PAUTA EJ #2

1) a) Régimen impermanente, planteamos la ecuación de Euler entre 1 y 2.

Como $h, H \ll L$, despreciamos h y H frente a la longitud L

$$\frac{L}{g}\frac{dv}{dt} + B_2 - B_1 = 0 \quad (1)$$ 1,0

$$B_1 = h - z + \frac{P_1}{\gamma} + \frac{v^2}{2g} \quad ; \quad B_2 = \frac{v^2}{2g} + P_{atm}$$

Pero P_1 no es cte, es función de z. Proceso isotérmico: $P_{ini}\forall_{ini} = P_{fin}\forall_{fin}$

$$\Rightarrow 2P_{atm}\cdot\frac{\pi d^2}{4}(H-h) = P_1\cdot\frac{\pi d^2}{4}(H-h+z) \Leftrightarrow P_1 = 2P_{atm}\frac{(H-h)}{(H-h+z)} = 2P_{atm}\left(\frac{1}{1+\frac{z}{H-h}}\right)$$ 1,0

$$h \ll H \Rightarrow \frac{z}{H-h} \ll 1 \Rightarrow 2P_{atm}\left(\frac{1}{1+\frac{z}{H-h}}\right) \approx 2P_{atm}\left(1-\frac{z}{(H-h)}\right)$$

Reemplazando en (1): $$\frac{L}{g}\frac{dv}{dt} + \cancel{\frac{v^2}{2g}} + \frac{P_{atm}}{\gamma} - h + z - \frac{2P_{atm}}{\gamma}\left(1 - \frac{z}{(H-h)}\right) - \cancel{\frac{v^2}{2g}} = 0$$

$$v = \frac{dz}{dt} \Rightarrow \frac{dv}{dt} = \frac{d^2z}{dt^2} \Rightarrow \frac{L}{g}\frac{d^2z}{dt^2} + z\left(1 + \frac{2P_{atm}}{\gamma(H-h)}\right) - \frac{P_{atm}}{\gamma} - h = 0 \quad (2)$$ 1,0

El caudal es $Q(t) = A\cdot v(t)$. Para encontrar $v(t)$, primero necesitamos $z(t)$, o sea, resolver (2)

Llamando $C = \left[1 + \frac{2P_{atm}}{\gamma(H-h)}\right]\frac{g}{L}$, $D = \left(-\frac{P_{atm}}{\gamma} - h\right)\frac{g}{L} \Rightarrow \frac{d^2z}{dt^2} + Cz + D = 0$

Cambio de variables: $Cz + D = Cy \Leftrightarrow z = y - \frac{D}{C} \Rightarrow \frac{dz}{dt} = \frac{dy}{dt}$; $\frac{d^2z}{dt^2} = \frac{d^2y}{dt^2}$

Resolvemos $\frac{d^2y}{dt^2} + Cy = 0$. Polinomio característico $\Rightarrow \lambda^2 + C = 0$

$$\Rightarrow \lambda = \frac{\pm\sqrt{-4C}}{2} = \pm\, i\sqrt{C} \Rightarrow y = C_1 \cos(\sqrt{C}t) + C_2\, \mathrm{sen}(\sqrt{C}t)\ ; \quad z = y - \frac{D}{C}$$

$$\Rightarrow z(t) = C_1 \cos(\sqrt{C}t) + C_2\, \mathrm{sen}(\sqrt{C}t) - \frac{D}{C}$$ 1,0

Condiciones de Borde. $z(t=0)=0 \Rightarrow C_1 - \frac{D}{C} = 0 \Rightarrow C_1 = \frac{D}{C}$

$$\left.\frac{dz}{dt}\right|_{t=0} = 0 \Rightarrow -C_1\sqrt{C}\,\mathrm{sen}(\sqrt{C}t) + C_2 \cos(\sqrt{C}t) = \frac{D}{C}\sqrt{C}\,\mathrm{sen}(\sqrt{C}t) + C_2\cos(\sqrt{C}t)$$

$$t=0 \Rightarrow C_2 = 0 \quad \Rightarrow z(t) = \frac{D}{C}\cos(\sqrt{C}t) - \frac{D}{C} = \frac{D}{C}\left[\cos(\sqrt{C}t) - 1\right]$$ 1,0

$$v(t) = \frac{dz}{dt} = -\frac{D}{C}\sqrt{C}\,\mathrm{sen}(\sqrt{C}t) = \frac{-D}{\sqrt{C}}\,\mathrm{sen}(\sqrt{C}t)$$ 0,8

$$Q = VA \Rightarrow Q(t) = \frac{\pi d^2}{4}\left[\frac{\frac{g}{2}\left(\frac{P_{atm}}{\gamma} + h\right)}{\sqrt{\left(1 + \frac{2P_{atm}}{\gamma(H-h)}\right)\frac{g}{2}}}\right] \mathrm{sen}\left(\sqrt{\frac{g}{2}\left(1 + \frac{2P_{atm}}{\gamma(H-h)}\right)}\,t\right)$$ 0,2