

Ejercicios sobre Propagación de errores.

• Resumen de fórmulas:

1- Suma / resta:

$$Z = A + B \quad // \quad \text{ó} \quad A - B$$

$$\Delta Z = \pm \sqrt{\Delta a^2 + \Delta b^2}$$

2- Producto / división:

$$Z = A \cdot B \quad // \quad \text{ó} \quad A / B$$

$$\Delta Z = \pm Z \cdot \sqrt{\left(\frac{\Delta a}{A}\right)^2 + \left(\frac{\Delta b}{B}\right)^2}$$

3- Variable elevada a Potencia:

$$Z = A^n$$

$$\Delta Z = \pm n \cdot A^{n-1} \cdot \Delta a$$

4- Variable \times cte:

$$Z = K \cdot A$$

$$\Delta Z = \pm K \cdot \Delta a$$

5- Sea $f(x)$ una función (Ej: $\sin(x)$, $\cos(x)$, e^x , $\ln(x)$, etc.) \Rightarrow El error asociado a esta función está dado por:

$$\Delta f = \pm \frac{1}{2} \left| f(x+\Delta x) - f(x-\Delta x) \right|$$

Con $\Delta x =$ error asociado a x .

• Problemas:

P1) $w = \frac{A+B}{C} = ?$

Si $A = 50.75 \pm 0.03$ [m]
 $B = 24.12 \pm 0.07$ [m]
 $C = 19.318 \pm 0.005$ [m]

Sol:

El problema se separa en 2 partes:

(2)

a) $Z = A + B$

• $Z = 50.75 + 24.12 = 74.87 \text{ cm}$

• $\Delta Z = \pm \sqrt{\Delta a^2 + \Delta b^2}$

$\Rightarrow \Delta Z = \pm \sqrt{0.03^2 + 0.07^2}$

$= \pm 0.076$

} se entrega con 1 cifra significativa!

$\Rightarrow [\Delta Z = \pm 0.08]$

$\Rightarrow \boxed{Z = 74.87 \pm 0.08}$

b) $W = \frac{Z}{C}$ con $Z = 74.87 \pm 0.08 \text{ (m)}$
 $C = 19.318 \pm 0.005 \text{ (m)}$

• $W = \frac{74.87}{19.318} = 3.8757$

• $\Delta W = \pm W \cdot \sqrt{\left(\frac{\Delta Z}{Z}\right)^2 + \left(\frac{\Delta C}{C}\right)^2}$

$= \pm 3.8757 \cdot \sqrt{\left(\frac{0.08}{74.87}\right)^2 + \left(\frac{0.005}{19.318}\right)^2}$

$= \pm 3.8757 \cdot \sqrt{1.142 \cdot 10^{-6} + 6.699 \cdot 10^{-8}}$

$= \pm 3.8757 \cdot 0.0011$

$\Rightarrow [\Delta W = \pm 0.004]$

\Rightarrow El resultado final es:

$\boxed{W = 3.876 \pm 0.004} \text{ cm}$

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P2] $W = \frac{5 \cdot \cos(x)}{\sin(y)}$ Con $X = 50.17 \pm 0.01$ [rad]
 $y = 70.01 \pm 0.02$ [rad]

El problema se separa en 5 partes:

a) $Z = \cos(x)$

• $Z = \cos(50.17) = 0.995$

• $\Delta Z = \pm \frac{1}{2} \cdot |\cos(50.17 + 0.01) - \cos(50.17 - 0.01)|$

$= \pm \frac{1}{2} \cdot |0.996 - 0.994|$

$\Rightarrow \Delta Z = \pm 0.001$

$\Rightarrow \boxed{Z = 0.995 \pm 0.001}$

b) $m = Z^3$ con $Z = 0.995 \pm 0.001$

• $m = 0.995^3 = 0.985$

• $\Delta m = \pm 3 \cdot 0.995^2 \cdot 0.001$

$= \pm 0.00297$ } Se aproxima!!

$\Rightarrow \Delta m = \pm 0.003$

$\Rightarrow \boxed{m = 0.985 \pm 0.003}$

c) $P = 5 \cdot m$ con $m = 0.985 \pm 0.003$

• $P = 5 \cdot 0.985 = 4.925$

• $\Delta P = \pm 5 \cdot 0.003 = 0.015$ } Se aproxima!!

$\Rightarrow \Delta P = \pm 0.02$

$\Rightarrow \boxed{P = 4.93 \pm 0.02}$

d) $T = \sin(y)$

• $T = \sin(70.01) = 0.780$

• $\Delta T = \pm \frac{1}{2} \cdot |\sin(70.01 + 0.02) - \sin(70.01 - 0.02)|$

$= \pm \frac{1}{2} \cdot |0.793 - 0.768|$

$= \pm 0.0125$ } Se aproxima!!

$\Rightarrow \Delta T = \pm 0.01$

$\Rightarrow \boxed{T = 0.78 \pm 0.01}$

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e) $w = \frac{P}{T}$ con $P = 4.93 \pm 0.02$ $T = 0.78 \pm 0.01$

• $w = \frac{4.93}{0.78} = 6.321$

• $\Delta w = \pm w \cdot \sqrt{\left(\frac{\Delta P}{P}\right)^2 + \left(\frac{\Delta T}{T}\right)^2}$

$= \pm 6.321 \cdot \sqrt{\left(\frac{0.02}{4.93}\right)^2 + \left(\frac{0.01}{0.78}\right)^2}$

$= \pm 6.321 \cdot \sqrt{1.646 \cdot 10^{-5} + 1.644 \cdot 10^{-4}}$

$= \pm 6.321 \cdot \sqrt{1.809 \cdot 10^{-4}}$

$= \pm 0.085$ } se aproxima!

⇒ El resultado final es:

$w = 6.32 \pm 0.09$

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