A TEST OF THE THEORY OF EXHAUSTIBLE RESOURCES*

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An empirical test of the theory of exhaustible resources requires an estimate of the time path of the shadow price of the unextracted resource that generally is not observable because of the prevalence of vertical integration in natural resource industries. In this paper we use duality theory to derive an econometric model that provides a statistical test of the theory of exhaustible resources. A restricted cost function is used to obtain estimates of the shadow prices of unextracted resources. The procedure is illustrated with data for the Canadian metal mining industry. For this industry the empirical implications of the theory of exhaustible resources are strongly rejected

I. INTRODUCTION

More than half a century ago Hotelling [1931] provided a rigorous theoretical model of the dynamic behavior of private markets for exhaustible resources. After a long period of relative neglect, the theory of exhaustible resources has received greatly increased attention since the early 1970s, and there now exists a large and well-developed literature based on the theoretical framework introduced by Hotelling. However, the ability of the theory of exhaustible resources to describe and predict the actual behavior of resource markets remains an open question.

The principal obstacle to empirical tests of the theory of exhaustible resources has been data availability. The implications of the theory for economic behavior are expressed in terms of the time path of the shadow price of the unextracted resource (also referred to as the resource's in situ price, scarcity rent, or net price). However, because of vertical integration in natural resource industries, market transactions generally occur only after a resource has been extracted and processed. In addition, the effect of cumulative extraction on the marginal cost of extraction, which is one of the major theoretical factors determining the time path of prices, is not directly observable.

In this paper we use duality theory to derive an econometric

model that provides a statistical test of the theory of exhaustible resources. Following Halvorsen and Smith [1984], a restricted cost function is used to obtain estimates of the shadow prices of unextracted resources from cost and production data for vertically integrated natural resource industries. The restricted cost function used here also provides estimates of the effects of cumulative extraction on the marginal cost of extraction.

The implications of the theory of exhaustible resources are expressed as parametric restrictions on the restricted cost function model and are tested using a Hausman [1978] specification test. The procedure is illustrated with data for the Canadian metal mining industry. For this industry the parametric restrictions implied by the theory of exhaustible resources are strongly rejected.

The following section reviews the implications of the theory of exhaustible resources for the behavior of vertically integrated natural resource firms. The results of previous attempts to test the empirical relevance of the basic theoretical framework are reviewed in Section III. The econometric model is described in Section IV, and the empirical results are discussed in Section V. Section VI contains concluding comments.

II. The Theory of Exhaustible Resources

This section reviews the implications of the theory of exhaustible resources for the dynamic behavior of private markets for exhaustible resources. Except for explicitly recognizing that firms may process as well as extract the resource, the model is a standard competitive market model of exhaustible resource extraction under conditions of certainty (see, e.g., Levhari and Liviatan [1977] and Weinstein and Zeckhauser [1975]). The principal feature of the model that distinguishes it from Hotelling's original competitive model is that extraction costs are assumed to be a function of cumulative extraction as well as of the current rate of output.

The resource-owning firm is assumed to be vertically integrated in that it engages in both the extraction and processing of an exhaustible resource. In each period the firm chooses the quantity of final output of the extracted and processed resource, Q, the quantity of the resource to be extracted, N, and the vectors of reproducible inputs, $\underline{X}^E$ and $\underline{X}^P$, to be used in the extraction and processing activities, respectively.

Assuming that the quantities of inputs used in extraction are separable from those used in processing, the firm's production

function can be written as

(1) $$Q = Q[\underline{X}^P, T, N(\underline{X}^E, Z, T)],$$

where Z is cumulative extraction, time T indexes the state of technology, and $N(*)$ is the extraction subproduction function.

The firm is assumed to maximize the wealth obtainable from its stock of the natural resource given input and output prices, the technological conditions governing extraction and processing, and the constraints,

(2) $$Z_t \leq S$$

(3) $$\dot{Z} = N_t,$$

where S is the firm's total stock of the resource.

The current-valued Hamiltonian for the firm's wealth maximization problem is

(4) $$H = P_Q Q(\underline{X}^P, N, T) - \sum_i P_i X_i^P - C^E(N, P_{\underline{X}}^E, Z, T) + \lambda N,$$

where P_Q is the price of output, P_X is a vector of input prices, $C^E(\cdot)$ is the minimal total cost function dual to the extraction subproduction function, and λ is a costate variable. Since λ is equal to the shadow value of the marginal unit extracted, it is equal to the negative of the shadow price of the marginal unit of the resource left in situ.

Defining $\mu = -\lambda$, the first-order conditions for an interior solution include

(5) $$P_Q \frac{\partial Q}{\partial X_i^P} = P_i,$$

(6) $$P_Q \frac{\partial Q}{\partial N} - \frac{\partial C^E}{\partial N} = \mu,$$

(7) $$\dot{\mu} = r\mu - \frac{\partial C^E}{\partial Z},$$

where r is the market rate of interest. Equations (5) and (6) are the static optimality conditions for reproducible inputs and the natural resource input, respectively. For a reproducible input, the value of the marginal product is equated to the price of the input, whereas for the natural resource, the value of the marginal product is

greater than the marginal extraction cost by an amount equal to the shadow price of the resource in situ.[1]

Equation (7) is the dynamic optimality condition for the natural resource. The right-hand side of (7) can be interpreted as the opportunity cost of deferring extraction, equal to the forgone interest $r\mu$, less the increase in future extraction costs from extracting the marginal unit ($\partial C^E/\partial Z > 0$). If the second term were zero, as assumed in Hotelling's original model, the price of the resource in situ should increase at the rate of interest, the famous "Hotelling Rule." However, when $\partial C^E/\partial Z > 0$, the price of the resource in situ is predicted to increase at less than the rate of interest, and may decrease over some time periods.

From equation (7), empirical testing of the implications of the theory of exhaustible resources for the dynamic behavior of resource-owning firms requires information on the shadow value of the resource in situ, μ, the effects of cumulative extraction on the marginal cost of extraction, $\partial C^E/\partial Z$, and the rate of interest r. As noted in the introduction, the difficulty in obtaining data on the first two of these variables accounts in large part for the paucity of information on the empirical validity of the theory of exhaustible resources. The following section reviews previous attempts to test the theory.

III. PREVIOUS TESTS OF THE THEORY

The first major empirical study of the time paths of natural resource prices was Barnett and Morse [1963]. Their purpose was to examine the hypothesis of increasing economic scarcity of natural resources, rather than to test the consistency of market data with the implications of the theory of exhaustible resources. Barnett and Morse graphed product prices for minerals and total extractive output for the period 1870–1957 and concluded that the trend was "approximately horizontal" [p. 211]. Subsequent econometric studies by Smith [1979] and Slade [1982] also failed to indicate any consistent upward trend in the prices of natural resource products.[2]

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1. With the first term on the right-hand side of equation (4) replaced by the firm's total revenue function, $R(\underline{X}^P,\underline{N},T)$, and P_Q in equations (5) and (6) reinterpreted as the firm's marginal revenue, the model applies as well to the case of a monopolized natural resource.
2. Smith [1979] concluded that for the period 1900–1973 the trend in mineral prices was negative with the rate of decline decreasing over time in absolute magnitude, while Slade [1982] concluded from a study of twelve major metals and fuels that the price paths for nonrenewable natural resources were U-shaped

Because these studies examined the prices of natural resource products, rather than the prices of natural resources in situ, they provide no direct evidence concerning the consistency of market data with the theory of exhaustible resources. Even if extraction costs were not affected by cumulative extraction, so that the time path of the in situ prices was predicted to obey the Hotelling Rule, product prices would in general increase at the rate of interest only if extraction and processing costs were zero. Furthermore, if extraction and processing costs decrease over time due either to technological change or to decreases in the prices of reproducible inputs, decreasing natural resource product prices are not inconsistent with increasing prices of the natural resource in situ.

The first published attempt to explicitly test the empirical relevance of the theory of exhaustible resources [Heal and Barrow, 1980] noted in its introduction that the price variable of interest is the price of the resource in situ. However, the subsequent empirical tests in this study are based instead on product prices. Therefore, although the results are quite negative (e.g., interest rate changes, but not levels, are found to be significantly related to metal prices), they do not provide a strong refutation of the standard theory of exhaustible resources.

Smith [1981] extended the Heal and Barrow analysis to a wider range of natural resource products and examined long-term rather than short-term price movements. Noting the severe constraints placed on the analysis by the available data, he tested and rejected a Hotelling-type model in which extraction costs were assumed to be zero, while finding that Heal-Barrow-type models incorporating changes in interest rates did have some predictive power.

Only two published studies have tested the theory of exhaustible resources using time series estimates of in situ prices of resources, and they have reached contrasting conclusions. The first study [Stollery, 1983] used annual data for the International Nickel Company for 1952 to 1973 to estimate a log-linear demand function and a Cobb-Douglas production function. The estimation results were used to calculate the price of the resource in situ as the difference between marginal revenue and marginal cost.[3] The null

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3 Neither study explicitly recognizes that firms generally process as well as extract natural resources. Instead, final output is treated as being equivalent to the extracted resource, and all production costs are treated as extraction costs. Marginal revenue rather than product price is used in calculating the implicit in situ price in the Stollery study because the firm is assumed to be a monopolist with a competitive fringe

hypothesis that the estimated time path of in situ prices was consistent with the time path implied by equation (7) was accepted, with the estimated rate of discount used by the firm being 15 percent.

The second study [Farrow, 1985] estimated a translog cost function for a U. S. metal mining firm using monthly data for 1975 through 1981. The estimated price of the resource in situ was calculated as the difference between the product price and the estimated marginal cost of output. Estimation of a number of alternative specifications of test equations based on equation (7) yielded results inconsistent with the theoretical model, including significantly negative estimates of the rate of discount.

Miller and Upton [1985] used cross-section estimates of in situ energy prices to test what they refer to as the Hotelling Valuation Principle. They noted that if the time path of the in situ price of a resource is expected by market participants to follow the Hotelling Rule, the asset value of a stock of the resource during any period will depend mainly on the current period product price and extraction cost. Using stock market valuations of the oil and gas reserves of a sample of U. S. companies, they found that the data were consistent with the Hotelling Valuation Principle.

Miller and Upton's finding that market forecasts of future resource prices are consistent with the Hotelling Rule suggests that the rule provides the best available predictions of the time paths of resource prices. However, as noted by Swierzbinski and Mendelsohn [1989], this is not equivalent to showing that the actual time paths of resource prices are consistent with the Hotelling predictions. Swierzbinski and Mendelsohn show that when the stock of a resource is uncertain, the Hotelling Rule may provide the best available prediction of future resource prices, even though unanticipated changes in expectations due to the arrival of information cause the actual time paths of resource prices to deviate from the Hotelling predictions.

We conclude that the empirical validity of the implications of the theory of exhaustible resources for the time paths of resource prices remains an open question. Previous empirical studies have clearly been severely constrained by the availability of time-series data on the prices of resources in situ and the effects of cumulative extraction on marginal extraction costs. In the following section, we describe an econometric model that is capable of providing estimates of both of these crucial variables, while simultaneously providing a parametric test of the implications of the theory of exhaustible resources.

IV. The Econometric Model

Our procedure for testing the theory of exhaustible resources is based on the estimation of a restricted cost function dual to the production function for final output (equation (1)). Specifically, the cost function corresponds to the minimization of the cost of the reproducible inputs given the optimal output Q and rate of extraction N in each period.[4]

Omitting time subscripts, the Lagrangian for the constrained cost minimization problem is

$$(8) \quad \mathscr{L} = \sum_i P_i \left(X_i^P + X_i^E \right) + \theta \left[Q - Q(\underline{X}^P, T, N) \right]$$
$$+ \delta \left[N - N(\underline{X}^E, Z, T) \right] .$$

The values of Q and N are taken as set at their wealth-maximizing levels. The first-order conditions for the cost-minimizing quantities of reproducible inputs in processing and extraction are, respectively,

$$(9) \quad P_i = \theta \frac{\partial Q}{\partial X_i^P} ,$$

$$(10) \quad P_i = \delta \frac{\partial N}{\partial X_i^E} .$$

The solution of this cost minimization problem yields the restricted cost function,

$$(11) \quad CR = CR(Q, \underline{P}_X, N, Z, T),$$

where P_X is the vector of reproducible input prices and CR is the minimal total expenditure on reproducible inputs given $Q, \underline{P}_X, N, Z$, and T. By the envelope theorem,

$$(12) \quad \frac{\partial CR}{\partial N} = \frac{\partial \mathscr{L}}{\partial N} = -\theta \frac{\partial Q}{\partial N} + \delta .$$

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4 Of course, firms will not explicitly solve the restricted cost minimization problem considered here, but instead will solve simultaneously for the wealth-maximizing quantities of Q and N together with the quantities of reproducible inputs that minimize total costs. However, the optimal quantities of reproducible inputs given by the solution to the restricted cost minimization problem will be identical to the quantities implied by the more general wealth maximization problem; see Lau [1976] It should be noted that if capital were a quasi-fixed factor, the relevant restricted cost function would be that corresponding to the minimization of the cost of the other reproducible inputs given the quantities of capital as well as N The null hypothesis that capital is a variable factor was tested and could not be rejected at the 0.01 level

The right-hand side of (12) can be interpreted by considering the solution of the total cost minimization problem with N unrestricted. The Lagrangian is

$$(13) \qquad \tilde{\mathscr{L}} = \sum_i P_i \left(X_i^P + X_i^E \right) + \mu N(\underline{X^E}, Z, T) + \tilde{\theta} \left[Q - Q(\underline{X^P}, T, N) \right],$$

where μ is the (unobserved) shadow price of the ore in situ. The first-order conditions for the reproducible inputs are

$$(14) \qquad\qquad\qquad P_i = \tilde{\theta} \, \frac{\partial Q}{\partial X_i^P}$$

$$(15) \qquad\qquad\qquad P_i = \left(-\mu + \tilde{\theta} \, \frac{\partial Q}{\partial N} \right) \frac{\partial N}{\partial X_i^E} .$$

The solution values, X^E and X^P, for this minimization problem will be identical to those derived for the restricted problem with N set equal to its wealth maximizing level. From (9) and (14),

$$(16) \qquad\qquad\qquad \theta = \tilde{\theta} .$$

From (10), (15), and (16),

$$(17) \qquad\qquad\qquad \delta = -\mu + \theta \, \frac{\partial Q}{\partial N} .$$

Substituting from (17) in (12),

$$(18) \qquad\qquad\qquad \frac{\partial CR}{\partial N} = -\mu .$$

Similarly, differentiating (8) and (13) with respect to Z and using (17) shows that the derivatives of restricted and total cost with respect to cumulative extraction are equal. Since cumulative extraction affects total cost through its effect on extraction costs,

$$(19) \qquad\qquad\qquad \frac{\partial CR}{\partial Z} = \frac{\partial C^E}{\partial Z} .$$

Thus, the estimates of μ and $\partial C^E/\partial Z$ required for testing the dynamic optimality condition (equation (7)) can be obtained by estimating the restricted cost function (equation (11)) and differentiating with respect to N and Z, respectively.

Following Schankerman and Nadiri [1986], we specify a generalized Cobb-Douglas functional form for the restricted cost

function:[5]

(20) $\ln CR - \ln Q = \alpha_0 + \sum_i \alpha_i \ln P_i + \alpha_N \ln N + \alpha_Z \ln Z + \alpha_T T$

$$+ \sum_i \gamma_{iN} \ln P_i \ln N + \sum_i \gamma_{iZ} \ln P_i \ln Z + \sum_i \gamma_{iT} (\ln P_i) T$$

$$+ \gamma_{NZ} \ln N \ln Z + \gamma_{NT}(\ln N)T + \gamma_{ZT}(\ln Z)T, \qquad i = K, L,$$

where the reproducible inputs are specified to be capital K and labor L. Linear homogeneity in prices is imposed on the restricted cost function by the restrictions,

$$\sum_i \alpha_i = 1.0,$$

$$\sum_i \gamma_{iN} = \sum_i \gamma_{iZ} = \sum_i \gamma_{iT} = 0, \qquad i = K, L.$$

The generalized Cobb-Douglas restricted cost function allows for both nonhomogeneity of the production function and biased technical change. The production function is homogeneous if and only if the restricted cost function satisfies the restrictions,

(21) $\qquad \gamma_{iN} = \gamma_{NT} = \gamma_{NZ} = 0, \qquad i = K, L.$

Technical change is Hicks neutral with respect to the reproducible inputs if and only if the restricted cost function satisfies the restrictions,

(22) $\qquad \gamma_{iT} = 0, \; i = K, L.$

Hicks neutrality with respect to the natural resource input requires the additional restriction $\gamma_{NT} = 0$.

Estimation of the cost shares of the reproducible inputs jointly with the restricted cost function increases the efficiency of the parameter estimates. Using Shephard's lemma,

(23) $\qquad M_i = \alpha_i + \gamma_{iN} \ln N + \gamma_{iZ} \ln Z + \gamma_{iT} T, \qquad i = K, L,$

where $M_i = P_i X_i / CR$ is the share of reproducible input i in restricted cost.

Estimation of equations (20) and (23) provides consistent estimates of the parameters of the restricted cost function whether or not the time path of the in situ price of the natural resource

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5 The translog form used in Halvorsen and Smith [1984] was found to be overparameterized for the model including cumulative extraction When tested against the full translog form, the generalized Cobb-Douglas cannot be rejected at the 10 percent level The computed F is 1 44 against a critical value of 1 93.

conforms to the dynamic optimality condition (equation (7)). However, under the null hypothesis that this condition is satisfied, more efficient estimates can be obtained by adding to the system of equations an additional equation incorporating the restrictions on the parameters implied by dynamic optimality.

Equation (7) can be expressed in discrete form as

$$(24) \qquad \mu_t = (1 + r)\mu_{t-1} - \frac{\partial C_t^E}{\partial Z_t}.$$

From equations (18) and (20),

$$\mu_t = -\left[\alpha_N + \sum_i \gamma_{iN} \ln P_{it} + \gamma_{NZ} \ln Z_t + \gamma_{NT}T\right]\left(\frac{CR_t}{N_t}\right) = -M_{N,t}CRN_t,$$

(25)

where $M_{N,t}$, the expression in square brackets, is equal to the ratio of the shadow value of the natural resource input to restricted cost, and CRN_t is average restricted cost per unit of current extraction. From equations (19) and (20),

$$\frac{\partial C_t^E}{\partial Z_t} = \left[\alpha_Z + \sum_i \gamma_{iZ} \ln P_{it} + \gamma_{NZ} \ln N_t + \gamma_{ZT}T\right]\left(\frac{CR_t}{Z_t}\right) = M_{Z,t}CRZ_t,$$

(26)

where $M_{Z,t}$ is equal to the ratio of the shadow rental value of cumulative extraction to restricted cost, and CRZ_t is average restricted cost per unit of cumulative extraction. Substituting from (25) and (26) in (24) and rearranging terms,

$$(27) \qquad CRZ_t = \frac{M_{N,t}CRN_t - (1+r)M_{N,t-1}CRN_{t-1}}{M_{Z,t}}.$$

Under the null hypothesis that the time path of the natural resource's in situ price conforms to the dynamic optimality condition of the theoretical model, the parameters embedded in equation (27) are a subset of the parameters in equation (20). Estimation of the system of equations, (20), (23), and (27), with and without the corresponding parameter restrictions imposed on (27) would permit a standard likelihood ratio test of the null hypothesis. However, for this test to be valid, the unconstrained estimates must be consistent under both the null and alternative hypotheses. The latter requirement would be met only in the unlikely case that the time path of in situ prices under the alternate hypothesis was a function of only $P_K, P_L, N, Z, T,$ and r.

A valid test can be developed by recalling that estimation of equations (20) and (23) provides consistent estimates of the parameters of the restricted cost function under both the null and alternative hypotheses. Because estimation of the full system of equations, (20), (23), and (27), provides consistent and asymptotically efficient estimates of the parameters of the restricted cost function under the null hypothesis but inconsistent estimates under the alternative hypothesis, a Hausman [1978] specification test can be used to test the null hypothesis that the dynamic optimality condition is satisfied.

The Hausman specification test involves comparison of the estimates of the parameters obtained by estimating the full system of equations, (20), (23), and (27), with the estimates obtained by estimating only equations (20) and (23). The test statistic is a quadratic form computed by differencing the two sets of parameter estimates and standardizing the vector of differences by the difference in their covariance matrices. The test statistic is asymptotically distributed as chi-square with degrees of freedom equal to the number of parameters being tested.

V. Data and Empirical Results

The econometric model is estimated with annual time series data for the Canadian metal mining industry for 1954 through 1974.[6] Final output Q is the dollar value of ore concentrate deflated by the wholesale price index for metal mining. The quantity of capital is calculated using the perpetual inventory method, and the price of capital, P_K, is a modified Christensen-Jorgenson [1969] service price index reflecting acquisition cost, the rate of interest, and the rate of depreciation.

The quantity of labor is the total number of workers and the price of labor, P_L, is equal to average wages plus indirect benefits. The quantity of ore extracted, N, is equal to the total number of tons of metallic ore hoisted. The value of Z in period t is the cumulative amount of ore hoisted from 1949, the first year for which extraction data are available, through period $t - 1$. The untransformed values of all variables entered in log form are normalized to equal unity in the median year, 1964. The time variable, T, is normalized to have the value zero in 1964.

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6. We are grateful to G Anders of the Ontario Ministry of Natural Resources for making the data available to us. The data are described in more detail in Smithson et al. [1979]

Classical additive error terms are appended to equations (20), (23), and (27) to reflect errors in cost-minimizing behavior. Because Q and N are endogenous variables, estimation is by iterative three-stage least squares [Berndt et al. 1974].[7] The set of instruments includes the log of the price of output as well as the functions of time and of the prices of reproducible inputs that appear in the restricted cost function. Because the cost shares sum to unity for each observation, one of the cost share equations is deleted from the system of equations. The estimation results are invariant to the choice of equation to be deleted.

Homogeneity of the production function and Hicks neutral technical change were tested by estimating equations (20) and (23) with and without the corresponding restrictions imposed. The test statistic is [Judge et al., 1980; Theil, 1971]

$$\frac{RSS_R - RSS_U}{J} \bigg/ \frac{RSS_U}{MT - K},$$

where RSS_R and RSS_U are the sum of square fitted residuals for the restricted and unrestricted equations, respectively, J is the number of restrictions, M is the number of equations, T is the number of observations, and K is the total number of parameters in the unrestricted equations. The test statistic is distributed asymptotically as F with degrees of freedom equal to J in the numerator and $(MT - K)$ in the denominator.

Homogeneity of the production function cannot be rejected at the 0.01 level.[8] The value of the test statistic is 3.8, and the critical value is 4.5. The restriction corresponding to Hicks neutral technical change with respect to the natural resource input, $\gamma_{NT} = 0$, is included in the set of restrictions for homogeneity and is therefore accepted. However, Hicks neutrality with respect to the reproducible inputs is rejected at the 0.01 level. The value of the test statistic is 12.5, and the critical value is 7.5.

Given the acceptance of the homogeneity restrictions, (21), they are imposed in the final form of the model used to test the null hypothesis that the time path of in situ prices satisfies the dynamic optimality condition implied by the theory of exhaustible resources. The dynamic optimality condition is tested assuming both

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7 Cumulative extraction Z does not include the current period's extraction and is therefore a predetermined variable

8. The acceptance of homogeneity implies that the degree of returns to scale is equal to $1 - \alpha_n$ From Table II, the estimate of returns to scale is 1 29 It should be noted that returns to scale for an industry, as estimated here, are not necessarily equal to returns to scale for individual firms in the industry

constant discount rates and discount rates pegged to actual real Canadian interest rates.[9] As shown in Table I.A, the null hypothesis is strongly rejected for constant discount rates ranging in value from 2 to 20 percent. As shown in Table I.B, the null hypothesis is also strongly rejected for variable discount rates ranging in value from one fourth to four times the actual real Canadian interest rate in each year.

Given the rejection of the null hypothesis that the dynamic optimality condition is satisfied, subsequent results are reported for the model excluding equation (27). The parameter estimates are shown in Table II. Eight of the eleven estimates are significant at the 0.01 level. The values of R^2 for equations (20) and (23) are 0.998 and 0.997, respectively.[10] Regularity conditions for the restricted cost function are that it be nondecreasing and concave in the prices of reproducible inputs and nonincreasing and convex in the quantity of ore [Lau, 1976]. The regularity conditions are satisfied for all 21 observations.

In addition to providing a test of the theory of exhaustible resources, the econometric model provides estimates of the in situ price of the natural resource, which several authors have proposed as the best single index of trends in resource scarcity (see, e.g., Brown and Field [1978] and Fisher [1981]). The time path of the in situ price of the natural resource, metallic ore, is shown in Table III, together with the time path of an alternative scarcity index, the price of final output.[11]

Summary descriptions of the behavior of the alternative scarcity indexes can be obtained by estimating semi-log trend equations. The regression results for the in situ price μ and output price P_Q are

$$(28) \qquad \ln \mu = 4.6201 \ + \ 0.0057T \qquad \overline{R}^2 = 0.05$$
$$(0.0057) \quad (0.0040)$$

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9. The interest rate used is an average of yields on Canadian government bonds with maturities of ten years or more. Real rates are obtained by subtracting the rate of change in the Canadian Consumer Price Index in each year from the nominal bond yields.

10. The values of R^2 are calculated as $R^2 = 1 - RSS/[(n-1)(SD)^2]$, where RSS is the sum of squared residuals, SD is the standard deviation of the dependent variable, and n is the number of observations.

11 Although the same basic data are used, the index of the in situ price obtained here is substantially different from the index obtained in Halvorsen and Smith [1984]. Differences in the model and estimating procedures used in the current paper include the incorporation of cumulative extraction in the restricted cost function, exclusion of an energy variable, use of a different functional form, and estimation by three-stage least squares rather than an iterative Zellner-efficient procedure

TABLE I
TESTS OF DYNAMIC OPTIMALITY CONDITION

A. Constant discount rates·

Discount rate	Test statistic
0.02	58 2[a]
0 05	48 8[a]
0.10	69 3[a]
0.15	291 8[a]
0.20	102 2[a]

B Variable discount rates·

Discount rate[b]	Test statistic
0.25∗ι	34 6[a]
0.50∗ι	29 3[a]
1.00∗ι	34.0[a]
2.00∗ι	178.1[a]
4 00∗ι	276.8[a]

a The null hypothesis is rejected at the 0 01 level The critical value of the test statistic is 24 7
b The discount rate is specified to be proportional to the actual real interest rate ι

$$(29) \qquad \ln P_Q = \underset{(0.0148)}{4.6590} + \underset{(0.0012)}{0.0038T} \qquad \overline{R}^2 = 0.33.$$

Both indexes suggest a slight upward trend in resource scarcity over the period, with estimated growth rates for μ and P_Q of 0.57 percent and 0.38 percent, respectively. However, the estimated growth rate for μ is not significant at the 10 percent level.

TABLE II
PARAMETER ESTIMATES

Parameter	Estimate	Standard error
α_0	0 0073	0 0116
α_K	0 2858*	0 0028
α_L	0 7142*	0 0028
α_N	−0 2861	0 1088
α_Z	1 7026*	0 2770
α_T	−0.1985*	0 0369
γ_{KZ}	0.0762*	0.0190
γ_{LZ}	−0 0762*	0.0190
γ_{ZT}	0.0495*	0.0071
γ_{KT}	−0.0037	0.0025
γ_{LT}	0 0037	0.0025

*Significant at the 0 01 level

TABLE III
ESTIMATED PRICE INDEXES

Year	In situ price[a]	Output price[b]
1954	100.00	100.00
1955	104.44	107 97
1956	105 17	113.25
1957	108.35	108 47
1958	124 90	105 28
1959	111 05	107 07
1960	113.87	108.07
1961	117 21	108.17
1962	103.80	107 17
1963	95.99	106.08
1964	92.49	108 86
1965	85 48	112 25
1966	98.48	112 45
1967	100 32	114.84
1968	101 38	115 84
1969	110 01	116.04
1970	111 36	111.85
1971	115 97	118 92
1972	132 17	112.75
1973	115 54	107 27
1974	136 06	111 06

a Calculated using equation (25)
b Metal mining wholesale price index divided by general wholesale price index

VI. CONCLUDING COMMENTS

Empirical tests of the theory of exhaustible resources have been hampered by the unavailability of data on the in situ prices of resources and the effects of cumulative extraction on the marginal cost of extraction. The econometric model developed here provides a direct test of the theory's implications for the dynamic behavior of vertically integrated resource industries.

Using data for the Canadian metal mining industry, the empirical implications of the theory of exhaustible resources are strongly rejected. Because the data used to estimate the econometric model are at a high level of aggregation, the empirical results obtained here should be considered as only tentative.[12] Estimation

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12. The output of the Canadian metal mining industry is an aggregate of several different minerals, and the aggregate in situ price estimated here may have been affected by shifts in the composition of output over the sample period

of the model with data for individual exhaustible resources or, preferably, individual resource firms is required to confirm the finding that the theory of exhaustible resources is not empirically valid.

Rejection of the empirical validity of the theory of exhaustible resources, if confirmed, would imply either that the theoretical model does not provide an adequate characterization of privately optimal behavior, or that resource firms do not behave in a privately optimal way. While it is possible that firm behavior is not privately optimal, perhaps due to the use of inappropriate rules of thumb [Farrow, 1985], inadequacy of the theoretical model seems a more likely reason for the theory to be rejected.

In particular, the theoretical model tested here assumes both complete certainty and perfect arbitrage. The effects of introducing uncertainty considerations while retaining the assumption of perfect arbitrage have been considered in a number of studies.[13] The resulting predictions of resource prices, which are conditional on the uncertain event not occurring, follow modified Hotelling rules in which the discount rate reflects the probability of the uncertain event as well as the interest rate. Given the rejection of the deterministic model's implications for a wide range of discount rates, generalizing the model in this way is unlikely to restore the predictive ability of the theory of exhaustible resources.[14]

A more promising approach may be to relax the assumption of perfect arbitrage in addition to allowing for uncertainty. This is the approach taken by Heal and Barrow [1980], who assume that arbitrage affects the flow market for natural resources but do not impose capital market equilibrium. Their model, which also incorporates uncertainty with respect to rates of return, results in the prediction that the rate of change in the in situ price will be a function of interest rate changes, rather than levels.

The Heal-Barrow model has not been tested with data on in

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13. The principal issues considered have been uncertainty with respect to the total size of the resource stock (e.g., Gilbert [1979]), discoveries of new reserves (e.g., Pindyck [1980]), and the discovery of a perfect producible substitute for the resource (e.g., Dasgupta and Stiglitz [1981]). For a discussion of these and other studies in the context of a general model of uncertainty, see Desmukh and Pliska [1985].
14. Of course, one possible reason for the rejection of the model's predictions is that uncertain events such as the invention of substitutes or changes in beliefs about total resource stocks have in fact occurred over the sample period, resulting in shifts in the time path of in situ prices. Because the type of information necessary to control for such shifts is not available, the test in this paper is based on the maintained hypothesis that they did not occur.

situ prices, but does appear to have some predictive power with respect to product prices [Heal and Barrow, 1980; Smith, 1981; Agbeyegbe, 1989]. An interesting approach to pursue in future research would be the use of econometric estimates of in situ prices to test models of the Heal-Barrow type.

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