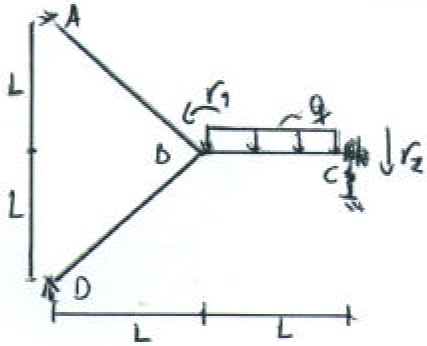
Guía de Ejercicios

P1)



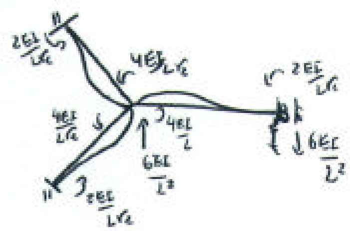
$EI = cte$
$EA = \infty$
$K = \frac{10EI}{L^3}$

Determine los diagramas de momento y corte. Dibuje la deformada.

Sol)

$GIE = 3$
$GL = 2$ $\Rightarrow$ rigidez

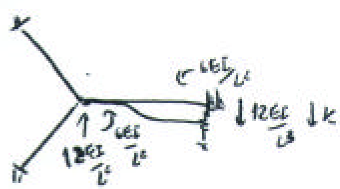
Si $r_1 = 1$ $r_2 = 0$



$\Rightarrow$ $K_{11} = 2 \times \frac{4EI}{L\sqrt{2}} + \frac{4EI}{L} = \frac{4EI}{L}\left(\sqrt{2}+1\right)$

$K_{21} = \frac{6EI}{L^2}$

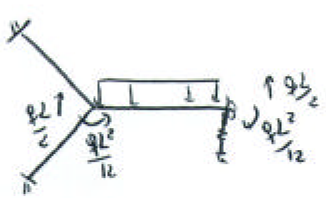
Si $r_1 = 0$ $r_2 = 1$



$\Rightarrow$ $K_{22} = K + \frac{12EI}{L^3} = 22\frac{EI}{L^3}$

$K_{12} = \frac{6EI}{L^2}$

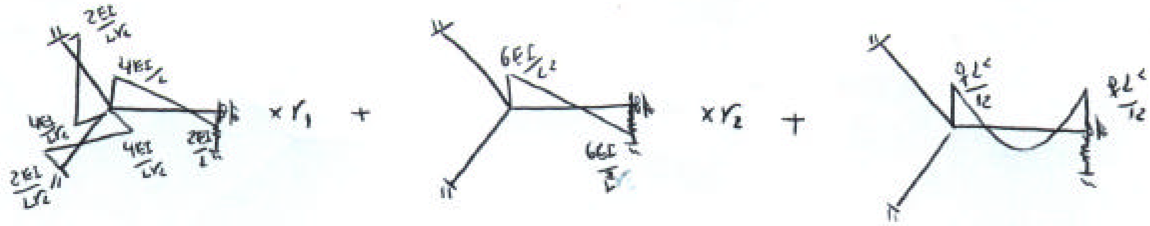
$$K = \frac{EI}{L^3}\begin{bmatrix} 4(\sqrt{2}+1)L^2 & 6L \\ 6L & 22 \end{bmatrix} \rightsquigarrow K^{-1} = \frac{L^3}{EI}\frac{1}{(88(\sqrt{2}+1)-36)L^2}\begin{bmatrix} 22 & -6L \\ -6L & 4(\sqrt{2}+1)L^2 \end{bmatrix} = \frac{L}{EI}\begin{bmatrix} 0{,}125 & -0{,}034L \\ -0{,}034L & 0{,}055L^2 \end{bmatrix}$$



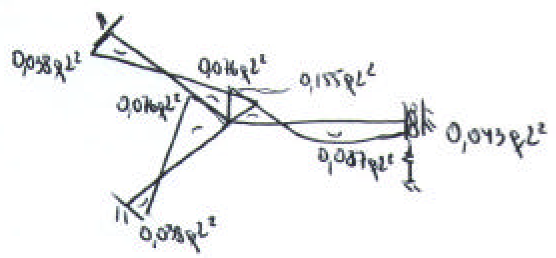
$\rightsquigarrow$ Vector de cargas nodales $R = \begin{Bmatrix} -\frac{qL^2}{12} \\ \frac{qL}{2} \end{Bmatrix}$

$$\rightsquigarrow r = K^{-1}R = \frac{L}{EI}\begin{bmatrix} 0{,}125 & -0{,}034L \\ -0{,}034L & 0{,}055L^2 \end{bmatrix}\begin{Bmatrix} -\frac{qL^2}{12} \\ \frac{qL}{2} \end{Bmatrix} = \begin{Bmatrix} -0{,}027\,\frac{qL^3}{EI} \\ 0{,}03\,\frac{qL^2}{EI} \end{Bmatrix}$$

Ahora el momento total es:

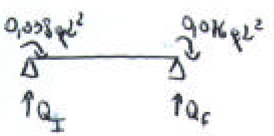


M)



Para el corte

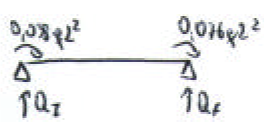
Tramo AB



$$Q_I = -\frac{(0{,}076 + 0{,}038)\,qL^2}{L\sqrt{2}} = -0{,}081\,qL$$

$$Q_F = -Q_I = +0{,}081\,qL$$

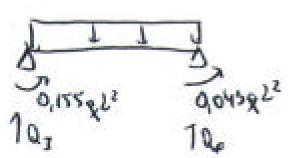
TRAMO BD



$$Q_I = \frac{M_{BD} + M_{DB}}{L\sqrt{2}} = -0{,}081\,qL$$

$$Q_F = 0{,}081\,qL$$

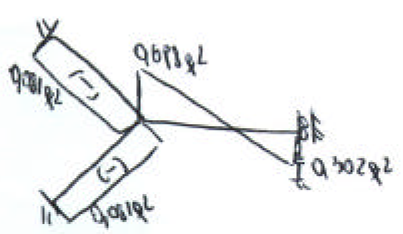
TRAMO BC



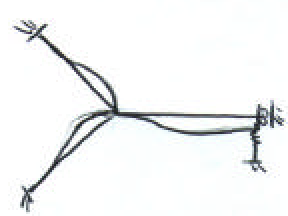
$$Q_I = \frac{qL}{2} + (0{,}155 + 0{,}043)\,qL = 0{,}698\,qL$$

$$Q_F = \frac{qL}{2} - (0{,}155 + 0{,}043)\,qL = 0{,}302\,qL$$

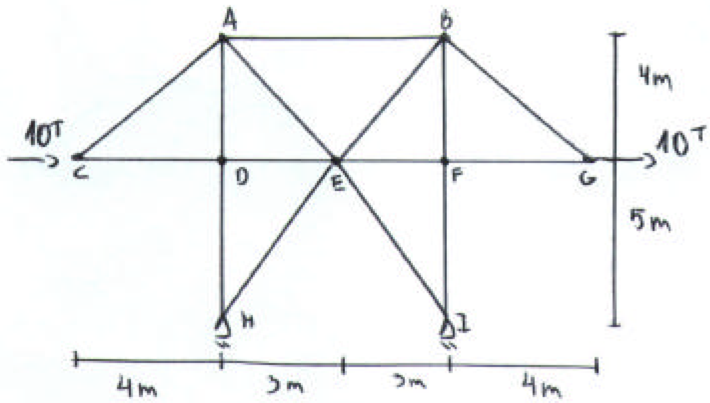
Q)



D)



P2/ Determina el esfuerzo axial en las barras de la figura.

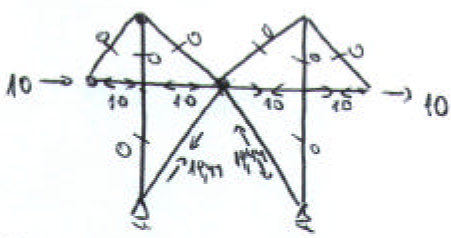


EA = cte

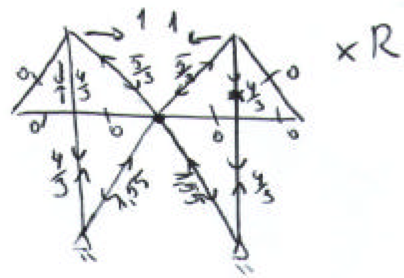
15 Barras
21 Rotulas
4 apoyos

$\Rightarrow GIE = -15 \times 3 + 21 \times 2 + 4 = 1$

Por flexibilidad

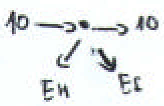


+



$\times R$

Nudo E/ → Estructura Cargas Reales

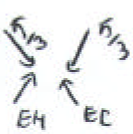


$\Sigma F_V \quad -EH \times \frac{5}{\sqrt{25+9}} - EI\,\frac{5}{\sqrt{34}} = 0 \Rightarrow EH = -EI$

$\Sigma F_H \quad -EH\,\frac{3}{\sqrt{34}} + EI \cdot \frac{3}{\sqrt{34}} + 20 = 0 \Rightarrow EH = \frac{\sqrt{34}}{6} \times 20 = 19{,}44^T$

$\Rightarrow EI = -19{,}44^T$

Nudo E/ → Estructura con carga unitaria



$\Sigma F_H \Rightarrow EH = EI$

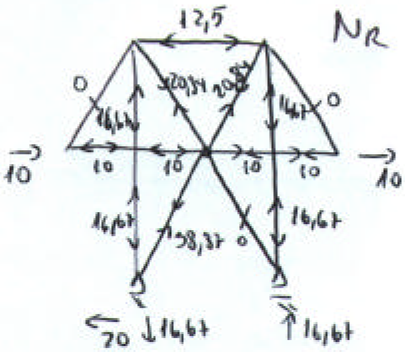
$\Sigma F_V \Rightarrow 2EH \cdot \frac{5}{\sqrt{34}} = \frac{5}{3} \times 2 \times \frac{4}{5} \Rightarrow EH = 1{,}55$

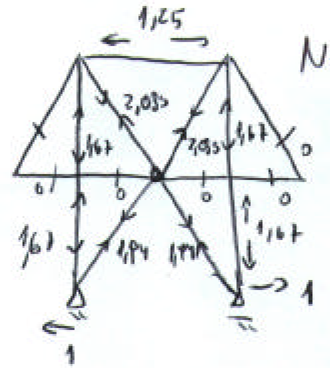
Matriz de flexibilidad

$$f_{11} = \Sigma \frac{N_i^1 \cdot N_i^1}{EA} \cdot L_i = \frac{2}{EA}\left(\left(\frac{4}{3}\right)^2 \times 5 + \left(\frac{4}{3}\right)^2 \times 4 + \left(\frac{5}{3}\right)^2 \times 5 + (1{,}55)^2 \times \sqrt{34}\right) + \underbrace{\frac{6 \times 1^2}{EA}}_{\text{Barra AB}} = \frac{93{,}8}{EA}$$

$$\Delta_{1q} = \Sigma \frac{N_i^1 \times N_{ei} \cdot L_i}{EA} = 0 \Rightarrow \boxed{R = 0}$$

Otra forma de hacer el problema es:





$\times R$

$\Sigma M_H: -10\times 2\times 5 + V_I \times 6 = 0$

$V_I = 16{,}67^T$

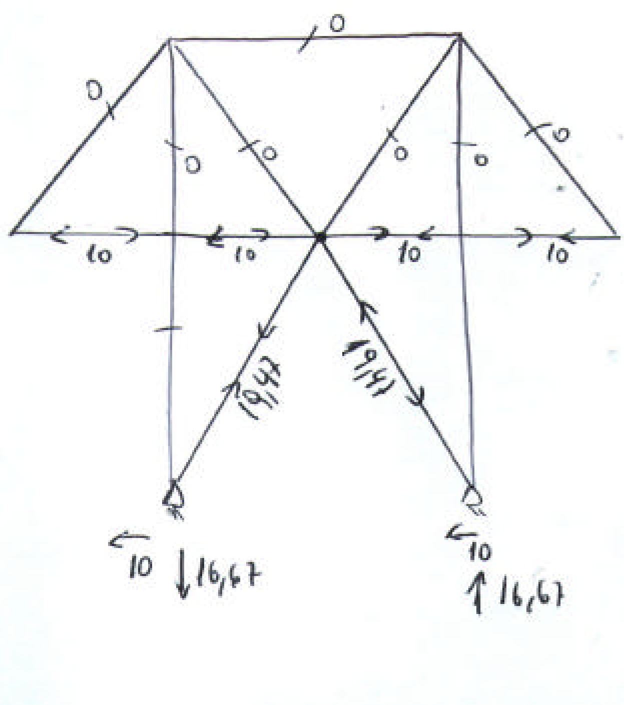
$$f_{11} = \frac{2}{EA}\left(1{,}67^2\times 5 + 1{,}94^2\times\sqrt{34} + 1{,}67^2\times 4 + 2{,}083^2\times 5\right) + \frac{1{,}25^2\times 6}{EA} = \frac{146{,}85}{EA}$$

$$\Delta_{1R} = \frac{1}{EA}\left(12{,}5\times 1{,}25\times 6 + 2\times 16{,}67\times 1{,}67\times 4 + 20{,}84\times 2{,}084\times 2\times 5 + 1{,}67\times 16{,}67\times 2\times 5 + 38{,}87\times 1{,}94\times\sqrt{34}\right) = \frac{1468{,}9}{EA}$$

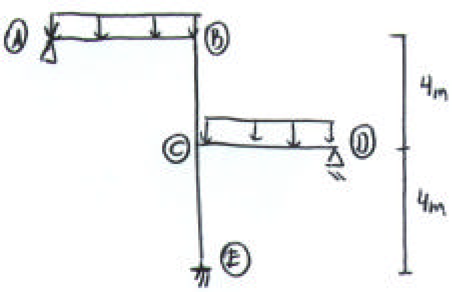
$$R = -\frac{1468{,}9}{146{,}85} = -10$$

Entonces las fuerzas normales sobre cada barra es:

$N_T = N_R + N'\times R$



P3)



EI = cte

$q = 2^t/m$

Por Cross

Factores de distribución

$$F_{BA} = \frac{\frac{3}{4}\frac{EI}{4}}{\frac{3}{4}\frac{EI}{4} + \frac{EI}{4}} = \frac{3/16}{7/16} = \frac{3}{7} \qquad F_{BC} = 1 - F_{BA} = \frac{4}{7}$$

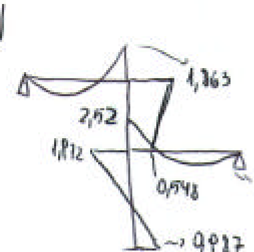
$$F_{CB} = \frac{1/4}{1/4 + \frac{1}{4} + \frac{3}{4}\cdot\frac{1}{4}} = \frac{1/4}{11/16} = \frac{4}{11} \qquad F_{CE} = \frac{4}{11} \qquad F_{CD} = \frac{3}{11}$$

Momentos de empotramiento perfecto

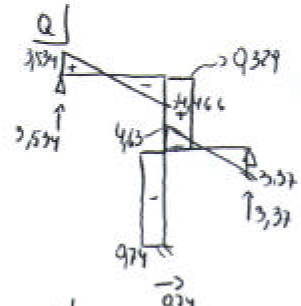
$$\frac{ql^2}{8} = \frac{2\cdot 4^2}{8} = 4^{TM}$$

	BA	BC	CB	CD	CE	EC
FD	3/7	4/7	4/11	3/11	4/11	1
M_{emp}	−4			4		
	+1,714	+2,286 →	+1,143			
		−0,935 ←	−1,870	−1,403	−1,870	→ −0,935
	0,401	0,534 →	0,267			
		−0,049 ←	−0,097	−0,073	−0,097	→ −0,049
	0,021	0,028 →	0,014			
		−0,003 ←	−0,005	−0,004	−0,005	→ −0,003
	0,001	0,002 →	0,001			
			−0,001			
	−1,863	1,863	−0,548	2,520	−1,972	−0,987

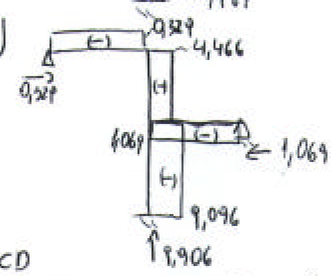
M)



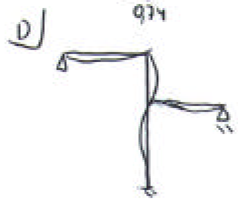
Q)



N)

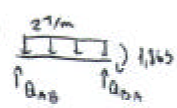


D)



Corte:

AB



$$Q_{BA} = \frac{1,863 + 2 \times 4^2/2}{4} = 4,466^T$$

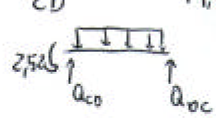
$$Q_{AB} = 2\cdot 4 - 4,466 = 3,534^T$$

BC

$$Q_{BC} = \frac{(1,863 - 0,548)}{4} = 0,329^T$$

$$Q_{CB} = -0,329^T$$

CD



$$Q_{CD} = \frac{(2,52 + 2 \times 4^2/2)}{4} = 4,63^T$$

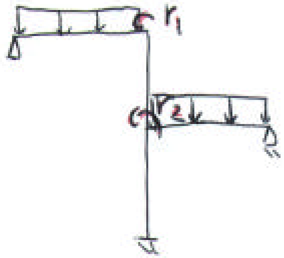
$$Q_{DC} = 4\cdot 2 - 4,63 = 3,37$$

CE

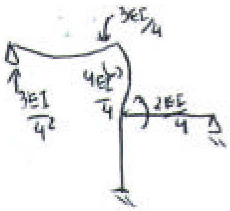
$$Q_{CE} = -(1,972 + 0,987)/4 = -0,740^T$$

$$Q_{EC} = +0,740^T$$

Haciendo el mismo problema pero ahora por rigidez



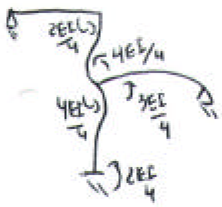
$r_1 = 1$; $r_2 = 0$



$$\Rightarrow \quad K_{11} = \frac{7EI}{4}$$

$$K_{12} = \frac{2EI}{4} = \frac{1}{2}EI$$

$r_2 = 1$; $r_1 = 0$

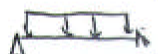


$$K_{22} = \frac{11EI}{4}$$

$$K_{21} = \frac{2EI}{4} = \frac{EI}{2}$$

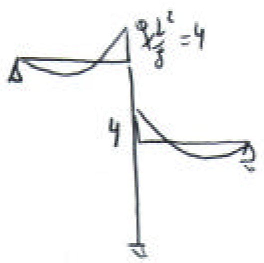
$$\Rightarrow \quad [K] = \frac{EI}{4}\begin{bmatrix} 7 & 2 \\ 2 & 11 \end{bmatrix} \quad \Rightarrow \quad [K]^{-1} = \frac{4}{EI}\,\frac{1}{(77-4)}\begin{bmatrix} 11 & -2 \\ -2 & 7 \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} 0{,}603 & -0{,}110 \\ -0{,}110 & 0{,}384 \end{bmatrix}$$

Vector de cargas

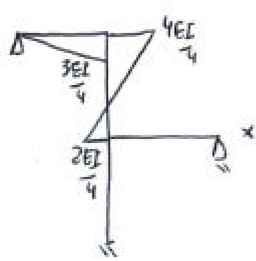


$$\frac{ql^2}{8} = 4^{tm} \Rightarrow \quad R = \begin{Bmatrix} 4 \\ -4 \end{Bmatrix} \quad \Rightarrow \quad r = K^{-1}R = \begin{Bmatrix} 2{,}849 \\ -1{,}973 \end{Bmatrix}\frac{1}{EI}$$

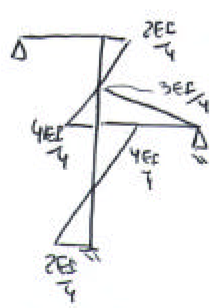
M_T)



$+$

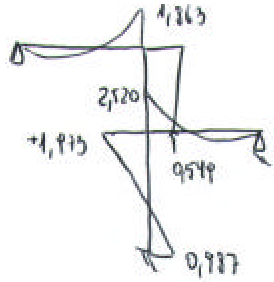


$\times \frac{2{,}849}{EI} +$



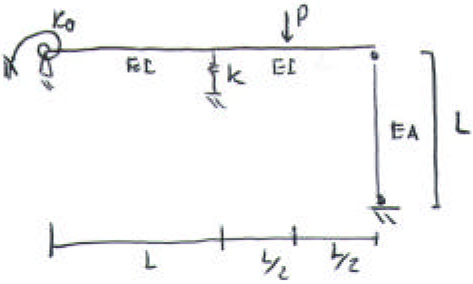
$\times \frac{-1{,}973}{EI}$

M_T



Como puede verse ambos métodos dan un resultado prácticamente idéntico (diferencias menores 0,001 TM)

P4



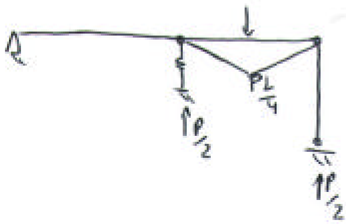
$EA = \frac{3EI}{L^2}$

$K = \frac{3EI}{2L^3}$

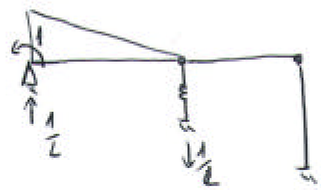
$k_\theta = \frac{EI}{L}$

GIE = 2
GL = 4 (2 giros y 2 desplazamientos) ⇒ flexibilidad

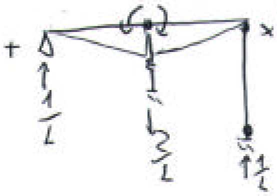
Entonces



+



$\times R_1$ +



$\times R_2$

$$f_{11} = \frac{1}{3}\cdot 1\cdot 1\,\frac{L}{EI} + \left(\frac{1}{L}\right)^2\frac{1}{k} + \frac{1}{K_\theta} = \frac{L}{3EI} + \frac{2L^3}{L^2\,3EI} + \frac{L}{EI} = \frac{2L}{EI}$$

$$f_{12} = -\frac{1}{2}\cdot 1\cdot\frac{L}{EI}\cdot\frac{1}{3} + \frac{1}{L}\cdot\frac{2}{L}\cdot\frac{1}{k} = \frac{-L}{6EI} + \frac{4L^3}{L^2\,3EI} = \frac{7}{6}\frac{L}{EI}$$

$$f_{22} = 2\cdot\frac{1}{3}\frac{L}{EI} + \left(\frac{2}{L}\right)^2\frac{1}{k} + \underbrace{\left(\frac{1}{L}\right)^2\frac{L}{EA}}_{\text{Biela}} = \frac{2}{3}\frac{L}{EI} + \frac{8}{3}\frac{L}{EI} + \frac{1}{L^2}\frac{L^3}{3EI} = \frac{11}{3}\frac{L}{EI}$$

$$f = \frac{L}{EI}\begin{bmatrix} 2 & 7/6 \\ 7/6 & 11/3 \end{bmatrix} \qquad f^{-1} = \frac{EI}{L}\begin{bmatrix} 11/3 & -7/6 \\ -7/6 & 2 \end{bmatrix}\frac{1}{\left(\frac{22}{3} - \frac{49}{36}\right)} = \frac{EI}{L}\begin{bmatrix} 0{,}614 & -0{,}195 \\ -0{,}195 & 0{,}335 \end{bmatrix}$$

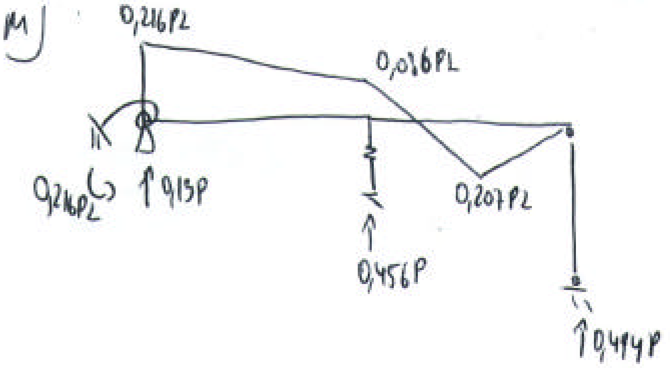
$$\Delta_{q1} = -\frac{P}{2}\cdot\frac{1}{L}\cdot\frac{1}{k} = -\frac{1}{3}\frac{PL^2}{EI}$$

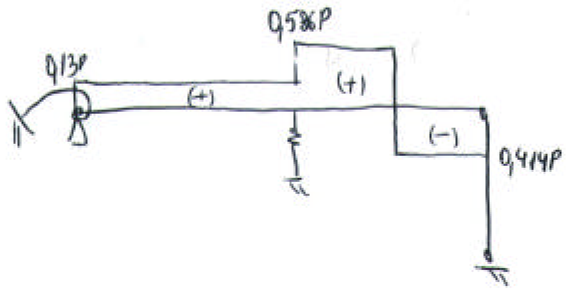
$$\Delta_{q2} = \frac{1}{2}\frac{PL}{4}\frac{L}{EI}\cdot\frac{1}{2} - \underbrace{\frac{2}{L}\cdot\frac{P}{2}\frac{1}{k}}_{\text{resorte}} + \underbrace{\frac{P}{2}\cdot\frac{1}{L}\cdot\frac{L}{EA}}_{\text{Biela}} = \frac{PL^2}{EI}\left(\frac{1}{16} - \frac{2}{3} + \frac{2}{3}\right) = \frac{PL^2}{16EI}$$

$$\{\Delta\} = \{\Delta_q\} + [f]\{R\}$$

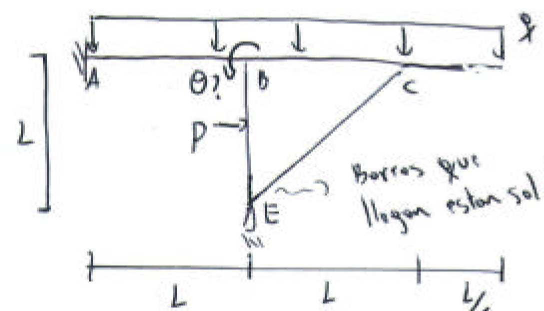
$$\Rightarrow \{R\} = -[f]^{-1}\{\Delta_q\} = -\frac{EI}{L}\begin{bmatrix} 0{,}614 & -0{,}195 \\ -0{,}195 & 0{,}335 \end{bmatrix}\frac{PL^2}{EI}\begin{Bmatrix} -1/3 \\ 1/16 \end{Bmatrix} = \begin{Bmatrix} 0{,}216 \\ -0{,}036 \end{Bmatrix} PL$$

M)





P5)



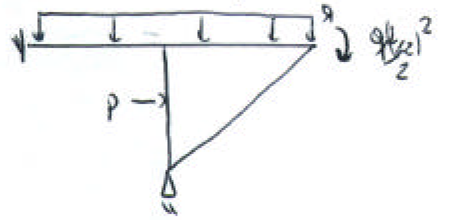
P = 2 ton
q = 2 ton/m
L = 5 m
EI = cte

Calcular diagramas de momento y corte, y el giro θ.

GIE = 4

GL = 3 ⇝ 3 giros en B, C y E

Utilizamos Cross y transformamos la estructura a:



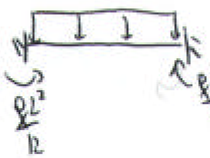
Factores de distribución

$$F_{BA} = \frac{\frac{1}{3}}{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} = \frac{1}{3} \qquad F_{BE} = \frac{1}{3} = F_{BC}$$

$$F_{CB} = \frac{\frac{1}{3}}{\frac{1}{3}+\frac{1}{3\sqrt{2}}} = 0{,}586 \qquad F_{CE} = \frac{\frac{1}{3\sqrt{2}}}{\frac{1}{3}+\frac{1}{3\sqrt{2}}} = 0{,}414 \qquad \begin{array}{l} F_{EB} = F_{CB} = 0{,}586 \\ F_{EC} = F_{CE} = 0{,}414 \end{array}$$

$$M_{ext\ en\ C} = -\frac{q(l/2)^2}{2} = -\frac{2\cdot(3/2)^2}{2} = -2{,}25\ tm$$

Momentos de empotramiento

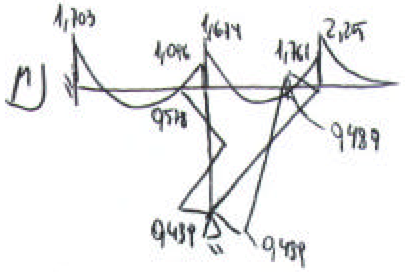


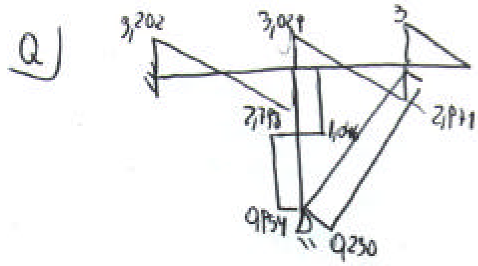
$$\frac{ql^2}{12} = \frac{2\cdot 9}{12} = 1{,}5 \qquad \Rightarrow \quad \begin{array}{l} M_{emp\,AB} = +1{,}5\ tm = M_{emp\,BC} \\ M_{emp\,BA} = -1{,}5\ tm = M_{emp\,CB} \end{array}$$

$$\frac{PL}{8} = \frac{2\cdot 3}{8} = \frac{6}{8} \qquad \Rightarrow \quad \begin{array}{l} M_{emp\,EB} = 0{,}75\ tm \\ M_{emp\,BE} = -0{,}75\ tm \end{array}$$

	AB	BA	BE	BC	CB	CE	EC	EB
Mext					-2,25			
FD		1/3	1/3	1/3	0,586	0,414	0,414	0,586
				-0,660 ←	-1,319	-0,931	→ -0,466	
Memp	1,5	-1,5	-0,75	1,5	-1,5			0,75
	0,235 ←	0,47	0,47	0,47	→ 0,235			→ 0,235
				0,371 ←	0,741	0,524	→ 0,262	
			-0,229 ←			-0,162 ←	-0,323	-0,458
	-0,024 ←	-0,048	-0,047	-0,047	→ -0,024			→ -0,024
				0,055 ←	0,109	0,077	→ 0,039	
			-0,005			-0,003 ←	-0,006	-0,009
	-0,008 ←	-0,017	-0,017	-0,016	→ -0,008			→ -0,008
				0,003 ←	0,006	0,005	→ 0,003	
			0,002			0,001 ←	0,002	0,003
		-0,001	-0,002	-0,002	→ -0,001			
	1,703	-1,096	-0,578	1,674	-1,761	-0,489	-0,489	0,489

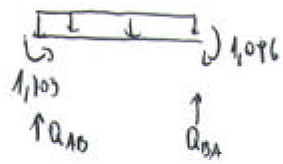
Valores en círculo para el cálculo del giro al final ⇒





Cortes

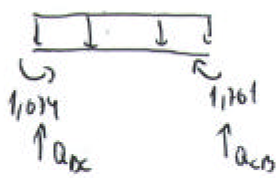
AB



$$Q_{AB} = \frac{(1{,}703 - 1{,}096)}{3} + \frac{2 \cdot 3}{2} = 3{,}202\ T$$

$$Q_{BA} = 2 \cdot 3 - 3{,}202 = 2{,}798$$

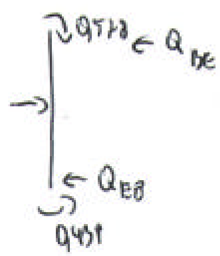
BC



$$Q_{CB} = \frac{(1{,}761 - 1{,}674)}{3} + 3 = 3{,}029\ T$$

$$Q_{BC} = 6 - 3{,}029 = 2{,}971\ T$$

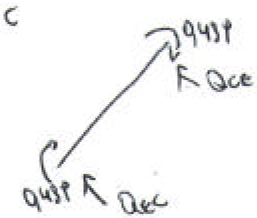
EB



$$Q_{BE} = \frac{(0{,}578 - 0{,}439)}{3} + \frac{2}{2} = 1{,}046\ T$$

$$Q_{EB} = 2 - 1{,}046 = 0{,}954\ T$$

EC



$$Q_{CE} = \frac{(0{,}489 + 0{,}438)}{3\sqrt{2}} = 0{,}230$$

$$Q_{EC} = -0{,}230$$

<u>Cálculo del giro</u>

$$\frac{4EI}{L}\theta = 0{,}47 + 0{,}048 - 0{,}017 - 0{,}001 = 0{,}405 \Rightarrow \theta = 0{,}405 \cdot \frac{3}{4EI} = \frac{0{,}304}{EI}$$

Rigidez barra AB al giro en B