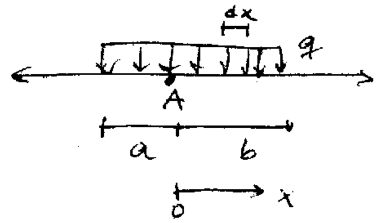
Caso c Viga infinita con carga repartida.

1) Punto A dentro del área cargada.




Debemos suponer que aplicamos infinitas cargas $q'dx$ ubicados a una distancia $x$ del pto A.

La flexión producida por $q'dx$ será

$$w = \frac{q'dx\,\lambda}{2k'} A_{\lambda x}$$

Así $$w_A = \frac{q'\lambda}{2k'}\left[\int_0^a A_{\lambda x}\,dx + \int_0^b A_{\lambda x}\,dx\right]$$

pero $$D_{\lambda x} = \cos\lambda x\, e^{-\lambda x}$$

$$\frac{d D_{\lambda x}}{dx} = -\lambda \operatorname{sen}\lambda x\, e^{-\lambda x} - \lambda \cos\lambda x\, e^{-\lambda x}$$

$$= -\lambda A_{\lambda x}$$

$$\Rightarrow \int A_{\lambda x}\,dx = -\frac{1}{\lambda} D_{\lambda x}$$

entonces

$$w_A = \frac{q'\lambda}{2k'}\left(-\frac{1}{\lambda}\right)\left\{D_{\lambda a} - D_{\lambda 0} + D_{\lambda b} - D_{\lambda 0}\right\}$$

$$W_A = \frac{q'}{2k'}\left\{2 - D_{\lambda}(a) - D_{\lambda}(b)\right\}$$

de manera similar

$$\theta_A = \frac{q'\lambda}{2k'}\left[A\lambda a - A\lambda b\right]$$

$$M_A = \frac{q'}{4\lambda^2}\left[B\lambda a + B\lambda b\right]$$

$$V_A = \frac{-q'}{2}\left[C\lambda a - C\lambda b\right]$$

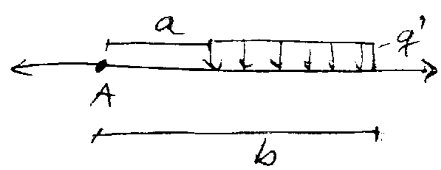
2. Punto A a la izquierda del área de carga

$$W_A = \frac{q'}{2k'}\left(D\lambda(a) - D\lambda(b)\right)$$

$$\theta_A = \frac{q'\lambda}{2k'}\left[A\lambda a - A\lambda b\right]$$

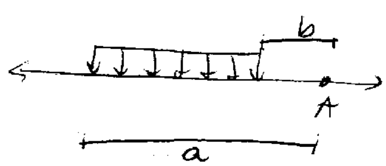
$$M_A = \frac{-q'}{4\lambda^2}\left[B\lambda a - B\lambda b\right]$$

$$V_A = \frac{q'}{4\lambda}\left[C\lambda a - C\lambda b\right]$$




$q' = q \cdot B$

3 Punto A a la derecha de la zona de carga




$$W_A = \frac{-q'}{2k'}\left[D\lambda a - D\lambda b\right]$$

$$\theta_A = \frac{q'\lambda}{2k'}\left[A\lambda a - A\lambda b\right]$$

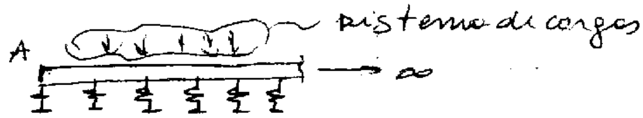
$$M_A = \frac{q'}{4\lambda^2}\left[B\lambda a - B\lambda b\right]$$

$$V_A = \frac{q'}{4\lambda}\left[C\lambda a - C\lambda b\right]$$

Vigas de largo semi infinito

Tienen un extremo finito que se modela por una condición de borde particular.

a) Viga semi-infinita con extremo libre

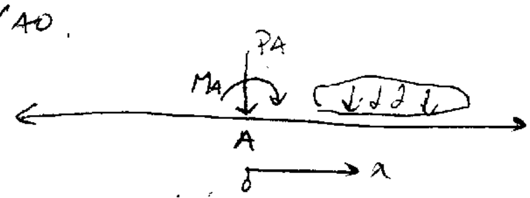



condiciones de borde $V|_{x=0} = 0$

$M|_{x=0} = 0$

Para una viga infinita dado un "sistema de cargas" se obtiene un $M_{A0}$ y un corte $V_{A0}$ en el punto A.
Para eliminar estos esfuerzos y representar la condición de borde aplico un momento $M_A$ y una carga $P_A$ infinitamente cerca del punto A tal que estos produzcan un momento $-M_{A0}$ y corte $-V_{A0}$.

$\Rightarrow$




De esta manera obtenemos

$$\Sigma M|_{x=0}: \quad M_{A0} + \frac{P_A}{4\lambda} \cancel{C_{\lambda x}}^{1} + \frac{M_A}{2} \cancel{D_{\lambda x}}^{1} = 0$$

$$\Rightarrow \quad M_{A0} + \frac{P_A}{4\lambda} + \frac{M_A}{2} = 0$$

$$\Sigma F_V|_{x=0}: \quad V_{A0} - \frac{P_A}{2} \cancel{D_{\lambda x}}^{1} - \frac{M_A \lambda}{2} \cancel{A_{\lambda x}}^{1} = 0$$

$$V_{A0} = \frac{P_A}{2} + \frac{M_A}{2} \lambda$$

$$\Rightarrow \quad M_A = -4M_{A0} - 2\frac{V_{A0}}{\lambda}$$

$$P_A = 4(\lambda M_{A0} + V_{A0})$$

b) Viga semiinfinita con extremo rotulado


Condiciones de borde $M|_{x=0} = 0$
$w|_{x=0} = 0$

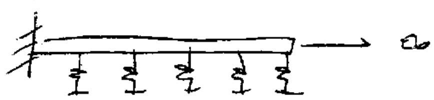
$$\sum M|_{x=0} \rightarrow M_{A0} + \frac{P_A}{4\lambda} + \frac{M_A}{2} = 0$$

$$\sum w|_{x=0} \rightarrow w_{A0} + \frac{P_A\lambda}{2k'}A_{\lambda 0} + \frac{M_A\lambda^2}{k'}B_{\lambda 0} = 0$$

$$\Rightarrow w_{A0} + \frac{P_A\lambda}{2k'} = 0 \rightarrow \boxed{P_A = \frac{-2k'w_{A0}}{\lambda}}$$

$$\rightarrow \boxed{M_A = \frac{k'w_{A0}}{\lambda^2} - 2M_{A0}}$$

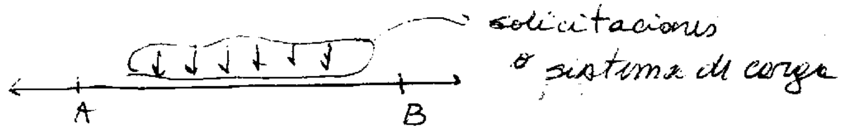
c) Viga semi-infinita con un extremo empotrado



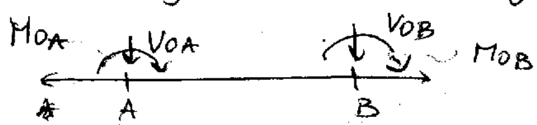

$w|_{x=0} = 0$
$\theta|_{x=0} = 0$

$$\Rightarrow \boxed{P_A = \frac{-2k'}{\lambda}w_{A0}} \qquad \boxed{M_A = \frac{-k'}{\lambda^2}\theta_{A0}}$$

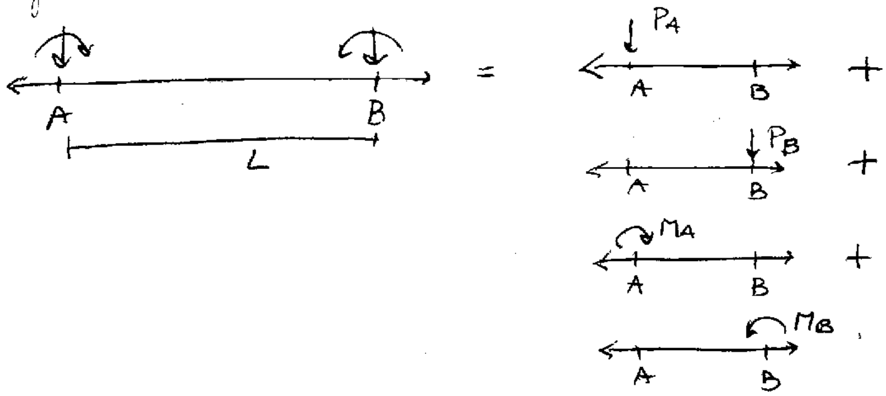
## Viga finita $\quad \frac{\pi}{4} < \lambda L < \pi$.




Sistema de carga generan corte y momento en A y B.




Para contrarestar estas solicitaciones y mantener los condiciones de borde ~~se~~ deben aplicar corte y momento en los bordes.




$$\Rightarrow -V_{0A} = -\frac{1}{2} P_A \cdot \cancel{D_{\lambda 0}}^{1} + \frac{D_{\lambda L}}{2} P_B - \frac{\lambda}{2} M_A \cancel{A_{\lambda 0}}^{1} + \lambda \frac{A_{\lambda L}}{2} M_B$$

$$-V_{0B} = -\frac{D_{\lambda L}}{2} P_A + \frac{1}{2} \cancel{D_{\lambda 0}}^{1} P_B - \lambda \frac{A_{\lambda L}}{2} M_A + \frac{\lambda}{2} A_{\lambda 0} M_B$$

$$-M_{0A} = \frac{1}{4\lambda} P_A + \frac{C_{\lambda L}}{4\lambda} P_B + \frac{1}{2} M_A + \frac{D_{\lambda L}}{2} M_B$$

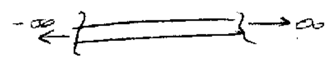
$$-M_{0B} = \frac{C_{\lambda L}}{4\lambda} P_A + \frac{1}{4\lambda} P_B + \frac{D_{\lambda L}}{2} M_A + \frac{1}{2} M_B$$

Matriz a resolver para encontrar $P_A, P_B, M_A, M_B$

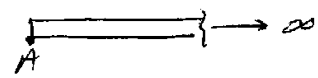
$$\begin{bmatrix} -\frac{1}{2} & \frac{D_{\lambda L}}{2} & -\frac{\lambda}{2} & \frac{\lambda A_{\lambda L}}{2} \\ -\frac{D_{\lambda L}}{2} & \frac{1}{2} & -\frac{\lambda A_{\lambda L}}{2} & \frac{\lambda}{2} \\ \frac{1}{4\lambda} & \frac{C_{\lambda L}}{4\lambda} & \frac{1}{2} & \frac{D_{\lambda L}}{2} \\ \frac{C_{\lambda L}}{4\lambda} & \frac{1}{4\lambda} & \frac{D_{\lambda L}}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} P_A \\ P_B \\ M_A \\ M_B \end{bmatrix} = \begin{bmatrix} -V_{0A} \\ -V_{0B} \\ -M_{0A} \\ -M_{0B} \end{bmatrix}$$

## Resumen

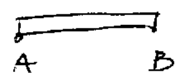
viga infinita $\lambda L > \pi$




viga semiinfinita $\lambda L > \pi$




viga finita $\frac{\pi}{4} < \lambda L < \pi$




viga rígida o corta $\lambda L < \pi/4$


(Navier)