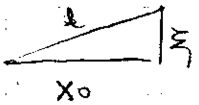
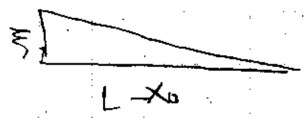


$$l = \sqrt{\xi^2 + x_0^2} \rightarrow dl = \frac{\xi\, d\xi}{\sqrt{\xi^2 + x_0^2}}$$



$$l' = \sqrt{\xi^2 + (L-x_0)^2} \rightarrow dl' = \frac{\xi\, d\xi}{\sqrt{\xi^2 + (L-x_0)^2}}$$

$$\rightarrow U(\xi_0) = \tau \int_0^{\xi_0} \frac{\xi\, d\xi}{\sqrt{\xi^2 + x_0^2}} + \tau \int_0^{\xi_0} \frac{\xi\, d\xi}{\sqrt{\xi^2 + (L-x_0)^2}}$$

$$= \tau\left(\sqrt{\xi_0^2 + x_0^2} - x_0\right) + \tau\left(\sqrt{\xi_0^2 + (L-x_0)^2} - (L-x_0)\right)$$

$$\simeq \tau x_0\left(\cancel{1} + \frac{\xi_0^2}{2x_0^2} - \cancel{1}\right) + \tau(L-x_0)\left(\cancel{1} + \frac{\xi_0^2}{2(L-x_0)^2} - \cancel{1}\right)$$

$$= \frac{\tau \xi_0^2}{2}\left(\frac{1}{x_0} + \frac{1}{L-x_0}\right)$$

$$\langle \xi_0^2 \rangle = \frac{\int_{-\infty}^{\infty} \xi_0^2\, e^{-\frac{U(\xi_0)}{k_B T}}\, d\xi_0}{\int_{-\infty}^{\infty} e^{-\frac{U(\xi_0)}{k_B T}}\, d\xi_0} = \frac{\cancel{2}\int_0^{\infty} \xi_0^2\, e^{-\alpha \xi_0^2}\, d\xi_0}{\cancel{2}\int_0^{\infty} e^{-\alpha \xi_0^2}\, d\xi_0} \quad \text{cm } \alpha = \frac{\tau L}{2 k_B T x_0 (L - x_0)}$$

$$\langle \xi_0^2 \rangle = \frac{1}{4\pi}\left(\frac{\pi}{\alpha}\right)^{3/2} 2\sqrt{\frac{\pi}{\alpha}} = \; = \frac{1}{2\alpha} = \frac{k_B T\, x_0 (L - x_0)}{\tau L}$$