor changes from 0.9 to 0.85 in the second row group) shrinks quantities and exploration levels.

A decline in initial stock reserves for firm 2 or plant 2 (S^2 is reduced from 2.4 to 2.3) induces more exploration by both players (firms or plants), reduces Q^2 output in period 1 slightly and induces Q^1 to rise relative to the base case. Increased initial scarcity of one player's stock induces greater search activity and greater production in period 1 by the other player.

A rise in player 2's extraction costs (parameter c^2 increases from 0.10 to 0.11) induces a decline in 2's research effort and first-period production but an expansion in firm 1's research effort and production in period 1.

A rise in player 2's exploration costs (parameter w^2 rises from 0.55 to 0.56) induces less exploration by 2 and less production in period 1, while inducing more exploration by player 1 and more production in period 1.

Finally a decline in the productivity of research effort for firm 2 (E^2 declines from 1.0 to 0.99) induces less exploration by player 2 and less production in period 1, while at the same time inducing more exploration by player 1 and more production in period 1.

IV. EXTENSIONS

In this section we extend our results on strategic rivalry to the case where exploration is stochastic. The easiest way to extend our model to allow for stochastic exploration is to imagine a three stage game, where in the first stage nature informs the players that exploration is uncertain. The players know the only source of uncertainty is via a random variable ϵ^i where $\epsilon^i \sim$ IID $(0, \sigma^2), i = 1, 2$. The players know the distribution of ϵ^i. The players don't know, however, what realization ϵ^i will take on. In the second stage, firms choose their desired level of exploration activity, and in the third stage they choose extraction levels conditional on the chosen levels of exploration. Now consider the equations in the paper. If uncertainty is introduced additively to the exploration production function then we have $S^i + S^i(x^i) + \epsilon^i, i = 1, 2$. Then equation (1) for profits becomes expected profits $E(\pi^i)$, where 'E' denotes the expectation operator. All our general results, equations (1) to (11), go through with the addition of an 'E' on the right-hand side of these equations. Our first-order conditions are now interpreted in terms of expected marginal benefits and expected marginal costs. Our discussion following equation (9) is still valid.

Now let's consider the linear demand and quadratic extraction cost example. The only source of randomness is through the additive disturbance term on the exploration production function. But, by construction, all our first-order conditions (FOC) are linear in the choice variables. The expectation operator, E, is a linear operator, and assuming $E(\epsilon^i) = 0$, the terms $E(\epsilon^i)$ drop out of the first-order conditions. As an example consider equation (14). Introducing additive uncertainty into the model yields (14′).

$$\hat{Z}^i = S^i + S^i(x^i) + \epsilon^i$$

but

$$E(\hat{Z}^i) = E(S^i + S^i(x^i) + \epsilon^i) = S^i + S^i(x^i) = Z^i.$$

Hence, the expected value of (14′) is just (14). In the case of a linear demand and quadratic extraction costs with additive exploration uncertainty, all our results in equations (14)–(24) go through and our results in table 1 remain valid. If instead of $E(\epsilon^i) = 0$ we had $E(\epsilon^i) = \mu$, then equations (14)–(24) would by augmented by an additional constant term that contained the non-zero mean, μ.

Finally, let's consider the case in which the stochasticity affects the exploration function multiplicatively. We assume ϵ^i and $S^i(x^i)$ are independent. In this case $\tilde{Z}^i = S^i + S^i(x^i)\epsilon^i$, $i = 1, 2$. If $E(\epsilon^i) = 0$, $\forall i$, $i = 1, 2$ then $E(\tilde{Z}^i) = S^i$ and the terms Z^i in (14)–(24) would be replaced by S^i. If ϵ^i and $S^i(x^i)$ are independent, but $E(\epsilon)^i = \mu$, then our Z^i terms in equations (14)–(24) would be replaced by $S^i + \mu S^i(x^i)$.

In summary, additive randomness of the form $\epsilon^i \sim \text{IID}(0, \sigma^2)$ in the linear demand and quadratic extraction cost example does not alter our results in table 1. A non-zero mean for the disturbance term would change the entries of table 1 (if $\mu \neq 1$) but not the rankings of the three cases we reported on. And finally, multiplicative uncertainty of the form $S^i(x^i)\epsilon^i$ would not alter our basic results, provided $E(S^i(x^i)\epsilon^i) = S^i(x^i)E(\epsilon^i)$, but would of course alter the numerical values of table 1 if $\mu \neq 1$.

The issue of strategic uncertainty is considerably more complicated, since one has to have assumptions on what each player knows and believes and how the play of the game is to be organized. In the appendix we report some results from a game with strategic stock size uncertainty but no exploration.

V. CONCLUDING REMARKS

We have considered a market failure in exploration for exhaustible resources arising from the large size of players relative to the market. We considered duopoly outcomes. In a two- stage duopoly game we observed more exploration and production early on than occurs in a pure monopoly or partial monopoly. Duopoly in output markets did not affect exploration levels in a mixed model relative to a pure monopoly model for a particular example. Though our analysis may not have clear policy implications it is of interest in comparing R&D activity and exploration activity, since the two-stage game framework has been made use of recently in R&D economics (e.g., Spencer and Brander 1983 and d'Aspremont and Jacquemin 1987).

In our model, where firms undertake exploration activity before extraction begins, successful exploration when output markets are imperfectly competitive not only leads to increased market shares, but also inflicts a capital loss on the rival's current known reserves. We assume that market shares depend on own and rival's marginal revenue. It is also assumed that a higher marginal revenue leads to a higher market share while a higher marginal revenue for rivals leads to a reduction in own market shares. When discoveries are large, exploration can affect future as well as current output prices. Current discoveries add to existing supplies and depress future selling prices. But since current prices are tied to future prices through the resource stock constraint, one firm's large discovery may drive down future and present prices whereby inflicting capital losses on rival owners of known but unsold stock.

In the R&D literature, one of the primary motives for undertaking R&D is to reduce costs. Spencer and Brander 1983 show that in an imperfectly competitive output market, where firms undertake R&D before production takes place, firms may use R&D for strategic purposes rather than just to minimize costs. In the Spencer and Brander model firms' market shares depend on their own and rivals' marginal cost. It is assumed that a lower own marginal cost leads to a higher market share while a lower marginal cost for rivals leads to a reduction in own market shares. Thus the difference between our model and the Spencer and Brander model is that in our model firms may use exploration for strategic purposes not only to increase own market shares but also to inflict capital losses on rivals.

The Spencer-Brander R&D game can be readily reformulated and made similar to our game of exploration and extraction. In the R&D game, there will be one open-ended period. Firm i's output is produced with a variable input L^i and knowledge capital K^i as in $Q^i = f^i(L^i, K^i)$. Variable costs are w^i per unit of L^i, and knowledge capital is augmented by investment I^i costing s^i per unit and increasing K^i by $g^i(I^i)$. Let there be an industry inverse demand schedule $p(Q^1 + Q^2)$. Firm i's profit is then $p(Q^1 + Q^2)f^i(L^i, K^i + g^i(I^i)) - w^iL^i - s^iI^i$. In a sense of priorness, an R&D game is played conditional on a game in output levels being played downstream. We suppose implicitly that investment in new knowledge is not leaked to the rival, although such a model could be investigated (d'Aspremont and Jacquemin). Thus property rights problems are not essential here. Two differences in this model and the exploration-extraction model are (1) in the latter, even in perfect competition, these are rents to be used to pay for exploration activity, whereas in the R&D game there is a shadow price corresponding to the value of additional knowledge. Output price will lie above current operating or variable costs. Exploration and R&D investment are paid for in somewhat different ways in the two models. (2) In the R&D game there is only an implicit capital loss on existing knowledge as competition increases; that is, the shadow price of K^i changes as competition increases or decreases, whereas in the exploration-extraction game the capital losses on existing reserves from more competition are directly observed as output price changes. Presumably the K^i in the R&D game is marketable as, say, patents, and its value depends on the marginal product of the knowledge *and* the price of output produced with the patents. The shadow price $p\ \partial f^i/\partial\ [K^i + g^i(I^i))$ will change with the intensity of competition (duopoly vs monopoly for example.) These differences in the models are really of a second-order significance. The Spencer-Brander formulation focuses on cost functions rather than the production function and contains no potential capital loss term.

APPENDIX

Strategy in the context of uncertainty is considerably more complicated, since one has to have assumptions about what each player knows and believes and how the 'play' is to be organized. See for example Kreps (1984) and Harris (1987). Lewis, Lindsey, and Ware (1986) present a subtle, two-person, three-period game between owners of the backstop and owners of a stock of a mineral. Among a family of possible outcomes of the game, one has to decide on plausible *equilibrium* outcomes.

Consider an example of a game involving pure stock size uncertainty (i.e., one with no exploration activity). Firm 1 knows its stock size before it 'delivers,' but firm 2 does not. Firm 1 moves first and knows firm 2's stock size. Firm 2 moves second with its initial quantity delivered. In the second period, both firms 'dump' their remaining stock. We restrict each firm to one of two initial 'deliveries' (i.e., large or small). Firm 2 has priors on the likely size of Firm 1's total stock. Firm 1's first move constitutes a 'signal' to Firm 2 upon which Firm 2 updates its priors on whether the stock size is large or small.

Our example is based on a linear demand schedule known to both firms. We calculated actual profits, given nature's move, for possible player sequences.[8] The game is illustrated in figure 1.

Firm 1's stock size is uncertain. It becomes known to Firm 1 who moves first with action L (large extraction) or S (small extraction). Firm 2 responds with action L (large extraction from its stock size) or S (small extraction from its stock size). Pay-offs are listed for Firm 1 above Firm 2 at each terminal node.

Firm 1's pay-offs are recorded above those of Firm 2. Firm 1 does better playing S, whether his stock is large or small. Thus Firm 2 can glean no information from Firm 1's first move. The reasonable response for Firm 2 is not to revise its priors after Firm 1 moves. If Firm 2 knew 1's stock were small, playing S (small) would be its best response, and if Firm 2 knew 1's stock were large, playing L (large) would be its best response. However, given its beliefs (probabilities), given its uncertainty about Firm 1's stock, its expected pay-off is higher from playing L (namely $0.6(486.9) + 0.4(473.4)$). Given priors 0.99 on 1 having a small stock

8 The inverse industry demand curve is $p = 50 - Q$, where p is price and Q is the sum of the current outputs of the two firms. Firm 1 has a large extraction in the initial period of twelve units or a small extraction of ten units. Firm 2 has a small extraction in the initial period of eight units or a large extraction of nine units. Firm 2's stock size is fifteen units and the discount factor $1/(1 + r)$ is 0.9. What is not extracted in period 1 is completely extracted in the second period. Nature deals Firm 1 a stock size of twenty units with probability 0.6 or eighteen units with probability 0.4.

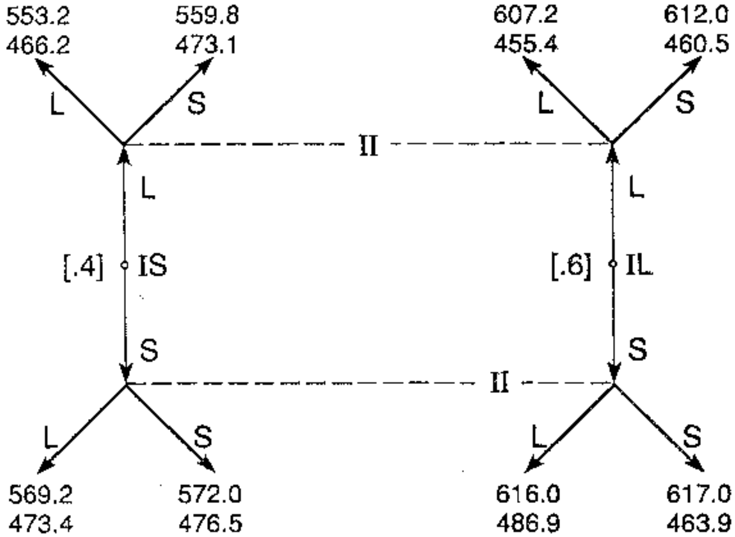

FIGURE 1

and 0.01 on 1 having a large stock, playing *S* would then be 2's best response, on average. This game is simple to analyse because Firm 2 playing *S* is a dominant strategy. Determining the best play sequence is complicated when a variety of play sequences (sequential equilibria) are plausible candidates for being the 'optimal' solution (play sequence).

Figure 2 shows the same game as in Figure 1 but with different pay-offs. We note that Firm 1 always does better when Firm 2 responds with *S*. Hence Firm 1 would like to 'communicate' by his first move to induce Firm 2 to respond with *S*. However, Firm 2 does better playing *L* when it thinks 1's stock size is in fact *L* and does better playing *S* when it thinks 1's stock size is in fact *S*. Thus there is the potential for conflict, something we did not observe in the extraction game above. Note also that if Firm 2 knew 1's stock size for certain, whether Firm 1 plays *L* or *S* does not affect 2's prospective pay-offs (600 or 601 for 1*S* and 602 or 601 for 1*L*).

In Kreps's words, this extensive game has 'two sorts of sequential equilibria.' In the first sort, 1 always plays *S*, irrespective of whether 1*S* or 1*L* is true, and 2 responds with *S* if it thinks 1*S* is true and with *L* if it thinks 1*L* is true. Firm 2's belief 'structure' for this sort of equilibria is of course supported by the off-the-equilibrium-path belief that if 1 played *L*, then 1 must have a large stock. There are randomized versions of this sort of equilibrium (see Harris 1987, 107–9).

The second sort of equilibrium has 1 play *L*, irrespective of whether 1*S* or 1*L* is true, and 2 responds with *L* if 1 were to play *S* and with *S* if 1 were to play *L*.

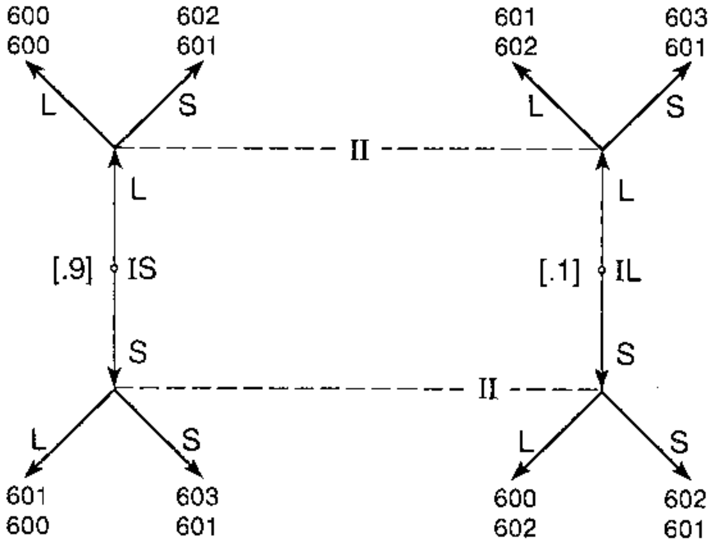

FIGURE 2

This requires out-of-equilibrium beliefs: if 1 plays S, one is most likely to have a large stock. These second sorts of equilibrium (including randomized versions) can be ruled out as 'unintuitive,' that is, invoking the intuitive criterion (Kreps 1984). See also Grossman and Perry (1986). For a more detailed analysis of this and very similar examples, see Kreps (1984) and Harris (1987, chap. 5).

We observe then that duopoly with stock size uncertainty takes us to the frontiers of game theory, to the frontiers of optimal choice action in the face of uncertainty about the best interpretation of one's rival's action. The interesting case of uncertainty about the pay-off to exploration and duopoly in extraction would be a significant extension of the framework above, since resources would be committed to purchase information in the face of downstream strategic uncertainty.

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