On the Nonexistence of Market Equilibria in Exhaustible Resource Markets with Decreasing Costs

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This paper examines the existence of competitive equilibria in markets for exhaustible resources where there are initial economies of scale in either the extraction of the resource or the utilization of the resource as an input in production. In such instances, which are fairly common, we find that the classic Hotelling rule for competitive extraction does not apply, since competitive price equilibria generally do not exist. This is in marked contrast to static markets where the usual textbook example of firms with U-shaped average cost curves is not inconsistent with the existence of competitive equilibria. Furthermore, oligopolistic market equilibria in which resource firms act as Nash producers may also fail to exist when there are returns to scale in production.

I. Introduction

In most of economic theory, convexity of technologies is assumed to ensure that optimal production decisions exist and that they are well behaved. Beginning with Hotelling (1931) much of the work on ex-

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haustible resources has followed in this tradition. But the extraction and utilization of particular exhaustible resources may very well display various sorts of nonconvexities. For example, there may be startup costs when a new mine is opened and shutdown costs when the deposit is exhausted. There may be initial scale economies in mining the resource or in utilizing the resource as an input to produce a final consumption good.

Generally, nonconvexities are known to present problems in economic theory. Competitive market equilibria need not exist when technologies are nonconvex. Furthermore, even if equilibria exist, it may not be possible to achieve socially optimal outcomes in competitive markets. While these problems have been extensively analyzed for static markets, the effects of nonconvexities on production in intertemporal markets for natural resources are not yet fully understood.

This paper deals with the existence of competitive equilibria in markets for exhaustible resources where there are decreasing unit costs of production over some range of output. Various authors including Clark (1976), Vousden (1977), and Lewis, Matthews, and Burness (1979) have analyzed the optimal use of exhaustible resources with nonconvexities in production. Our concern here, however, is to examine competitive behavior of resource producers under decreasing cost conditions. It is convenient to classify possible sources of nonconvexities and decreasing costs as arising from (1) fixed (flow) costs and/or setup (or shutdown) costs, and (2) economies of scale in either the resource extraction sector or the sector utilizing the resource.

When nonconvexities are of type 1, it seems intuitively clear that competitive equilibria may fail to exist and that production will be inefficient. For example, in competitive markets with identical firms, each facing fixed flow costs and constant marginal costs of extraction, the existence of an equilibrium will require that price net of marginal cost rise at the rate of discount. But in such cases each firm will want to delay producing until the very end in order to minimize its present value fixed costs, so that a competitive equilibrium will not exist (see Lewis et al. 1979, pp. 227–28). Equilibria will also fail to exist in situations where there are fixed setup costs to operating each mine (see Hartwick, Kemp, and Long 1980) and where the output of each firm is constrained by initial investment (see Olewiler 1981).

Our concern in this paper is with the type 2 source of decreasing costs. The importance of initial scale economies in resource extraction is well documented in Campbell and Scott (1980). Following Scott (1967) we will assume a U-shaped average cost-of-extraction curve to reflect initial increasing returns to scale in resource production. As-

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suming the resource is used as an input in the production of some other good, we will allow also for the possibility that there are scale economies in the resource-using sector. In either case we find that competitive equilibria, in which all producers take input and output prices as given, generally do not exist.

This result is of interest to us for several reasons. First, the existence of scale economies in resource extraction is well documented. Accordingly, various authors have modeled extractive firms as having U-shaped average cost curves in analyzing competitive resource markets. However, the question of the existence of equilibria in such markets has, until now, been ignored. Second, our results suggest that production nonconvexities pose more serious problems for existence of equilibria in intertemporal resource markets than they do in static markets. Recall that the usual textbook example of firms with U-shaped average cost curves is not necessarily inconsistent with competitive equilibria existing in static markets.

The failure of equilibrium is a result of the presence of scale economies in production which cause the profit function for each firm to be nonconcave over an initial range of output. We show that this lack of concavity is sufficient to preclude the existence of competitive equilibria in resource markets. We also argue that the same existence problem may arise under noncompetitive conditions. That is, in oligopolistic resource markets where firms act as Cournot producers, equilibria can fail to exist when scale economies are sufficiently important. Similar existence problems are encountered in static oligopoly analysis.

Section II of this article treats the case of type 2 nonconvexities, and we present our conclusions in Section III.

II. Decreasing Costs Due to Scale Economies

A. *Economies of Scale in the Resource Extraction Sector*

To illustrate our point we want to consider the simplest Hotelling-type model that is modified slightly to allow for increasing returns to scale in extraction.

Consider a society that derives utility $U(Q)$ from the direct consumption of an exhaustible resource which is extracted and consumed at the rate Q. Assume $U' > 0$ and $U'' < 0$. Suppose that there is an industry that supplies the resource and that producers face an inverse demand schedule $p(Q) = U'(Q)$, where Q represents total industry output. Assume that the industry consists of a sufficiently large number of identical firms so as to warrant price-taking behavior. For simplicity we presume that all firms have the same initial reserves,

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R, and face the same costs of extraction, $C(q)$, as a function of output, q, with $C(0) = 0$. We emphasize that the nonexistence of equilibria result that we are about to prove does not depend on the assumptions that all firms are identical and that they are price takers. The consequences of relaxing these assumptions are discussed below.

Initial increasing returns to scale in the resource extraction technology implies that $C(q)$ is not convex and that the average cost curve $C(q)/q$ is U-shaped, as depicted in figure 1, top and bottom. Let $\bar{q}$ be the output rate that minimizes the average cost. We further assume that there are nonzero adjustment costs to changing the extraction rate by some discrete amount from one instant to the next. Were this not the case, then if we vary the extraction rate infinitely often between 0 and $\bar{q}$, any cost level on the ray OA of figure 1, top, could be attained in a time-average sense. In other words, by engaging in a "chattering control" we could make the cost function convex.[1]

If it is assumed that firms take prices, $p(t)$, as given, the optimization problem facing the representative firm i is

$$\max_{\{q_i\}, T_i} \int_0^{T_i} e^{-\delta t}\{p(t)q_i(t) - C[q_i(t)]\}dt \tag{1}$$

subject to $R_i(0) = R(0) < \infty$, $R_i(t) \geq 0$, and $\dot{R}_i(t) = -q_i(t)$, for all firms $i = 1, 2, \ldots, N$. The rate of discount, δ, is assumed to be positive and common to all firms.

In equilibrium all firms are simultaneously maximizing profits, given the time path of prices, $p(t)$, and at each instant of time $p(t) = p[Q(t)]$.[2] It follows from first principles that if an equilibrium exists with all firms maximizing profits we must have

$$p(t) \text{ continuous.} \tag{2}$$

If, for example, $p(t)$ were to increase discontinuously at some time t_0, then by (1), profit-maximizing firms would produce more just after the jump increase in price at t_0 than they would just before the jump. But this would imply that total industry output was greater after the

[1] Formally, this is equivalent to assuming that q must be a piecewise continuous function of time. If it were possible to vary the extraction rate "infinitely often" between 0 and $\bar{q}$, e.g., to spend half the time with $q = 0$ and half the time with $q = \bar{q}$, then a cost of $C(\bar{q})/2$ could be obtained in a time-average sense. In fact, by varying the amount of time spent at $q = 0$ and $q = \bar{q}$ one could obtain any level of cost on the ray OA. This method of convexification is similar to that of using mixed strategies in game-theoretic models. See Davidson and Harris (1981) and Lewis and Schmalensee (in press) for a discussion of chattering controls in the context of investment and renewable resource management models, respectively.

[2] Here, as in almost all resource extraction models, we are implicitly assuming the existence of a complete set of forward markets in order to sidestep the difficult issue of expectations formation.

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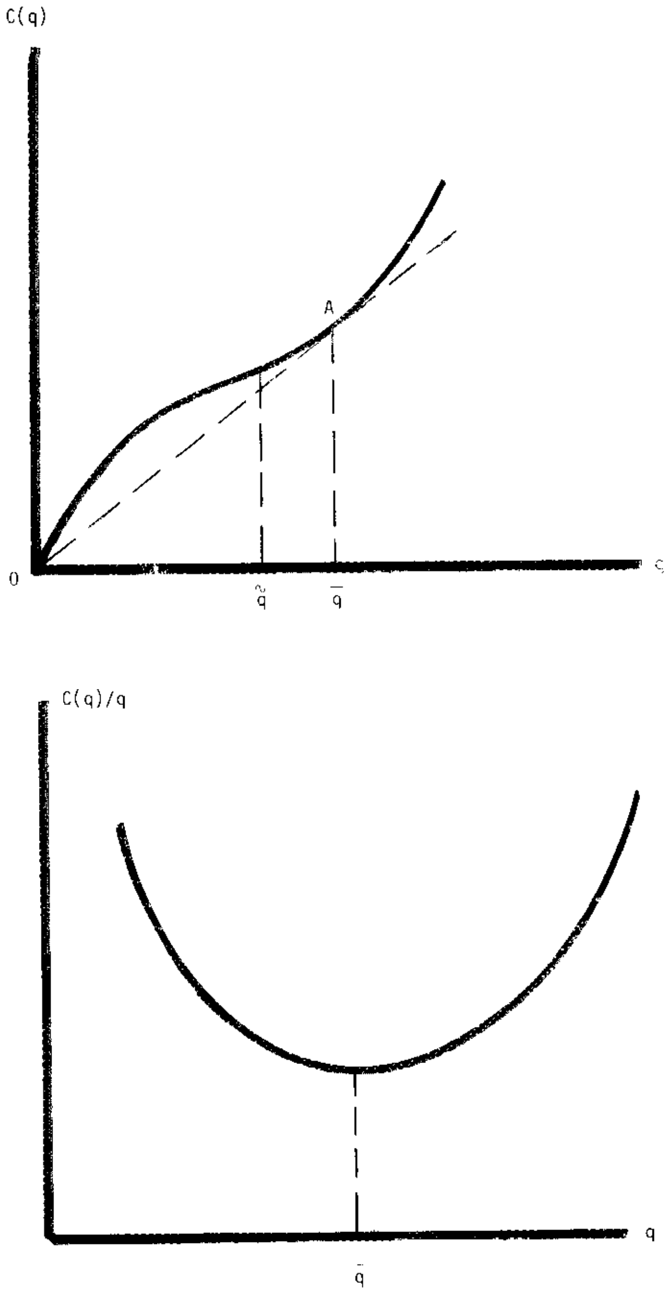

FIG. 1

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jump than before and that market price (which is inversely related to output) must have decreased discontinuously at t_0, which violates our original supposition. A similar argument also establishes that prices cannot jump down at any time during extraction. Hence, market price must be continuous.

In addition to (2) an equilibrium is further characterized by the solution to (1). Before proceeding we note that using the assumption that all firms are identical, we can show that if a market equilibrium exists, it must involve each firm extracting at the same rate. The proof is tedious and is neither mathematically nor economically profound, so we have relegated it to the Appendix. The control-theoretic solution to (1) requires that the representative firm choose q in order to maximize the present value Hamiltonian, $H(q, \lambda, t) \equiv \epsilon^{-\delta t}\{p(t)q(t) - c[q(t)] - \lambda q(t)\}$, at each instant of time.[3] This implies

$$p(t) - C'[q(t)] \leq \lambda(t),\ q(t) > 0, \tag{3a}$$

and

$$C''[q(t)] > 0,\ q(t) > 0. \tag{3b}$$

According to (3b), firms always produce on the upward-sloping portion of their marginal cost curve. The solution to (1) is further characterized by

$$\dot{\lambda}(t) = \delta\lambda(t), \tag{4}$$

$$\lim_{t \to T} e^{-\delta t}\{p(t)q(t) - C[q(t)] - \lambda(t)q(t)\} = 0, \tag{5}$$

where $\lambda(t)$ is a positive and continuous function. Differentiating (3a) with respect to time and using (4) yields the familiar Hotelling rule,

$$\frac{d}{dt}\{p(t) - C'[q(t)]\} = \delta\{p(t) - C'[q(t)]\}, \tag{6}$$

that price net of marginal cost rises at the discount rate whenever extraction is positive.

With $q(t)$ bounded away from zero, it follows that T is finite, because firms have only finite reserves $R(0)$ to produce from. Further, the continuity of $\lambda(t)$ and $q(t)$ implies from (3a) that $p(T) - C'[q(T)] - \lambda(T) = 0$. This together with (5) and the fact that T is finite yields

$$C[q(T)] - C'[q(T)]q(T) = 0, \tag{7}$$

which is satisfied by $q(T) = 0$ or $q(T) = \bar{q}$. Since $q(t)$ continuous and (3a) imply $C''[q(t)] > 0$ whenever $q(t) > 0$, it follows that $q(T) = \bar{q}$ is

[3] Since all firms produce identically, we can dispense with subscripting variables in the analysis to follow.

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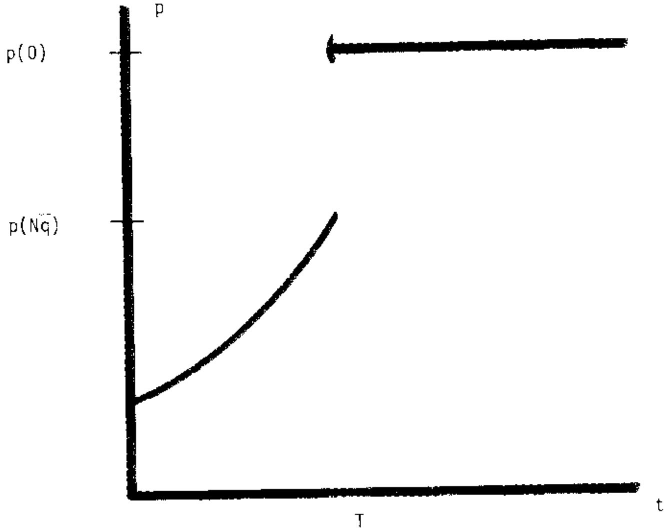

FIG. 2

the relevant solution to (5). Hence, the firm's output rate at the terminal time must be such that it minimizes the average cost.[4]

Let us summarize our results thus far. Equation (6) implies that price rises continuously while firms extract. At the terminal time T, each firm is producing at the finite rate $\bar{q}$, so that total industry output is $N\bar{q}$, and market price, expressed as a function of industry output, is given by $p(Nq)$. However, at the instant after T, industry output is zero and the market price is $p(0) > p(N\bar{q})$, as illustrated below. But this implies a violation of our equilibrium condition, (2), that $p(t)$ be continuous, since the price of the resource jumps discontinuously to the choke price, $p(0) = U'(0)$, at T. This, in turn, implies that individual firms (taking the price path as given) can profit from the discontinuous jump by holding back production prior to T in order to produce after T. Hence, it is not possible to satisfy conditions (2) and (3)–(5) simultaneously, so that an intertemporal market equilibrium does not exist.[5]

This is an unexpected result, since scale economies are not necessarily inconsistent with the existence of competitive equilibria in static models. There, in fact, the presence of initial increasing returns to

[4] Burness (1976) obtains a similar result.

[5] It is easy to see that with full convexity of the cost curves, market equilibria do exist. In this case $\bar{q} = 0$, so that industry output tends to zero and price rises continuously to the choke price as t approaches T.

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scale has the virtue of rendering the size of the firm finite. In the intertemporal problem being examined here, the finiteness of the resource stock requires the price to be rising over time. In order for price to rise continuously—a necessary condition for competitive equilibrium—industry output and hence the output of each firm must eventually decline smoothly to zero. But the presence of scale economies in extraction causes each firm's profit schedule to be nonconcave over an initial range of output. Since firms refuse to operate in this nonconcave region (see [3b]), it is not possible to support a continuous price profile; so competitive equilibria do not exist.

This reasoning suggests also that the same nonexistence problem may arise when we relax our assumption that all producers are identical price-taking firms. For example, allowing producers to have different U-shaped average cost curves should not change the analysis. Firms' profits will still be nonconcave over an initial range of output, so that the rate of extraction and the price of the resource will jump discontinuously when the last set of firms stops producing. In the case of an oligopolistic industry where each resource firm acts as a Cournot producer, market equilibria typically require that each firm's profit be a concave function of its own output (see Lewis and Schmalensee [1980] and the references cited therein). Again the presence of scale economies is likely to eliminate concavity of firm profits, so that equilibria in oligopolistic markets may also not exist.

The nonexistence of a competitive equilibrium considered above arises because the nonconvexity of the cost function causes the extraction rate and the price of the resource to jump discontinuously at the terminal time. One might conjecture that the equilibria could be restored if firms could store extracted resources above the ground. Firms would then respond to an anticipated jump in prices by accumulating inventories to be sold at the time of the price increase. Presumably, this would smooth consumption so that jumps in price would be eliminated. In order to investigate this possibility, we assume that firms can store the resource above the ground at zero cost. The profit-maximizing problem facing the representative firm is then given by

$$\max_{\{q\},\{s\},T,S} \int_0^T e^{-\delta t}\{p(t)s(t) - C[q(t)]\}dt \tag{8}$$

subject to $\dot{R} = -q$ and $\dot{I} = q - s$, and the nonnegativity of q, s, R, and I, where q and s are the rates of extraction and sales, respectively, and I is the stock of the resource stored above the ground. The terminal times of extraction and sales, respectively, are represented by T and S. The necessary conditions for the maximization in (8) require the exis-

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tence of continuous functions $\lambda(t)$ and $\mu(t)$ such that the constraints and the following conditions are satisfied:[6]

$$s(t) = \begin{cases} 0 & < \mu(t) \\ s \in (0, \infty) \text{ if } p(t) = \mu(t) \\ \infty & > \mu(t) \end{cases} \tag{9a}$$

$$-C'[q(t)] - \lambda + \mu \leq 0 \quad [= \text{if } q(t) > 0];$$

$$C''[q(t)] > 0 \qquad q(t) > 0; \tag{9b}$$

$$\dot{\lambda} = \delta\lambda; \tag{9c}$$

$$\dot{\mu} = \delta\lambda; \tag{9d}$$

$$\lim_{t \to T} e^{-\delta t}\{C'[q(t)]q(t) - C[q(t)]\} = 0. \tag{9e}$$

We will now show that if a solution to (8) exists, it must involve $T = S$. Suppose $T < S$ and let $\{q(t)\}$ and $\{s(t)\}$ be the solution to (8). If $T < S$ we must have $I(T) > 0$. Condition (9b) implies $q(t)$ is bounded above so that there must exist a period just prior to T when $I(t)$ is also strictly positive. Let $\tau < T$ be an instant at which $I(\tau) > 0$. Consider a variation in $\{q(t)\}$ satisfying

$$\tilde{q}(t) = \begin{cases} q(t) - \gamma & t \in (\tau, \tau + \Delta) \\ \gamma & t \in (T, T + \Delta) \\ q(t) & \text{otherwise}; \end{cases} \tag{10a}$$

$$\tilde{s}(t) = s(t) \qquad \text{for} \qquad t \leq S. \tag{10b}$$

The paths $\{\tilde{q}(t)\}$ and $\{\tilde{s}(t)\}$ are feasible for small $\gamma > 0$ and $\Delta > 0$ since sales in excess of production can be taken directly from inventories during the interval $(\tau, \tau + \Delta)$ when $\tilde{q}(t) = q(t) - \gamma$. Since the sales profile is unchanged, so is the present value of the firm's revenues. But the variation in output reduces present value costs, since some production is delayed until after time T. However, this implies that the original extraction and sales profiles could not have been optimal, and thus T must equal S.

Conditions (9b)–(9d) imply that whenever extraction is positive $q(t)$ is a continuous decreasing function which is bounded away from zero. Consequently, T must be finite, since $R(0)$ is finite. It follows from (9e) that $C'[q(T)]q(T) - C[q(T)] = 0$, which implies that $q(T) = \bar{q}$. But recall $T = S$ so that $s(T) \geq q > 0$. This means that sales are strictly positive at the terminal time S and zero afterward. As in the no-storage case, price must jump discontinuously at the terminal time, so that an intertemporal equilibrium does not exist.

[6] See Takayama (1974, p. 655, theorem 8.C.3). We shall assume that the constraint $I(t) \geq 0$ is not binding except at the initial and terminal times.

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B. Economies of Scale in the Resource-utilizing Sector

It turns out that intertemporal competitive equilibrium may still fail to exist even when the technology for extracting the resource is well behaved, that is, convex. The problem can arise when there is a nonconvexity somewhere else in the economy, in particular in a manufacturing sector that uses the resource as an input in production.

A sketch of the argument used for showing this is as follows.[7]

Suppose that the extracted resource is used as an input to manufacture a final product y. Assume that both the resource-extraction sector and the production sector consist of competitive price-taking firms. For simplicity we shall assume that the resource is the only input used by the manufacturing sector. Denote by $y = g(q)$ the production function for each manufacturer, and suppose that there are initial increasing returns to scale in producing y as shown in figure 3, top and bottom. Denote $\hat{q}$ as the input rate that maximizes average productivity.

Arguing as we did in the previous section, we can show that if a competitive equilibrium exists, then production of the manufactured good Y declines monotonically up until time T, when output $Y(T) = g(\hat{q}) > 0$ and the resource is exhausted. Prior to T, the price of the resource rises smoothly at the rate δ (if we assume for simplicity that extraction costs are zero). This means that $P(t)$ jumps discontinuously to $P(0)$ at T. But such jumps in price cannot occur in equilibrium, since individual price-taking resource firms will desire to delay extraction until after T to take advantage of higher prices. Since it is not possible to have $P(t)$ continuous and satisfy our equilibrium conditions simultaneously, a competitive equilibrium must not exist.

Again we find that the presence of scale economies causes competitive equilibria in multiperiod resource markets not to exist. But here the production nonconvexities affect resource extraction indirectly through the manufacturing sector. As we demonstrated above, it is possible also to show here that allowing producers the option of costlessly maintaining inventories of the final good as a means of eliminating price jumps is not sufficient to restore equilibrium.

III. Conclusion

We have studied the existence of competitive equilibria in exhaustible resource markets where there are decreasing costs due to scale economies or learning by doing. The existence question would seem to be

[7] The complete formal argument is available from the authors.

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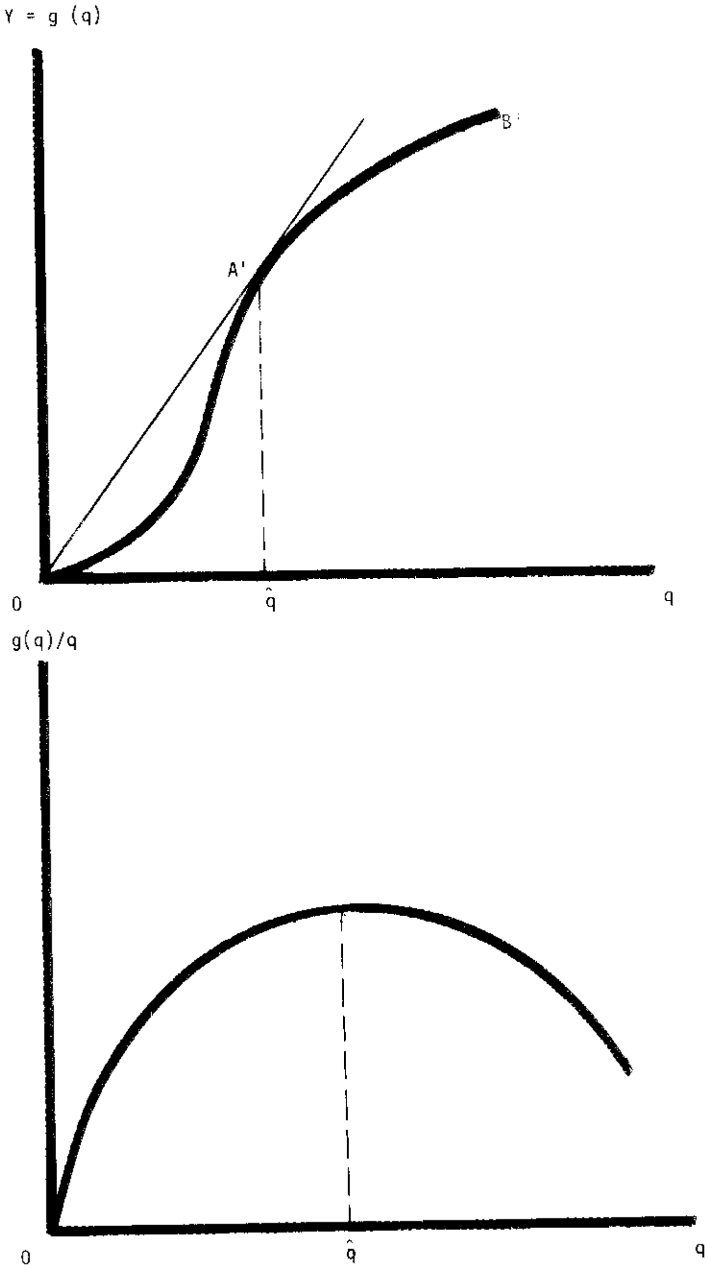

FIG. 3

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important since many of the mined resources are subject to decreasing cost technologies, at least for small outputs. Our results on this matter are primarily negative. In Hotelling-type models where producers and consumers can contract for current and future deliveries of the resource at specified prices, competitive equilibria fail to exist when there are scale economies in extracting the resource or in using it to manufacture a final product. Technically, the problem arises because each firm's profit function is nonconcave over an initial range of output. This lack of concavity presents a problem in establishing the existence of oligopolistic equilibria in resource markets as well.

Unfortunately, given our assumptions, there appears to be no easy solution for the nonexistence problem. In the game theory literature, nonexistence problems are typically dealt with by "convexifying" the problem through the use of mixed strategies. A similar option exists here. We could allow the firms to eliminate the increasing returns of production by having them engage in chattering controls (see Sec. II*A*, para. 3). However, we believe the use of such controls would have to be taken as a mathematical convenience rather than as an appropriate way to model economic alternatives of behavior.

As a practical matter, it is difficult to assess the importance of our nonexistence result and its implications for performance in exhaustible resource industries. In reality there are several factors that may mitigate the problem we have uncovered. First, our result that all producers stop at once, thus causing a large discontinuous price increase, will disappear if the resource stock is heterogeneous, so that the extraction cost curves differ across mines. Second, the jump in price at the end of the extraction horizon will depend on the degree of scale economies in mining, which may be small for some resources. The incorporation of these factors into our theory suggests that there may be a series of many small jumps in prices as one resource mine closes after another. To the extent that these variations in price are small and hard to detect, conditions consistent with market equilibrium may be approximated.

These considerations suggest also that it would be useful to look for less demanding notions of intertemporal equilibria in resource markets. This might involve relaxing the stringent assumption that the entire time path of resource prices is known and is determined in a complete set of future markets. In reality, most natural resources are traded in short-term futures and spot markets. Naturally, there is often significant uncertainty about future resource prices. In these instances sudden fluctuations in price are not necessarily inconsistent with the market being in equilibrium when information about future prices is imperfect and costly to obtain.

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Appendix

Proof That All Firms Extract at the Same Rate

First we show that all firms begin and terminate production at the same time. Suppose, contrarily, that there is a firm j which begins producing after the other firms at some time $S_j > 0$. Then prior to S_j price must be strictly increasing; otherwise firm j would not find it profitable to wait until S_j before producing. Since $\dot{p}(t) = (dp/dQ)\dot{Q} > 0$, it follows that there is at least one firm, i, operating prior to S_j with $\dot{q}_i < 0$ (subscripts refer to firms, and dotted variables are time derivatives). The profit-maximization conditions for each firm are given by (3) with the appropriate substitution for subscripts. Differentiating (3a) with respect to time and using (3b) one obtains

$$\dot{p}(t) = \delta\{p(t) - C'[q_i(t)]\} + C''[q_i(t)]\dot{q}_i(t). \tag{A1}$$

Since $\dot{q}_i(t) < 0$, (A1) implies

$$\dot{p}(t) < \delta\{p(t) - C'[q_i(t)]\}. \tag{A2}$$

Given our assumptions about $C(q)$ as illustrated in figure 1, there exists a unique $\bar{q}$, located at the point of inflection in figure 1, top, which minimizes $C'(q)$. Notice that $C''(\bar{q}) = 0$. From (3b) it follows that $q_i(t) > \bar{q}$. Let $\bar{C}' = C'(\bar{q})$. Then (A2) implies

$$\frac{d}{dt}[p(t) - \bar{C}'] = \dot{p}(t) < \delta\{p(t) - C'[q_i(t)]\} < \delta[p(t) - \bar{C}']. \tag{A3}$$

The present value marginal return to firm j at the moment it begins producing satisfies

$$e^{-\delta S_j}\{p(S_j) - C'[q_j(S_j)]\} < e^{-\delta S_j}[p(S_j) - \bar{C}'], \tag{A4}$$

since $q_j(t) > \bar{q}$ by (3b). But (A3) and (A4) imply that firm j could have earned a higher present value return by beginning at some time before S_j and extracting at a rate equal to $\bar{q}$. Hence, delaying extraction cannot be optimal for any firm. Suppose firm j stops extracting before the other firms. Given the continuity of $p(t)$, $\lambda_i(t)$ for all firms i, it follows from (3a) that each firm's output is continuous during production except at the instant it stops producing, when its output jumps from a strictly positive level to zero. But this implies that industry output, and hence price, must jump discontinuously at the time firm j stops producing, which violates our equilibrium condition (2).

To complete our proof, suppose that $q_i(t) \neq q_j(t)$ even though both firms extract over the same time horizon. Differentiating (3a) with respect to time and using (3b) one obtains

$$\frac{d}{dt}\{C'[q_i(t)] - C'[q_j(t)]\} = \delta\{C'[q_i(t)] - C'[q_j(t)]\}. \tag{A5}$$

From (A5) it follows that $q_i(0) \neq q_j(0)$; otherwise the two extraction paths would coincide. Assume $q_i(0) > q_j(0)$. Then (A5) and the fact that $C''[q(t)] > 0$ for both firms implies $q_i(t) > q_j(t)$ for all t. But this is not possible, since both firms end up extracting the same amount, $R(0)$, of the resource over the same time horizon. Hence all firms must extract at the same rate.

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