

# Taxation of non-renewable resources at various stages of production

MARGARET E. SLADE University of British Columbia and  
Université Catholique de Louvain

*Abstract.* A general method of assessing tax-policy effects in exhaustible-resource industries is derived. The model includes both extraction and processing by a vertically integrated firm, and taxation can occur at any stage of production. Many of the standard results of the theoretical resource-taxation literature are derived in a unified framework. When stages of production are introduced, it is seen that tax-policy effects can differ dramatically from those predicted by the simpler model. After the method of determining the effects of existing taxes is derived, a procedure for designing taxes that achieve desired goals is analysed.

*Imposition des ressources non-renouvelables à divers stages de production.* On dérive une méthodologie générale pour évaluer les effets de politique fiscale sur les industries de ressources épuisables. Le modèle contient et le stage de l'extraction et le stage de la transformation dans une entreprise intégrée verticalement; l'imposition peut se faire à l'un ou l'autre stage. L'auteur peut dériver beaucoup des résultats standards dans la littérature théorique spécialisée sur la fiscalité et les ressources grâce à ce cadre d'analyse intégré. Avec l'introduction des stages de production, on voit cependant que les politiques fiscales peuvent avoir des effets fort différents de ceux prédits par le modèle plus simple. Après la dérivation d'une méthodologie pour déterminer les effets des taxes existantes, on suggère une procédure pour confectionner une fiscalité susceptible d'atteindre les objectifs désirés.

## INTRODUCTION

Tax-policy instruments are used by many levels of government – international, federal, state or provincial, and even local – in pursuing diverse and sometimes conflicting goals. The number of instruments employed is large and includes severance taxes, royalties, depletion allowances, profits taxes, and price and quantity controls. In addition, these instruments can be applied at any stage of production. For example, property taxes may be levied on ore in the ground, severance taxes on ore as it leaves the ground, royalties may be collected at first

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sale (after some degree of processing), and price controls may be placed on a final product.

Tax-setting authorities have ample reason to be concerned about the effects of their actions. If it is felt that private markets in the absence of taxation result in optimal extraction patterns, governing bodies will want to design neutral taxes. There are, however, many reasons that private markets may not result in optimal extraction patterns. For example, extraction may proceed too slowly if the resource is monopolized or too quickly if there are common-pool problems in exploration or exploitation. In addition, social and private discount rates can differ and extraction can create externalities. Governments may therefore want to design taxes that achieve particular objectives such as conservation or more rapid depletion.

The emphasis in this paper is on neutrality, not because it is felt that neutrality is a desirable goal, but because neutrality is the boundary between conservation and accelerated extraction. If this boundary cannot be characterized, it is not possible to make unambiguous statements about tax-policy effects.

Many authors have studied the effects of taxes and subsidies in extractive industries and have derived rules for determining them. (See, e.g., Burness, 1976; Sweeney, 1977; Dasgupta, Heal, and Stiglitz, 1980; Conrad and Hool, 1981; Heaps, 1985.) These rules, however, are not as general as they might seem. They are derived under the simplifying assumption that unprocessed ore is sold in a market and that taxes are levied directly on extraction, an assumption that is often violated in practice. This paper differs from the above in assuming that taxation can occur at any stage of production. When stages of processing are introduced, tax-policy effects can be dramatically different from those predicted by the simpler model.

The organization of the paper is as follows. In the next section, the standard model of resource taxation is discussed. The third section presents the general stages-of-production model and the fourth section analyses special cases. The final section summarizes and concludes.

#### TAXATION OF ORE

The analysis of tax-policy effects presented below deals with an extractive firm that is a price taker in both input and output markets. To abstract from the effects of erratic price movements on extraction profiles, undiscounted prices are assumed to appreciate at constant rates.

Tax policy is assumed to have no effect on prices. Most mineral-commodities are sold in world markets. Taxes, however, are often levied by smaller jurisdictions such as a state or province. A tax imposed locally should therefore not affect the price at which the firm can sell its output.

Two additional simplifying assumptions are made. First, it is assumed that the size of the stock of remaining reserves does not affect extraction cost. This

assumption, though unrealistic, is made in most theoretical resource-taxation studies.

Second, it is assumed that the entire stock of ore is ultimately depleted. Tax policy can therefore shift extraction from present to future and vice versa but cannot influence ultimate recovery. This assumption is needed if the meaning of increased conservation or more rapid exploitation is to be unambiguous.<sup>1</sup>

The firm is assumed to maximize the present value of its stream of profits. Under the above assumptions, an equilibrium condition for this maximization is that *the present value of the profit on the marginal unit of ore mined be equal in all time periods*. If this were not so, producers would have an incentive to shift extraction from periods of low- to periods of high-discounted profits.

Suppose that the firm is in equilibrium prior to the imposition of a tax. *Pre and after-tax extraction patterns will be the same if and only if the present value of the tax on the marginal unit of ore extracted is equal in all periods*. If the tax on the marginal unit appreciates at a rate less than the rate of interest, it falls in present value and thus gives producers an incentive to retard extraction. And if it appreciates at a rate greater than the interest rate and thus increases in present value, firms have an incentive to extract more rapidly to reduce the present value of tax payments.

These conditions can be stated formally as follows. Let  $R$  be the rate at which ore is extracted,  $r$  be the constant interest rate (the firm's private discount rate), and  $T$  be the tax per unit of ore mined, which varies with time but not with  $R$ . Then, by the above reasoning, if

$$\begin{aligned} &< r && \text{extraction is delayed} \\ \dot{T}/T = r && \text{the tax is neutral} \\ &> r && \text{extraction is accelerated.} \end{aligned} \tag{1}$$

In conditions (1), a dot over a variable denotes its time derivative. It should be clear that a subsidy per unit of ore mined (a negative tax) affects extraction in the opposite direction.

Conditions (1) can be used to derive tax effects when taxes fall directly on the unprocessed ore.<sup>2</sup> Their use results in simple, well-known rules. For example, consider a severance tax which is a constant amount  $\Gamma_1$  per unit of ore extracted. In this case,  $T = \Gamma_1$ ,  $\dot{T}/T = 0 < r$ , and extraction is delayed.

Second, consider a royalty that is a constant fraction  $\Gamma_2$  of revenues. In this case,  $T = \Gamma_2 P$ ,  $\dot{T}/T = \dot{P}/P$ , and extraction is delayed, unchanged, or accelerated depending on whether the rate of ore-price appreciation is less than, equal to, or greater than the interest rate.

1 Conservation usually means forgoing present for future consumption. If, however, present consumption forgone does not result in higher future consumption (i.e., if more ore remains in the ground at  $t = \infty$ ) it is not clear if tax policy 'conserves' the resource. In addition, given the assumption that cumulative extraction does not affect extraction cost, the second assumption does not seem unreasonable.

2 Conditions (1) are used by Burness (1976) and Sweeney (1977).

A profits tax, which is a constant fraction  $\Gamma_3$  of profits, is neutral because in equilibrium profit on the marginal unit is increasing at the rate of interest. If, however, the profits tax is combined with a percentage depletion allowance, under which a fixed proportion  $\Gamma_4$  of revenues is exempt from profits taxes, the combined tax is in general not neutral. The effect of these taxes is equivalent to a negative royalty equal to  $\Gamma_4\Gamma_3(1 - \Gamma_3)$  combined with a profits tax (Sweeney, 1977). A depletion allowance thus retards, has no effect on, or speeds up extraction depending on whether the rate of ore-price appreciation is greater than, equal to, or less than the interest rate.

Finally, consider a price control  $\bar{P}$  that can vary with time. With a price control, the effective tax per unit of ore extracted is the difference between the uncontrolled and the controlled price,  $T = P - \bar{P}$ . Therefore,  $\dot{T}/T = (\dot{P} - \dot{\bar{P}})/(P - \bar{P})$ . A price control thus retards, has no effect on, or speeds up extraction depending on whether the difference between the uncontrolled and controlled prices is appreciating at a rate less than, equal to, or greater than the rate of interest.

#### TAXATION AT VARIOUS STAGES OF PRODUCTION

When taxes are levied at various stages of production, rules for determining tax effects cease to be so simple. First, we must clarify the meaning of neutrality and second, we must determine how downstream taxes affect extraction patterns.

With the model of the previous section, the firm had only one decision variable,  $R$ . Neutrality was therefore defined with respect to  $R$ . When processing is introduced, the firm must choose not only  $R$  but also the amounts of variable inputs to use and the quantity of output to produce.

Let  $X$  be a vector of variable inputs with prices  $V$  and  $\phi(V, R)$  be the extraction-cost function. In this model, the extracted ore is not sold in a market; it is transferred within a vertically integrated firm. At the second stage, ore is combined with variable inputs to produce metal  $Q$  according to a processing-production function  $Q = f(X, R)$ . The words 'ore' and 'metal' are used generically. That is, 'ore' might be crude petroleum, in which case 'metal' would be a petroleum product. Finally, metal is sold in a market at a price  $P$ .

The firm's problem is to maximize variable profit, where its single-period before-tax variable-profit function is

$$\pi(P, V, R) = \max_X \{Pf(X, R) - V^T X\} - \phi(V, R). \quad (2)$$

A tax can be placed on ore as it leaves the ground (a severance tax), or it can be placed downstream on metal production or metal revenues. The general form of the firm's after-tax variable-profit function is  $\bar{\pi}(P, V, R, \Gamma)$ , where  $\Gamma$  is a vector of tax variables.

A tax policy  $\Gamma$  is said to be absolutely neutral if  $\Gamma$  does not change the extraction rate  $R$ , the marginal rate of technical substitution between  $R$  and all inputs in  $X$ , and the output rate  $Q$ .

In the stages-of-production model, lump-sum and cash-flow or profits taxes are absolutely neutral. I choose, however, to concentrate on a somewhat weaker form of neutrality. In this paper a tax is said to be (relatively) neutral if it leaves the rate of extraction – and therefore the stock of the resource that remains in the ground in each period – unchanged. The extraction rate is singled out because it is the belief of the author that the fundamental problem in the economics of exhaustible resources is the intergenerational allocation of a finite resource stock. A tax is therefore said to be neutral if it leaves this allocation unchanged. Thus, in what follows, the effects of taxation on extraction  $R$  are analysed.

Consider a severance tax, which by definition falls on the unprocessed ore. With this tax, and only with this tax, conditions (1) can be used directly to determine its effect on extraction. The derivation of the effects of downstream taxes such as royalties, depletion allowances, and price controls is more complex. In order to use conditions (1) when there are stages of production, taxes on metal production or metal revenues must be traced backwards to see how they translate into equivalent taxes on extraction.

Let  $\tau_1$  be a tax on revenues from the sale of metal  $PQ$  and  $\tau_2$  be a tax on the production of metal  $Q$ .<sup>3</sup> The idea behind the assessment of downstream tax effects is to transform  $\tau_1$  and  $\tau_2$  into equivalent taxes  $T_1$  and  $T_2$  on extraction  $R$ . When these mappings have been found, conditions (1) can be used to determine the effects of  $\tau_1$  and  $\tau_2$ .

If a tax per unit of ore mined varies with the rate of extraction, it is virtually impossible to determine its effect. In addition, because taxes that vary with extraction rates create incentives for producers to choose production levels for which taxes are low, such taxes cannot in general be neutral.<sup>4</sup> The analysis is therefore limited to technologies for which the tax pass through from revenue or output to extraction is independent of the rate of extraction. That is, a mapping such that

$$\tau_1 PQ = T_1 R = h_1(\tau_1, P, V)R, \quad (3)$$

is sought. This leads to

**PROPOSITION 1.** *If  $f$  is concave, differentiable, and increasing, a tax  $\tau_1$  ( $\tau_2$ ) on metal revenue  $PQ$  (on metal output  $Q$ ) can be transformed into an implicit tax*

3 Only two types of taxes are considered, because, as is shown below, the many different taxes analysed later are all special cases of these two.

4 Taxes that are functions of extraction rates have been adopted with disastrous consequences. For example, U.S. tax law, such as the Crude Oil Windfall Profit Tax of 1980, gives special treatment to 'stripper' wells, wells that produce less than ten barrels a day. The problem is that any well can, by cutting back production, become a stripper well. The result is a large increase in production from small inefficient wells.

$T_1 = h_1(\tau_1, P, V)(T_2 = h_2(\tau_2, P, V))$  on extraction  $R$  that is independent of  $R$  if and only if  $f$  is positively linearly homogeneous.

*Proof.* See appendix A.

In what follows, we therefore assume that  $f$  is linearly homogeneous. Linear homogeneity may be less restrictive than it initially appears. First, it is required of the processing (not the extraction) production function. A priori, there is no reason to believe that there are inputs to processing that are fixed in the long run and thus subject to diminishing returns. And second, linear homogeneity is required only in the range of output that the firm might consider producing.

When  $f$  is linearly homogeneous, the profit function can be written as

$$\pi(P, V, R) = g(P, V)R - \phi(V, R). \quad (4)$$

Hence

$$\tau_1 PQ = \tau_1 P \pi_P(P, V, R) = \tau_1 P g_P(P, V)R,$$

$$T_1 = \tau_1 P g_P(P, V),$$

and

$$\dot{T}_1/T_1 = \dot{\tau}_1/\tau_1 + \dot{P}/P + \dot{g}_P/g_P. \quad (5)$$

Let  $s_Q$  be output's share in profit,  $s_Q = P g_P/g$ . If  $V$  is constant, then (5) becomes

$$\dot{T}_1/T_1 = \dot{\tau}_1/\tau_1 + \dot{s}_Q/s_Q + s_Q \dot{P}/P. \quad (6)$$

By similar reasoning,

$$T_2 = \tau_2 g_P(P, V)$$

and

$$\dot{T}_2/T_2 = \dot{T}_1/T_1 - \dot{P}/P. \quad (7)$$

Equations (6) and (7) are the mappings sought. They show how downstream taxes and subsidies translate into equivalent taxes on ore extraction and give formulas for the rates of appreciation of the equivalent taxes. These equations can be combined with conditions (1) to determine downstream tax effects. When, for example,

$$\begin{aligned} &< r && \text{extraction is delayed} \\ \dot{T}_1/T_1(\dot{T}_2/T_2) &= r && \text{the tax is neutral} \\ &> r && \text{extraction is accelerated.} \end{aligned} \quad (8)$$

In addition, we can use equations (6) and (7) to design neutral taxes. Neutral taxes are solutions to the differential equation

$$\dot{T}/T = r. \quad (9)$$

Neutrality, however, is often not the goal. Taxing authorities may want to

change the firm's decisions. Suppose that, through a welfare analysis, the government has decided that it prefers the extraction path  $\bar{R}_t$  to the pre-tax path  $R_t$ . The firm can be induced to produce  $\bar{R}_t$  if a tax  $\bar{\Gamma}$  is chosen such that

$$\begin{aligned} \frac{d\bar{\pi}}{dt}(P, V, \bar{R}, \bar{\Gamma}) / \bar{\pi}(P, V, \bar{R}, \bar{\Gamma}) \\ = \frac{d}{dt} \{ \bar{g}(P, V, \bar{\Gamma})\bar{R} - MC \} / ( \bar{g}(P, V, \bar{\Gamma})\bar{R} - MC ) = r, \end{aligned} \quad (10)$$

where  $MC = \phi_R$ .

Equations (8) and (10) are the basic results of the model. The first gives a rule for determining the effect of a given tax and the second shows how to design a tax that achieves a particular objective. These equations are completely general. It should be obvious, however, that specific effects depend on the technology of processing. In the next section, therefore, downstream tax effects are derived under various technological assumptions.

#### TAX-POLICY EFFECTS UNDER DIFFERENT TECHNOLOGIES

In what follows, it is assumed that there is one variable input  $X$  with price  $V$ . Of interest is how output price  $P$  appreciates relative to  $V$ . For simplicity, it is assumed that  $V$  is constant and that  $P$  appreciates at a constant rate  $a > 0$ .<sup>5</sup> Then

$$P_t / V_t = P_0 e^{at} / V_0. \quad (11)$$

Without loss of generality, units can be chosen such that  $P_0 / V_0 = 1$  (so that  $\ln(P_t / V_t) = at$ ).

##### *Fixed-coefficient processing technology*

Suppose that  $f$  is a fixed coefficient. Then

$$Q = f(X, R) = \min \{ \alpha_1 R, \alpha_2 X \}. \quad (12)$$

If producers minimize cost, they choose inputs such that

$$Q = \alpha_1 R = \alpha_2 X. \quad (13)$$

It should be obvious that this case reduces to the standard case where ore is taxed directly (i.e., units can be chosen such that  $\alpha_1 = 1$ ). Tax effects for the fixed-coefficient technology are therefore the same as those derived in the second section.

To illustrate the use of equations (9) and (10), we design a time-varying

5 The parameter  $a$  should be interpreted as the rate of increase of output price relative to other prices;  $a$  is assumed to be positive, because in most exhaustible-resource models the undiscounted price of the resource rises over time. Resource prices can fall initially (see, e.g., Pindyck, 1978 and Slade, 1982) but eventually they rise. Equations (1), (6), and (7) can still be used to assess tax effects when  $a$  is a function of time or is negative.

royalty that is neutral and one that induces the firm to select a specified extraction path  $\bar{R}$ . The neutral royalty requires that  $\dot{T}_1/T_1 = r$ , which can easily be shown to yield

$$\dot{\tau}_1/\tau_1 = r - a. \quad (14)$$

Integrating we obtain the neutral royalty

$$\tau_1(t) = \tau_0 e^{(r-a)t}. \quad (15)$$

Suppose instead that the taxing authority wishes to induce the firm to select an extraction path  $\bar{R}$ . We assume that  $\bar{R}$  declines exponentially,

$$\bar{R}_t = R_0 e^{-\delta t}. \quad (16)$$

It is shown in appendix B that, if marginal-extraction cost is linear,  $MC = cR$ , then the royalty

$$\begin{aligned} \bar{\tau}_1(t) &= 1 - e^{(r-a)t} \{ 1 - \bar{\tau}_0 + \theta/(r + \delta)(1 - e^{-(r+\delta)t}) \}, \\ \bar{\tau}_0 &= \bar{\tau}_1(0), \\ \theta &= -cR_0(\delta + r)/P_0, \end{aligned} \quad (17)$$

induces the firm to produce  $\bar{R}$ .

#### *Cobb-Douglas processing technology*

Suppose that  $f$  is Cobb-Douglas. Then

$$Q = F(X, R) = AR^\alpha X^{(1-\alpha)} \quad 0 < \alpha < 1. \quad (18)$$

Without loss of generality,  $A$  can be set equal to one.

When there is no processing,  $\alpha$  (the resource share) is one. As more and more stages of production are added, the resource share falls. The Cobb-Douglas technology can thus be thought of as an approximation to a more general stages-of-production model where  $\alpha$  measures the degree of processing prior to the imposition of the tax.

Consider the effect of a constant royalty  $\tau_1$  levied on metal revenues. Using equation (6) and the fact that  $s_Q = 1/\alpha$ , which is constant, we see that

$$\dot{T}_1/T_1 = a/\alpha. \quad (19)$$

Similarly, equation (7) shows that

$$\dot{T}_2/T_2 = a(1 - \alpha)/\alpha. \quad (20)$$

Equation (19) shows that a royalty causes extraction to proceed more slowly, at the same rate, or more rapidly, according to whether

$$a/\alpha <, =, \text{ or } > r. \quad (21)$$

6 This result is similar to one noted by Lewis and Slade (1984) for the discrete-time case.

When there is no processing, in contrast, the effect of a royalty depends on whether  $a < , = ,$  or  $> r$ . With the Cobb-Douglas case, because  $\alpha < 1$ , a royalty is less likely to lead to conservation. The conventional wisdom, however, holds that royalties conserve the resource.<sup>7</sup> It is therefore more likely that the effect of a royalty will be counter-intuitive when processing is introduced. In addition, as more and more stages of processing are added (as  $\alpha$  falls) the probability of counter-intuitive tax effects increases.<sup>8</sup>

Calculations similar to those used to obtain equation (15) show that a time-varying royalty  $\tau_1(t)$  such that

$$\tau_1(t) = \tau_0 e^{(r-a/\alpha)t} \quad (22)$$

is neutral.

Here also, we can use equation (10) to design a royalty that induces the firm to choose a particular extraction path  $\bar{R}$ . For example, suppose that desired extraction is again given by equation (16). It is shown in appendix B that if

$$\bar{\tau}_1(t) = 1 - \{e^{(r-a/\alpha)t} [(1 - \tau_0)^{1/\alpha} + \theta'/(r + \delta)(1 - e^{-(r+\delta)t})]\}^\alpha, \\ \text{where } \theta' = -cR_0(r + \delta)/d \text{ and } d = KP_0^{1/\alpha}V^{(\alpha-1)/\alpha}, \quad (23)$$

the firm will extract  $\bar{R}$ . ( $K$  is a constant defined in appendix B.)

To consider the effects of price controls, it is necessary to specify the form of the control. A typical policy is to maintain the controlled price at some fraction  $\eta$ ,  $0 < \eta < 1$ , of the uncontrolled price. For example, a recent policy of the Canadian government was to keep the domestic price of natural gas at 75 per cent of the world price. I call this form of control a constant-relative-price control.

Let  $\bar{P}$  be the controlled price. With a constant-relative-price control,

$$\bar{P}_t = \eta P_t, \quad 0 < \eta < 1. \quad (24)$$

The tax per unit of metal produced is then

$$\tau_1 = P_t - \bar{P}_t = P_t - \eta P_t = (1 - \eta)P_t. \quad (25)$$

A constant-relative-price control is thus equivalent to a royalty of size  $\tau_1 = (1 - \eta)$  whose effects have already been analysed.

Suppose instead that the controlled price is maintained at  $P_t - \tau_2$ , where  $\tau_2$  is a positive constant. I call such a scheme a constant-price-difference control.

<sup>7</sup> For example, Dasgupta, Heal, and Stiglitz (1980, 171) state that 'A depletion allowance at a constant rate increases the rate of extraction, and its gradual removal increases it still further. A sales tax (royalty) has precisely the opposite effect.' It is clear from the context that the authors realize that the truth of this statement depends on the path of resources prices. In virtually all theoretical exhaustible-resource models where price is endogenous, however, price rises at a rate less than or equal to the rate of interest.

<sup>8</sup> The economic intuition for these results is as follows. As  $P/V$  increases, so does  $Q/R$ ; so that a constant tax on  $PQ$  means that the tax on  $R$  increases faster than  $\dot{P}/P$ , giving an incentive to extract at earlier dates. And the smaller is  $R$ 's share, the larger is the tax on  $R$ , holding the tax on  $PQ$  constant.

The tax per unit of metal produced is then  $\tau_2$ . Equation (20) therefore shows the rate of change of  $T_2$ . The constant-price-difference control thus retards, has no effect on, or speeds up extraction, depending on whether  $a(1 - \alpha)/\alpha$  is less than, equal to, or greater than the rate of interest.

When there is no processing,  $\alpha = 1$  and  $a(1 - \alpha)/\alpha = 0 < r$ . In this case, the constant-price-difference control always retards extraction. When other inputs enter processing, however, the control can encourage more rapid extraction. At the opposite extreme where  $\alpha = 0$ , the control always encourages depletion. Because the conventional wisdom holds that price controls conserve the resource, the probability that a control has counter-intuitive effects increases as the resource share in output falls (as the degree of processing increases).

Calculations similar to those used to obtain equation (22) show that a time-varying price-difference control  $\tau_2(t)$  such that

$$\tau_2(t) = \tau_0 e^{[r + a(1 - 1/\alpha)]t} \quad (26)$$

is neutral. In addition, a result similar to equation (23) can be derived for this form of price control.

#### *Translog variable-profit function*

It is well known that a Cobb-Douglas technology is very restrictive. In particular, its functional form constrains the elasticity of substitution between inputs to be one. If this were a close approximation to real-world substitution possibilities, the constraint would be of little concern. Empirical studies, however, find wide variations in elasticities of substitution between natural-resource and other inputs. A flexible functional form that does not place any restriction on substitution elasticities is therefore considered. Many such functional forms exist; the translog (Christensen, Jorgenson, and Lau, 1971) is analysed here.

Suppose that the function  $g$  is translog. A translog with only two arguments can be expressed very concisely; its logarithm is a quadratic in the logarithm of the ratio of the two arguments. Thus,

$$\pi(P, V, R) = \exp \{ \alpha_0 + \alpha \ln (P/V) + \frac{1}{2} \gamma \ln (P/V)^2 \} R - \phi(V, R).^9 \quad (27)$$

With the translog variable-profit function, output's share in profit,  $s_Q$ , is

$$s_Q = \partial \ln g / \partial \ln P = \alpha + \gamma at. \quad (28)$$

Equation (6) then shows that

$$\dot{T}_1/T_1 = a((\alpha + \gamma at)^2 + \gamma)/(\alpha + \gamma at) \simeq c_0 + c_1 t.^{10} \quad (29)$$

9 The constant  $\alpha_0$  includes  $\ln V$ , which is constant by assumption. Linear homogeneity in prices has been imposed a priori.

The rate of appreciation of the tax per unit of ore extracted is no longer constant but is a function of time.

The left-hand side of equation (29) is equal in sign to

$$\dot{T}_1/T_1 \stackrel{s}{=} s_Q^2 + \gamma, \quad (30)$$

where the symbol  $\stackrel{s}{=}$  means equal in sign. The behaviour of output's profit share  $s_Q$  over time (the sign of  $\gamma$ ) is thus crucial in determining the intertemporal behaviour of the tax  $T_1$ . If  $\gamma > 0$ ,  $\dot{T}_1/T_1$  grows and will eventually be greater than  $r$ .<sup>11</sup> On the other hand, if  $\gamma < 0$ ,  $s_Q$  decreases over time until the firm does not cover variable costs. In either case, the results are counter-intuitive. For positive  $\gamma$ , it is likely that the imposition of a constant royalty will eventually speed up extraction for any values of  $a$  and  $r$ , and for negative  $\gamma$ , the firm will eventually cease processing, in spite of appreciating output price.

A time-varying royalty  $\tau_1(t)$  such that

$$\dot{\tau}_1/\tau_1 = r - \{[a(\alpha + \gamma at)^2 + \gamma]/(\alpha + \gamma at)\} \simeq r - c_0 - c_1 t \quad (31)$$

will be neutral. In this case

$$\tau_1(t) \simeq e^{(r-c_0-c_1/2t)t}. \quad (32)$$

For neutrality,  $\tau_1$  can no longer appreciate at a constant rate, and unless  $\gamma = 0$  (the Cobb-Douglas case)  $\tau_1$  will not be a monotonic function of time.

Similar results for price controls and depletion allowances can be derived in an obvious fashion. In the translog case, the mathematics required to design a tax that induces the firm to produce  $\bar{R}$  is more tedious. Nevertheless, the calculation is straightforward.

### *Other technologies*

The Cobb-Douglas and translog functions provide local first- (second-) order approximations to arbitrary once (twice) differentiable functions. The results of the two preceding sections generalize easily to  $n$ th-order logarithmic approximations to arbitrary  $n$ th-differentiable variable-profit functions.

In general, if a polynomial of degree  $n$  in logarithms is an accurate description of the variable-profit function, the following hold:

1. If  $\tau_1$  is constant over time,  $\dot{T}_1/T_1$  is approximately an  $n - 1$  degree polynomial in  $t$ .
2. A constant royalty  $\tau_1$  is apt to result in alternating periods characterized by increased conservation and more rapid exploitation. The boundaries

<sup>10</sup> This approximation is not exact unless the two polynomials factor exactly. The approximation, however, is always close for large  $t$ .

<sup>11</sup> It is assumed that the mine will not close before  $\dot{T}_1/T_1 > r$ .

between the periods occur at times  $t$  that are roots of the polynomial  $\dot{T}/T = r$ . There can therefore be at most  $n$  of these periods.

3. The form of a neutral time-varying royalty is approximately an exponential raised to an  $n$ th-degree polynomial in  $t$ .

The analysis of specific technologies is subject to diminishing returns. It should be clear by now that the method of analysing tax effects derived in the second section can be applied by any technology that satisfies the assumptions of proposition 1. In addition, the method can be used to assess the effects of a large number of tax-policy instruments other than those considered here. Whether the goal is to design taxes that achieve particular objectives or to assess the intertemporal response to specific tax proposals, the stages-of-production model provides a powerful tool.

## SUMMARY AND CONCLUSIONS

Theoretical studies of exhaustible-resource taxation usually consider the special case where the unprocessed ore is taxed and sold directly. Under this assumption, it is possible to derive simple rules for determining tax effects. Non-fuel minerals, however, are almost always subject to some degree of processing prior to sale. For example, many mining companies are vertically integrated into milling, smelting, and refining. With the fuels, processing is less important but not negligible. For example, natural gas must be brought up to pipeline specification before it can be shipped. When stages of production are introduced, simple rules no longer apply, and responses to tax policy can differ dramatically from those predicted by the simpler model.

In this paper a general method of assessing the effects of downstream taxes is derived. The method is then used to determine how specific taxes affect extraction profiles. The dynamic behaviour of extraction is singled out because it is felt that the intergenerational allocation of a finite stock  $S$  is the fundamental problem in the economics of exhaustible resources. It should be obvious, however, that similar methods can be used to determine how taxes affect the firm's other decision variables such as metal output.

In addition to examining the effects of existing taxes, the paper outlines a method of designing downstream taxes that achieve specific goals. Goals that might be of interest include neutrality, increased conservation, and accelerated depletion.

The methods of determining tax effects and of designing tax policy are completely general and can therefore be applied to any differentiable technology. The direction of tax effects, however, varies with the technology considered. To obtain concrete results, therefore, three special cases are considered – the fixed-coefficient and Cobb-Douglas production functions and the translog variable-profit function.

Several insights can be gained from the exercise. First, when processing occurs prior to sale, a tax that achieves a particular objective depends on all the

parameters of the processing technology. A consequence of technology-specific tax effects is that a tax that is neutral for an industry, for example, cannot be neutral for every firm in the industry unless all firms have identical technologies.

Second, the choice of stage of production to tax is often made arbitrarily. It is nevertheless an important determinant of the intertemporal supply response. In levying taxes, therefore, more thought should be given to this decision.

Finally, to avoid tax-policy effects that are counter-intuitive, it is desirable to tax a stage of production that is as close to extraction as possible. Recall that in the Cobb-Douglas case, as the degree of processing increases, the probability of counter-intuitive tax effects also increases. In addition, substitution possibilities are apt to be less important at the earlier stages. For example, there is no substitute for iron ore in the production of pig iron, but pig and scrap iron are substitutes in the production of steel. As substitution possibilities diminish, the technology approaches fixed coefficient, which was seen to be equivalent to the case where ore is taxed directly.

It is important for tax authorities to understand the ramifications of their actions. They can adopt a tax policy that achieves a desired goal only after analysing the technology of the industry that is being taxed. In this paper rules for performing this analysis are derived. It is hoped that through the use of these rules a better understanding of tax-policy effects will be achieved and that, as a consequence, the taxes and subsidies that are adopted will be more likely to foster desired goals.

#### APPENDIX A

**PROPOSITION 1.** *If  $f(X, R)$ , the processing production function, is concave, differentiable, and increasing, a tax  $\tau$  on metal revenues  $PQ$  can be transformed into an implicit tax  $h(\tau, P, V)$  on extraction  $R$  if and only if  $f$  is positively linearly homogeneous.*

*Proof.* Suppose that  $f$  is positively linearly homogeneous. Then

$$\begin{aligned}\pi(P, V, R) &= \max_X \{Pf(X, R) - V'X\} - \phi(R, V) \\ &= R \max_{X/R} \{Pf(X/R, 1) - V'(X/R)\} - \phi(R, V) \\ &= Rg(P, V) - \phi(R, V).\end{aligned}$$

The tax collected is thus

$$\tau PQ = \tau P\pi_P = \tau Pg_P(P, V)R,$$

where the first equality results from the application of Hotelling's Lemma. We thus have

$$h(\tau, P, V) = \tau Pg_P(P, V).$$

Conversely, suppose that

$$\tau PQ = h(\tau, P, V)R \quad \text{for all } Q = f(X, R). \quad (\text{A1})$$

Let

$$\begin{aligned} \pi(P, V, R) &= \pi^1(P, V, R) + \pi^2(R, V) \\ &= \max_X \{Pf(X, R) - V'X\} - \phi(R, V), \end{aligned} \quad (\text{A2})$$

where

$$\pi^2(R, V) = -\phi(R, V).$$

Then

$$\pi^1(P, V, R) = Pf(X^*, R) - V'X^*,$$

where  $X^*$  is the optimal solution to the maximization (A2). Using (A1),

$$\pi_P^1(P, V, R) = f(X^*, R) = \tilde{\pi}_P^1(P, V)R. \quad (\text{A3})$$

Integrating (A3) yields

$$\pi^1(P, V, R) = \tilde{\pi}^1(P, V)R + \hat{\pi}^1(V, R).$$

By duality,

$$f(X, R) = \max \{Q \mid PQ - V'X \leq \pi^1(P, V, R) \text{ for all } P > 0, V > 0_n\}.$$

Hence

$$f(X, R) = \max_{\tilde{X}, \hat{X}} \{\tilde{f}(\tilde{X}, R) + \hat{f}(\hat{X}, R) \mid \tilde{X} + \hat{X} = X\},$$

where  $\tilde{f}$  is dual to  $\tilde{\pi}^1$  and is positively linearly homogeneous, and  $\hat{f}$  is dual to  $\hat{\pi}^1$ . Recovering  $\pi^1$  from  $f$  yields

$$\begin{aligned} \pi^1(P, V, R) &= \max_X \{\max_{\tilde{X}, \hat{X}} [P(\tilde{f}(\tilde{X}, R) + \hat{f}(\hat{X}, R)) \\ &\quad - V'(\tilde{X} + \hat{X}) \mid \tilde{X} + \hat{X} = X]\} \\ &= \max_{\tilde{X}, \hat{X}} \{P\tilde{f}(\tilde{X}, R) - V'\tilde{X} + P\hat{f}(\hat{X}, R) - V'\hat{X}\} \\ &= \tilde{\pi}^1(P, V)R + \hat{\pi}^1(P, V, R), \text{ say.} \end{aligned} \quad (\text{A4})$$

But

$$\hat{\pi}^1(P, V, R) = P\hat{f}(\hat{X}^*, R) - V'\hat{X}^* = \hat{\pi}^1(V, R).$$

Hence

$$\begin{aligned} \partial \hat{\pi}^1(V, R)/\partial P &= \partial \hat{\pi}^1(P, V, R)/\partial P = 0 \\ &= \hat{f}(\hat{X}^*, R) + P \sum_i (f_i \partial \hat{X}_i / \partial P - V_i \partial \hat{X}_i / \partial P) \end{aligned}$$

$$= \hat{f}(\hat{X}^*, R) + \left( \sum_i P f_i - V_i \right) \partial X_i / \partial P$$

where the term in parentheses equals zero by the first-order conditions for the maximization (A4).

Therefore

$$\hat{f}(\hat{X}^*, R) = 0.$$

By the concavity of  $f$ , any  $X$  can be an optimal  $X$ . Therefore,

$$\hat{f}(X, R) = 0,$$

and

$$f(X, R) = \tilde{f}(\tilde{X}, R),$$

which is positively linearly homogeneous.

The proof for a tax on output  $Q$  is very similar.

## APPENDIX B

*Problem.* Given an extraction path  $\bar{R}$ , choose a royalty  $\bar{\tau}$  such that the firm will extract  $\bar{R}$ .

*Case 1.* Fixed-coefficient production function,  $\bar{R}_t = R_0 e^{-\delta t}$ ,  $MC = cR$ . Let  $x = 1 - \bar{\tau}$ . In this case  $\bar{g}(P, V, \bar{\Gamma}) = P(1 - \bar{\tau}) = Px$  and  $\dot{\bar{g}}/\bar{g} = a + (\dot{x}/x)$ . By equation (10),

$$\dot{\bar{g}}/\bar{g} = MC/\bar{g} + r - rMC/\bar{g},$$

or

$$a + \dot{x}/x = -(\delta c R_0 e^{-\delta t})/(P_0 e^{at} x) + r - (rc R_0 e^{-\delta t})/(P_0 e^{at} x),$$

which simplifies to

$$\dot{x} = (r - a)x + \theta e^{-(\delta+a)t},$$

where  $\theta = -cR_0(\delta + r)/P_0$ .

This is a differential equation of the form

$$dy/dt = Ay + f(t)$$

whose solution is

$$y_t = e^{At} y_0 + \int_0^t e^{A(t-s)} f(s) ds \quad (\text{B1})$$

(see for example Duff and Naylor, 1966, 36).

Here,  $y = x$ ,  $A = r - a$ , and  $f(t) = \theta e^{-(\delta+a)t}$ . After algebraic simplification we obtain

$$x_t = e^{(r-a)t} \{x_0 + \theta/(r + \delta)(1 - e^{-(r+\delta)t})\}.$$

Case 2. Cobb-Douglas production function,  $\bar{R}_t = R_0 e^{-\delta t}$ ,  $MC = cR$ . In this case,

$$g = KP^{1/\alpha} V^{(\alpha-1)/\alpha},$$

where  $K2(1 - \alpha)^{(1-\alpha)/\alpha} = (1 - \alpha)^{1/\alpha}$ , and

$$\bar{g} = d(e^{at}x)^\beta,$$

where  $\beta = 1/\alpha$ ,  $x = 1 - \bar{r}$ , and  $d = KP_0^\beta V^{(1-\beta)}$ . Finally,

$$\dot{\bar{g}}/\bar{g} = \beta(\dot{x}/x + a).$$

By equation (10),

$$\dot{\bar{g}}/\bar{g} = MC/\bar{g} + r - rMC/\bar{g},$$

or

$$\beta(\dot{x}/x + a) = -(c\delta R_0 e^{-\delta t})/d(e^{at}x)^\beta + r - (rcR_0 e^{-\delta t})/(d(e^{at}x)^\beta),$$

which simplifies to

$$(\dot{x}^\beta) = (r - \beta a)x^\beta + \theta'e^{-(\delta+a\beta)t},$$

where  $\theta' = -cR_0(\delta + r)/d$ . We use (B1) with  $y = x^\beta$ ,  $A = r - \beta a$ , and  $f(t) = \theta'e^{-(\delta+a\beta)t}$ . After algebraic simplification we obtain

$$x_t = \{e^{(r-a/\alpha)t}[x_0^{1/\alpha} + \theta'/(r + \delta)(1 - e^{-(r+\delta)t})]\}^\alpha.$$

#### REFERENCES

- Burness, H.S. (1976) 'On the extraction of replenishable natural resources.' *Journal of Environmental Economics and Management* 3:4
- Christensen, L.R., D.W. Jorgenson, and L.J. Lau (1971) 'Conjugate duality and the transcendental logarithmic function.' *Econometrica* 39:2
- Conrad, R.F. and B. Hool (1981) 'Resource taxation with heterogeneous quality and endogenous reserves.' *Journal of Public Economics* 16:1
- Dasgupta, P., G.M. Heal, and J.E. Stiglitz (1980) 'The taxation of exhaustible resources.' In G.A. Hughes and G.M. Heal, eds., *Public Policy and the Tax System* (London: George Allen & Unwin)
- Duff, G.F.C. and D. Naylor (1966) *Differential Equations of Applied Mathematics* (New York: Wiley)
- Heaps, T. (1985) 'The taxation of nonreplenishable natural resources revisited.' *Journal of Environmental Economics and Management* 12:1
- Lewis, T.R. and M.E. Slade (1984) 'The effects of price controls and taxes on ex-

- haustible-resource production.' In A.D. Scott, ed., *Progress in Natural-Resource Economics* (Oxford: Oxford University Press)
- Pindyck, R.S. (1978) 'The optimal exploration and production of nonrenewable resources.' *Journal of Political Economy* 86:5
- Slade, M.E. (1982) 'Trends in natural-resource commodity prices: an analysis of the time domain.' *Journal of Environmental Economics and Management* 9:2
- (1984) 'Tax policy and the supply of exhaustible resources: theory and practice.' *Land Economics* 60:2
- Sweeney, J.L. (1977) 'Economics of depletable natural resources: market forces and intertemporal biases.' *The Review of Economic Studies* 44:1