$$\vec{L}_0 = \Big(-mL\,\text{sen}\phi\, \frac{L\,\text{sen}\phi}{2}\dot{\phi} - \frac{mL}{2}(1+\text{cn}\phi)\,L\,\text{cn}\phi\,\dot{\phi}\Big)\hat{k}$$

$$= -\frac{mL^2}{2}\Big(\text{sen}^2\phi\,\dot{\phi} + \text{cn}\phi\,\dot{\phi} + \text{cn}^2\phi\,\dot{\phi}\Big)\hat{k}$$

$$= -\frac{mL^2}{2}(1+\text{cn}\phi)\dot{\phi}\,\hat{k}$$

$$\Rightarrow \frac{d\vec{L}_0}{dt} = -\frac{mL^2}{2}\Big(-\text{sen}\phi\,\dot{\phi}^2 + (1+\text{cn}\phi)\ddot{\phi}\Big)\hat{k} = N'\frac{L}{2}(1-\text{cn}\phi)\hat{k}$$

Reemplazando el valor de $N'(\phi)$

$$-\frac{m\cancel{L^2}}{\cancel{2}}\Big(-\text{sen}\phi\,\dot{\phi}^2 + (1+\text{cn}\phi)\ddot{\phi}\Big) = \frac{m\cancel{L^2}}{\cancel{2}}(1-\text{cn}\phi)(\text{sen}\phi\,\dot{\phi}^2 - \text{cn}\phi\,\ddot{\phi})$$

$$\Leftrightarrow \cancel{\text{sen}\phi\,\dot{\phi}^2} - \ddot{\phi} - \cancel{\text{cn}\phi\,\ddot{\phi}} = \cancel{\text{sen}\phi\,\dot{\phi}^2} - \text{sen}\phi\,\text{cn}\phi\,\dot{\phi}^2 - \cancel{\text{cn}\phi\,\ddot{\phi}} + \text{cn}^2\phi\,\ddot{\phi}$$

$$\Leftrightarrow (1+\text{cn}^2\phi)\ddot{\phi} = +\text{sen}\phi\,\text{cn}\phi\,\dot{\phi}^2 \qquad \text{pero } \ddot{\phi} = \frac{d\dot{\phi}}{d\phi}\dot{\phi}$$

$$\Rightarrow (1+\text{cn}^2\phi)\cancel{\dot{\phi}}\frac{d\dot{\phi}}{d\phi} = \text{sen}\phi\,\text{cn}\phi\,\dot{\phi}^{\cancel{2}}$$

$$\Rightarrow \frac{d\dot{\phi}}{\dot{\phi}} = \frac{\text{sen}\phi\,\text{cn}\phi}{1+\text{cn}^2\phi}d\phi \qquad /\int$$

$$\int_{\frac{V_0}{L}}^{\dot{\phi}} \frac{d\dot{\phi}}{\dot{\phi}} = \int_0^{\phi} \frac{\text{sen}\phi\,\text{cn}\phi}{1+\text{cn}^2\phi}d\phi \Rightarrow \ln\Big(\frac{\dot{\phi}L}{V_0}\Big) = -\frac{1}{2}\ln(1+\text{cn}^2\phi)\Big|_0^{\phi}$$

$$\Rightarrow \ln\Big(\frac{\dot{\phi}L}{V_0}\Big) = -\frac{1}{2}\ln(1+\text{cn}^2\phi) + \tfrac{1}{2}\cancelto{0}{\ln(1)} = \ln\Big(\frac{1}{\sqrt{1+\text{cn}^2\phi}}\Big)$$

$$\Rightarrow \boxed{\dot{\phi} = \frac{V_0}{L}\frac{1}{\sqrt{1+\text{cn}^2\phi}}}$$

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