Universidad de Chile
FCFM
FI10A-06
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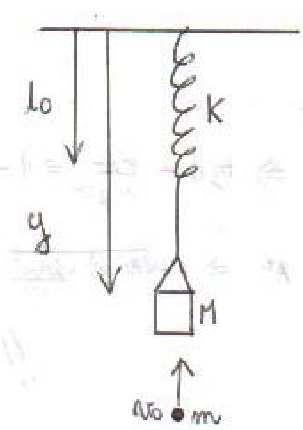
SOLUCIÓN AUXILIAR 12

Solución P1.



Aplicamos conservación de momentum inmediatamente antes y después del choque:

$p_i = m v_0$ $\quad p_f = (m+M) v$

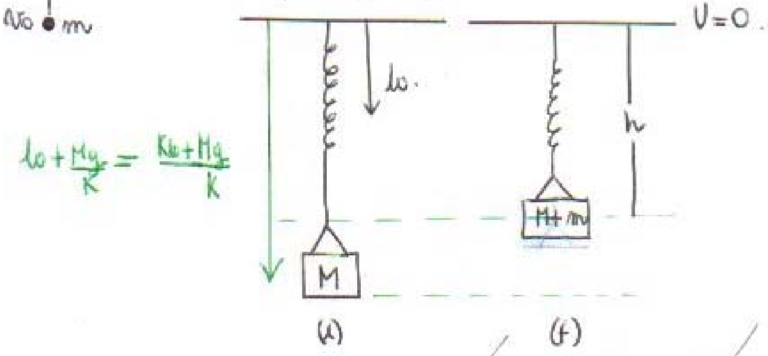
$p_i = p_f \Rightarrow m v_0 = (m+M) v \Rightarrow v = \frac{m v_0}{m+M}$.

Antes del choque, para el bloque M:

$\sum F_y : -K(y - l_0) + Mg = M\ddot{y}$

En equilibrio $\ddot{y} = 0 \Rightarrow y = [Mg + K l_0]/K$

Aplicamos ahora conservación de energía:



$E_i = -(M+m) g \left(l_0 + \frac{Mg}{K}\right) + \frac{1}{2}(M+m) v^2 \Rightarrow -\cancel{(M+m)} g \left(l_0 + \frac{Mg}{K}\right) + \frac{1}{2}\cancel{(M+m)} v^2 = -\cancel{(M+m)}$

$E_f = -(M+m) g h \Rightarrow h = \frac{g\left(l_0 + \frac{Mg}{K}\right) - \frac{1}{2} v^2}{g}$

Para el sistema bloque-bala:

$-K(y - l_0) + (M+m) g = (M+m)\ddot{y}$

pto de equilibrio: $\ddot{y} = 0 \Rightarrow -Ky + Kl_0 + (M+m) g = 0 \Rightarrow y_{eq} = \frac{K l_0 + (M+m) g}{K}$

$\therefore y_{eq} = l_0 + \frac{(M+m) g}{K}$.

Ocupando el cambio de variable $z = y - y_{eq}$

$-w^2 z = \ddot{z}$ con $w^2 = \sqrt{\frac{K}{M+m}}$

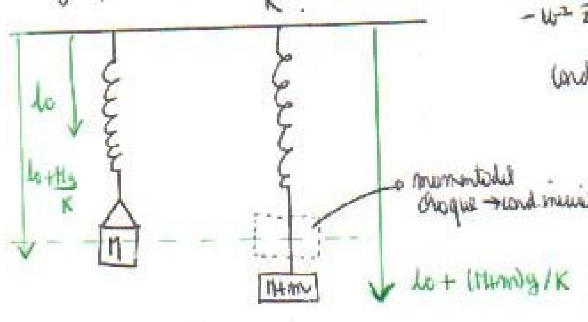
Condiciones iniciales:

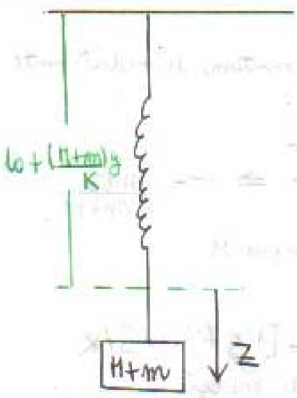
$z(t=0) = -\frac{mg}{K}$

$\dot{z}(t=0) = -v$

$z = A\cos(wt + \phi)$

$\dot{z} = -Aw\sin(wt + \phi)$.





$$Z = A\cos(\omega t + \phi) \quad (1)$$

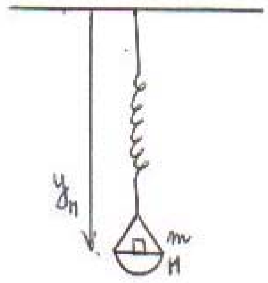
$$\dot{Z} = -A\omega \sin(\omega t + \phi) \quad (2)$$

$$\frac{(2)}{(1)} \Rightarrow -\omega\, tg(\omega t + \phi) = \frac{\dot{Z}}{Z}$$

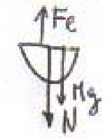
$$\text{para } t=0 \quad -\omega\, tg\,\phi = \frac{-v}{-\frac{mg}{K}} \Rightarrow tg\,\phi = \frac{Kv}{mg\omega} \Rightarrow \phi = tg^{-1}$$

$$(1)^2 + (2)^2 \Rightarrow Z^2 + \left(\frac{\dot{Z}}{\omega}\right)^2 = A^2 \Rightarrow A = \sqrt{Z(t=0)^2 + \frac{\dot{Z}(t=0)^2}{\omega^2}}$$

Solución P2.



DCL M.



$$\Sigma F_y: \; -K(y_M - l_0) + Mg + N = M\ddot{y}_M$$

DCL m

$$-N + mg = m\ddot{y}_m \Rightarrow N = mg - m\ddot{y}_m \quad (1)$$

pero $\ddot{y}_M = \ddot{y}_m = \ddot{y}$

$$\Rightarrow -K(y - l_0) + Mg + N = M\ddot{y}$$

$$-Ky + Kl_0 + Mg + mg - m\ddot{y} = M\ddot{y}$$

$$\Rightarrow -Ky + Kl_0 + (m+M)g = (M+m)\ddot{y}$$

$$\Rightarrow -\boxed{\frac{K}{M+m}}\, y + \frac{[Kl_0 + (m+M)g]}{M+m} = \ddot{y} \quad (2)$$

pto equilibrio: $\ddot{y} = 0 \Rightarrow y_{eq} = l_0 + \frac{(m+M)g}{K}$

Cambio variable: $Z = y - y_{equ} \Rightarrow -w^2 Z = \ddot{Z}$ con $w = \sqrt{\frac{K}{m+M}}$.

luego $Z = A\cos(wt + \phi)$

$$\ddot{Z} = \ddot{y}$$
$$y = Z + y \text{ equilibrio}.$$

De (1) $N = mg - m\ddot{y}(*) = mg - m\ddot{Z} = mg - m(-w^2 Z) = mg + mw^2 Z$

$$\Rightarrow N = mg + mw^2 A\cos(wt + \phi) = N(t).$$

De (*)

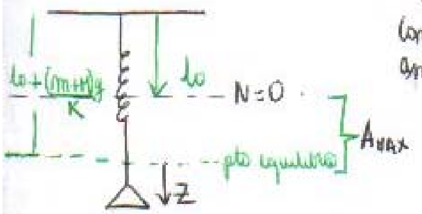
$N = mg - m\ddot{y}$ ocupando (2) $\Rightarrow N = mg - m\left[\frac{-Ky}{M+m} + \frac{Kl_0 + (m+M)g}{M+m}\right]$

Si $N = 0 \Rightarrow y = \ldots$

$$0 = mg - \frac{m}{M+m}[-Ky + Kl_0 + (m+M)g]$$

$$0 = \cancel{mg} + \frac{mKy}{M+m} - \frac{mKl_0}{M+m} - \cancel{mg}$$

$\Rightarrow y = l_0$ En $y = l_0$ se pierde contacto !!



Como el sistema oscila entorno a la posición de equilibrio, la amplitud máxima debe ser tq:

$$A \leq l_0 + \frac{(m+M)g}{K} - l_0.$$

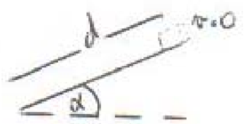
$$\Rightarrow A_{MAX} = \frac{(m+M)g}{K}.$$

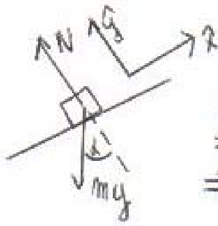
Solución P3.

Ocupando:

$E_f - E_i = W_{f\,roce}$

$W_{f\,roce} = -f_{roce} \cdot 2d$





$\Sigma F_y = 0$

$\Rightarrow N - mg\cos\alpha = 0$

$\Rightarrow N = mg\cos\alpha$.

Luego:

$E_f = \frac{1}{2}m(\alpha v)^2 \quad E_i = \frac{1}{2}mv^2$.

$\frac{1}{2}m(\alpha v)^2 - \frac{1}{2}mv^2 = -\mu mg\cos\alpha\, 2d$. ①

Falta determinar d:

Aplicamos energía en los siguientes instantes:

$E_f = mgh \qquad \operatorname{sen}\alpha = \frac{h}{d} \Rightarrow h = \operatorname{sen}\alpha \cdot d$

$E_i = \frac{1}{2}mv^2 \qquad W_{f\,roce} = -mg\cos\alpha\,\mu\, d$

Luego: $E_f - E_i = W_{f\,roce} \Leftrightarrow mg\operatorname{sen}\alpha\, d - \frac{1}{2}mv^2 = -mg\cos\alpha\,\mu\, d$

$\Rightarrow d(g\cos\alpha\,\mu + g\operatorname{sen}\alpha) = \frac{1}{2}v^2$

$\Rightarrow d = \frac{1}{2}v^2 / g\cos\alpha\,\mu + g\operatorname{sen}\alpha$. ②

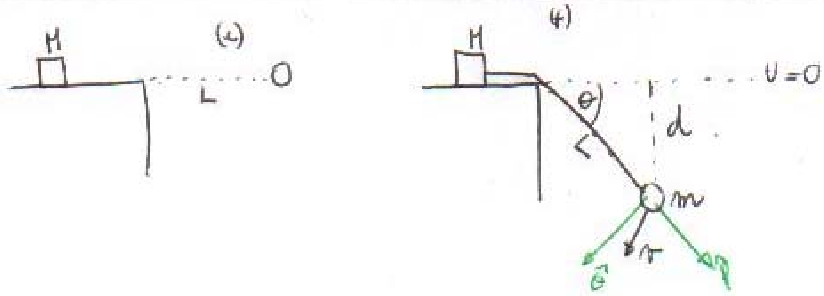
② en ① $\Rightarrow \frac{1}{2}m(\alpha v)^2 - \frac{1}{2}mv^2 = \frac{-\mu mg\cos\alpha\, 2 \cdot \frac{1}{2}v^2}{g\cos\alpha\,\mu + g\operatorname{sen}\alpha}$

$\Rightarrow \cancel{\frac{1}{2}mv^2}(\alpha^2 - 1) = \frac{-\cancel{\frac{1}{2}mv^2}\,\cancel{g}(2\mu\cos\alpha)}{\cancel{g}(\cos\alpha\,\mu + \operatorname{sen}\alpha)} \Rightarrow (1-\alpha^2)(\cos\alpha\,\mu + \operatorname{sen}\alpha) = 2\mu\cos\alpha$.

Despejando $\mu = \ldots$

!!

Solución P4.



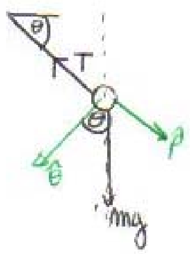
Aplicamos conservación de energía en los instantes (i) y (f)

$(i) \rightarrow E_i = 0$

$(f) \rightarrow E_f = -mgL\sin\theta + \frac{1}{2}mv^2$

$E_i = E_f = 0 \Rightarrow mv^2 = 2mgL\sin\theta$ ①

DCL.



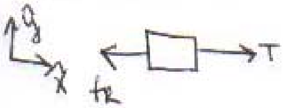
$\Sigma F_\rho: \ -T + mg\sin\theta = -\frac{mv^2}{L}$

$\Rightarrow T = mg\sin\theta + \frac{mv^2}{L}$

de ① $T = mg\sin\theta + 2mg\sin\theta$

$T = 3mg\sin\theta$ ②

Ahora hacemos (DCL) para M:



$\Sigma F_x \Rightarrow -f_R + T = Ma$

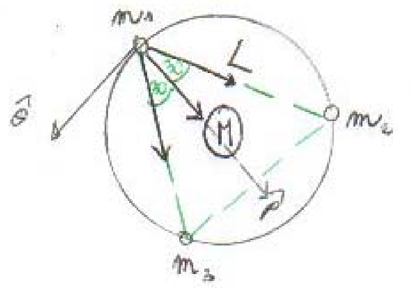
En reposo $a = 0 \Rightarrow T = f_R \leq \mu_e N$ condición de no deslizamiento

$\Rightarrow T \leq \mu_e M g$

Ocupando ② $\Rightarrow 3m\cancel{g}\sin\theta \leq \mu_e M \cancel{g}$

$\Rightarrow \sin\theta \leq \frac{\mu_e M}{3m}$

Solución P5.



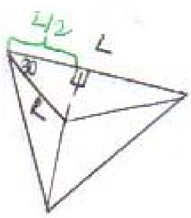
Para m_1 $\quad \Sigma F\hat{\rho} = -F_{m_2} - F_{m_3} - F_M = -m_1\frac{v^2}{R}.$

$$\Rightarrow -\frac{Gm^2}{L^2}\cos 30 - \frac{Gm^2}{L^2}\cos 30 - \frac{GmM}{R^2} = -m\frac{v^2}{R}.$$

$$\Rightarrow 2\frac{Gm^2}{L^2}\cos 30 + \frac{GmM}{R^2} = \frac{mv^2}{R}$$

$$\Rightarrow v^2 = \frac{R}{m}\left[2\frac{Gm^2}{L^2}\cos 30 + \frac{GmM}{R^2}\right]$$

Falta L.



$$\cos 30 = \frac{\frac{L}{2}}{R} \Rightarrow L = 2R\cos 30.$$