Pauta Ejercicio #4

$(x+y)^2 = 4(x-2y)$

$x - 2y = 1$

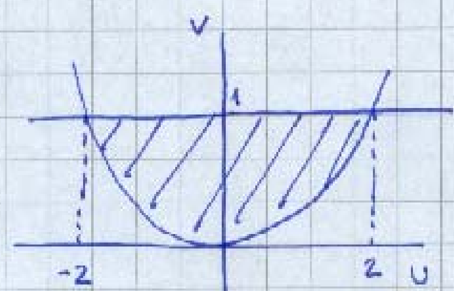
Sea $\quad u = x+y \qquad \dfrac{\partial u}{\partial x} = 1 = \dfrac{\partial u}{\partial y}$

$\qquad v = x-2y$

$\qquad\qquad \dfrac{\partial v}{\partial x} = 1 \qquad \dfrac{\partial v}{\partial y} = -2$

$u^2 = 4v$

$v = 1$



Además $\quad \begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} = 3 \qquad dudv = 3\,dxdy$

$$A = \frac{1}{3}\int_{-2}^{2}\int_{\frac{u^2}{4}}^{1} dv\,du$$

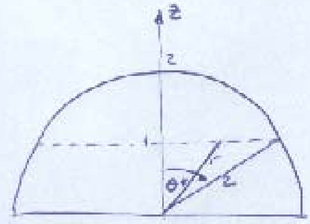
$$= \frac{1}{3}\int_{-2}^{2} 1 - \frac{u^2}{4}\,du = 3\left[u - \frac{u^3}{12}\right]_{-2}^{2}$$

$$= \frac{1}{3}\left[4 - \frac{16}{12}\right] =$$

$$= \frac{4}{3} - \frac{4}{9} = \boxed{\frac{8}{9} = A}$$

Ejercicio #5

$$V_0 = \{ (x,y,z) \,/\, x^2+y^2+z^2=4 \;,\; 1 \le z \le 2 \}$$



$$\cos\theta^* = \frac{1}{2} \Rightarrow \theta^* = \frac{\pi}{3}$$

$$r\,\cancel{2}\cos\theta = \frac{1}{\cancel{2}} \Rightarrow \boxed{r = \frac{1}{\cancel{2}\cos\theta}}$$

Luego el casquete queda definido por:

$$0 \le \varphi \le 2\pi$$

$$\frac{1}{\cos\theta} \le r \le 2$$

$$0 \le \theta \le \frac{\pi}{3}$$

$$V_0 = \int_0^{\pi/3}\int_0^{2\pi}\int_{1/\cos\theta}^{2} r^2 \operatorname{sen}\theta \cdot dr\, d\varphi\, d\theta = 2\pi \int_0^{\pi/3}\int_{\frac{1}{\cos\theta}}^{2} r^2 \operatorname{sen}\theta\, dr\, d\theta$$

(θ) (φ) (r)

$$= 2\pi \int_0^{\pi/3} \left.\frac{r^3}{3}\right|_{\frac{1}{\cos\theta}}^{2} \cdot \operatorname{sen}\theta\, d\theta = \frac{2\pi}{3}\int_0^{\frac{\pi}{3}} \left(2^3 - \frac{1}{\cos^3\theta}\right) \operatorname{sen}\theta\, d\theta$$

$$= \frac{2\pi}{3}\left[\int_0^{\pi/3} 8 \operatorname{sen}\theta\, d\theta - \int_0^{\pi/3} \frac{\operatorname{sen}\theta}{\cos^3\theta}\, d\theta\right] = \frac{2\pi}{3}\left[-8\cos\theta\Big|_0^{\frac{\pi}{3}} - \frac{1}{2\cos^2\theta}\Big|_0^{\pi/3}\right]$$

$$= \frac{2\pi}{3}\left(-8\left(\frac{1}{2}-1\right) - \left(\frac{1}{2\cdot\left(\frac{1}{2}\right)^2} - \frac{1}{2(1)^2}\right)\right) = \frac{2\pi}{3}\left[4 - \frac{3}{2}\right] = \frac{\cancel{2}\pi}{3}\cdot\frac{5}{\cancel{2}}$$

$$\boxed{V_0 = \frac{5\pi}{3}}$$