Pauta Control #3

P1)

a) $f(x,y) = \dfrac{x^2 y}{x^4+y^4+1}$ en $D = \{(x,y) \mid x \geq 0 \wedge y \geq 0\}$

$$\frac{\partial f}{\partial x} = \frac{2xy(x^4+y^4+1) - x^2y \cdot 4x^3}{(x^4+y^4+1)^2} = 0 \quad (1)$$

$$\frac{\partial f}{\partial y} = \frac{x^2(x^4+y^4+1) - x^2y \cdot 4y^3}{(x^4+y^4+1)^2} = 0 \quad (2)$$

$$2xy\,(x^4+y^4+1) - 4x^5y = 0$$

$$x^6 + x^2y^4 + x^2 - 4y^4x^2 = 0$$

entonces:

$$x^6 + x^2 - 3x^2y^4 = 0$$

$$2xy + 2xy^5 - 2x^5y = 0$$

$$\Rightarrow \quad \begin{aligned} x^2\,(x^4 - 3y^4 + 1) &= 0 \\ xy\,(1 + y^4 - x^4) &= 0 \end{aligned}$$

$$x, y \neq 0 \Rightarrow \quad \begin{aligned} x^4 - 3y^4 + 1 &= 0 \\ 1 + y^4 - x^4 &= 0 \\ 2 - 2y^4 &= 0 \Rightarrow \boxed{y = \pm 1} \end{aligned}$$

$$x^4 = 1 + y^4 \Rightarrow x^4 = 2 \Rightarrow \boxed{x = \pm (2)^{1/4}}$$

$\Rightarrow$ $\boxed{P = (\sqrt[4]{2}, 1) \in D}$ es un máximo.

Además la función se hace mínima en los ejes $x=0$ $y \in \mathbb{R}$ y $x \in \mathbb{R}, y=0$

con $f = 0$.

b) $f(x,y) = x^2+y^2$ con $D = \{(x,y) \mid x^2+4y^2 \leq 4\}$

∂D:

$L(x,y,\lambda) = x^2+y^2 - \lambda(x^2+4y^2-4)$

$$\frac{\partial L}{\partial x} = 2x - 2\lambda x = 0$$

$$\frac{\partial L}{\partial y} = 2y - 8\lambda y = 0$$

$$\frac{\partial L}{\partial \lambda} = -(x^2+4y^2-4) = 0$$

1) $x=0 \wedge y \neq 0$
$\Rightarrow y = \pm 1 \Rightarrow P_1 = (0, \pm 1)$

2) $y=0 \wedge x \neq 0 \Rightarrow x = \pm 2 \Rightarrow P_2 = (\pm 2, 0)$

Int D:

$P_3 = (0,0)$

$f(P_1) = 1$; $f(P_2) = 4$; $f(P_3) = 0$

$\Rightarrow$ P_2 es máximo
P_3 es mínimo.

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c) $f(x,y) = xy\, e^{-(x^2+y^2)}$

$$\frac{\partial f}{\partial x} = y\, e^{-(x^2+y^2)} - 2x^2 y\, e^{-(x^2+y^2)} = 0 \qquad (1)$$

$$\frac{\partial f}{\partial y} = x\, e^{-(x^2+y^2)} - 2y^2 x\, e^{-(x^2+y^2)} = 0 \qquad (2)$$

$$\Rightarrow \quad y - 2x^2 y = 0$$

$$x - 2y^2 x = 0$$

$$\Rightarrow \quad y(1-2x^2) = 0$$

$$x(1-2y^2) = 0$$

1) $\left.\begin{array}{l} x=0 \wedge y\neq 0 \\ y=0 \wedge x \neq 0 \end{array}\right\} \Rightarrow \quad P_1 = (0,0)$

2) $x \neq 0 \wedge y \neq 0 \Rightarrow \quad x = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{2}}{2}$

$$y = \pm\frac{\sqrt{2}}{2}$$

$$P = \left(\pm\frac{\sqrt{2}}{2}, \pm\frac{\sqrt{2}}{2}\right)$$

$\Rightarrow \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right); \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ son máximos

$\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right); \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ son mínimos.

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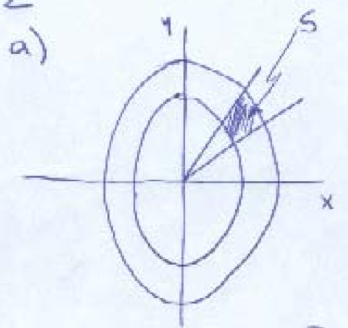
Control #3

2

a)

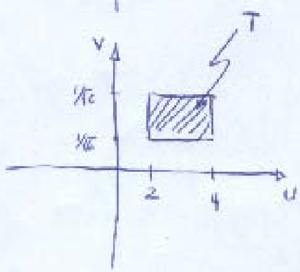


$x^2+2y^2=2 \qquad x^2+2y^2=4$

* $\frac{y}{x}=\frac{1}{\sqrt{6}} \qquad \frac{y}{x}=\frac{1}{\sqrt{2}}$

Sea $u=x^2+2y^2$

$v=\frac{y}{x}$ (1,0)



$$du\,dv = \begin{vmatrix} 2x & 4y \\ -\frac{y}{x^2} & \frac{1}{x} \end{vmatrix} dx\,dy$$

$$= 2+4\frac{y^2}{x^2}\,dx\,dy$$

$$du\,dv = 2+4v^2\,dx\,dy$$

$$\Rightarrow dx\,dy = \frac{1}{2+4v^2}\,du\,dv$$

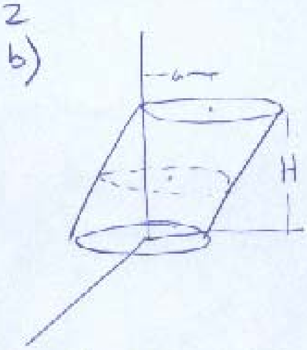
$$\text{Area} = \iint_S dx\,dy = \iint_T \frac{1}{2+4v^2}\,du\,dv \; (2.0) = \frac{1}{4}\int_2^4 \int_{1/\sqrt{6}}^{1/\sqrt{2}} \frac{1}{\frac{1}{2}+v^2}\,dv\,du = \frac{\pi\sqrt{2}}{24}$$

(3.0)

Control #3

2

b)



La posición del centro de cada circunferencia está dada por

$$\left(0, \frac{a}{H}z, z\right)$$

Luego el cilindro está definido por

$$x^2 + \left(y - \frac{a}{H}z\right)^2 = a^2 \qquad 0 \le z \le H$$

(1,0)

Por lo tanto

$$-a \le x \le a$$

$$0 \le z \le H$$

$$\frac{a}{H}z - \sqrt{a^2-x^2} \le y \le \frac{a}{H}z + \sqrt{a^2-x^2}$$

Luego

$$V = \underset{(x)}{\int_{-a}^{a}} \underset{(z)}{\int_{0}^{H}} \underset{(y)}{\int_{\frac{a}{H}z-\sqrt{a^2-x^2}}^{\frac{a}{H}z+\sqrt{a^2-x^2}}} dy\,dz\,dx = \int_{-a}^{a}\int_{0}^{H} 2\sqrt{a^2-x^2}\,dz\,dx$$

(2.0)

$$= 2H \cdot \int_{-a}^{a} \sqrt{a^2-x^2}\,dx \qquad \begin{array}{l} x = a\,\text{sen}\,\theta \\ dx = a\cos\theta\,d\theta \end{array} \qquad \begin{array}{l} x=-a \longrightarrow \theta = -\frac{\pi}{2} \\ x = a \longrightarrow \theta = \frac{\pi}{2} \end{array}$$

$$= 2H \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} a^2\cos^2\theta\,d\theta = 2a^2H \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2\theta\,d\theta = 2a^2H\left[\frac{\theta}{2} + \frac{\text{sen}\,2\theta}{4}\right]\Bigg|_{-\pi/2}^{\pi/2}$$

$$= \cancel{2}a^2H\left[\frac{1}{\cancel{2}}\left(\frac{\pi}{2}+\frac{\pi}{2}\right) + \frac{1}{4}\left(\cancel{\text{sen}\,\pi - \text{sen}-\pi}_{0}\right)\right]$$

$$V = \pi a^2 H$$

(3.0)

Control #3

3-

a) i) $\left.\begin{array}{c} \dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1 \\ x+y=1 \end{array}\right|$ $\quad y = 1-x \Rightarrow \dfrac{x^2}{a^2}+\dfrac{(1-x)^2}{b^2}=1$

$$\Rightarrow \frac{x^2}{a^2}+\frac{(1-2x+x^2)}{b^2}=1 \quad \Rightarrow \quad \frac{x^2}{a^2}+\frac{1}{b^2}-\frac{2x}{b^2}+\frac{x^2}{b^2}-1=0$$

$$\Rightarrow \left(\frac{1}{a^2}+\frac{1}{b^2}\right)x^2+\left(-\frac{2}{b^2}\right)x+\left(\frac{1}{b^2}-1\right)=0$$

Se desea que no haya intersección $\Rightarrow \left(-\frac{2}{b^2}\right)^2-4\left(\frac{1}{a^2}+\frac{1}{b^2}\right)\left(\frac{1}{b^2}-1\right)<0$

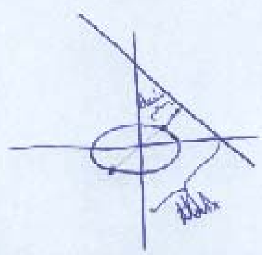
$$\Rightarrow \boxed{a^2+b^2<1}$$

ii) $L=(x_1-x_2)^2+(y_1-y_2)^2-\lambda\left(\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}-1\right)-\beta\left(x_2+y_2-1\right)$

$$\left.\begin{array}{l}
\dfrac{\partial L}{\partial x_1}=2(x_1-x_2)-\dfrac{2\lambda}{a^2}x_1=0 \\
\dfrac{\partial L}{\partial x_2}=-2(x_1-x_2)-\beta=0 \\
\dfrac{\partial L}{\partial y_1}=2(y_1-y_2)-\dfrac{2\lambda}{b^2}y_1=0 \\
\dfrac{\partial L}{\partial y_2}=-2(y_1-y_2)-\beta=0 \\
\dfrac{\partial L}{\partial \lambda}=-\left(\dfrac{x_1^2}{a^2}+\dfrac{y_1^2}{b^2}-1\right)=0 \\
\dfrac{\partial L}{\partial \beta}=-\left(x_2+y_2-1\right)=0
\end{array}\right\} \Rightarrow \quad x_1=\frac{a^2}{\sqrt{a^2+b^2}} \quad y_1=\frac{b^2}{\sqrt{a^2+b^2}}$$

$$x_2=\frac{1}{2}+\frac{b^2-a^2}{\sqrt{a^2+b^2}} \quad y_2=\frac{1}{2}+\frac{a^2-b^2}{\sqrt{a^2+b^2}}$$

$$d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$



$$\Rightarrow d=\left(\frac{1}{2}+\frac{(5a^4+5b^4-8a^2b^2)}{a^2+b^2}-\sqrt{a^2+b^2}\right)^{1/2}$$

Control #3

3-

b) $f(x,y,z) = x^4 + y^4 + z^4$

s.a. $x^2+y^2+z^2 = 1$

$x+y+z = 1$

$$L = x^4+y^4+z^4 - \lambda(x^2+y^2+z^2-1) - \beta(x+y+z-1)$$

$$\left.\begin{aligned}
\frac{\partial L}{\partial x} &= 4x^3 - 2x\lambda - \beta = 0 \\
\frac{\partial L}{\partial y} &= 4y^3 - 2y\lambda - \beta = 0 \\
\frac{\partial L}{\partial z} &= 4z^3 - 2z\lambda - \beta = 0 \\
\frac{\partial L}{\partial \lambda} &= -(x^2+y^2+z^2-1) = 0 \\
\frac{\partial L}{\partial \beta} &= -(x+y+z-1) = 0
\end{aligned}\right\}
\begin{aligned}
P_1 &= (1,0,0) \\
P_2 &= (0,1,0) \\
P_3 &= (0,0,1) \\
P_4 &= \left(-\tfrac{1}{3}, \tfrac{2}{3}, \tfrac{2}{3}\right) \\
P_5 &= \left(\tfrac{2}{3}, -\tfrac{1}{3}, \tfrac{2}{3}\right) \\
P_6 &= \left(\tfrac{2}{3}, \tfrac{2}{3}, -\tfrac{1}{3}\right)
\end{aligned}$$

$$f(P_1) = f(P_2) = f(P_3) = 1$$

$$f(P_4) = f(P_5) = f(P_6) = \frac{33}{81}$$

$\Rightarrow$ Máximos: $(1,0,0)$ $(0,1,0)$ $(0,0,1)$

Mínimos: $\left(-\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right)$ $\left(\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}\right)$ $\left(\frac{2}{3}, \frac{2}{3}, -\frac{1}{3}\right)$