

Parte Ej 8

Parte a)

$$\vec{F} = \left(-\frac{GMm}{r^2} + \frac{\beta}{r^3} \right) \hat{r}$$

$$[F] = [masa] \cdot \frac{[distancia]}{[tiempo]^2} = \frac{[\beta]}{[distancia]^3}$$

$$\Rightarrow \boxed{[\beta] = \frac{[masa] [distancia]^4}{[tiempo]^2}}$$

Parte b)

En coord. polares:

$$m\vec{a} = \vec{F}$$

$$\Rightarrow m(\ddot{r} - r\dot{\theta}^2) \hat{r} + m(2\dot{r}\dot{\theta} + r\ddot{\theta}) \hat{\theta} = \vec{F} = \left(-\frac{GMm}{r^2} + \frac{\beta}{r^3} \right) \hat{r}$$

$$\Rightarrow m(\ddot{r} - r\dot{\theta}^2) = \left(-\frac{GMm}{r^2} + \frac{\beta}{r^3} \right)$$

Como dicen órbita circular $\Rightarrow r = \text{cte} \Rightarrow \dot{r} = \ddot{r} = 0$.

$$\Rightarrow -m r \dot{\theta}^2 = -\frac{GMm}{r^2} + \frac{\beta}{r^3}$$

$$\text{pero } \dot{\theta} = \frac{2\pi}{T}$$

$$\Rightarrow -m r \frac{4\pi^2}{T^2} = -\frac{GMm}{r^2} + \frac{\beta}{r^3}$$

$$\Rightarrow \frac{4_{II}^2 m r^4}{T^2} = GMmr - \beta$$

$$\Rightarrow \frac{r^4}{T^2} = \frac{GMmr - \beta}{4_{II}^2 m}$$

$$\Rightarrow \boxed{\frac{T^2}{r^4} = \frac{m 4_{II}^2}{GMmr - \beta}}$$

Nueva ley.

Para $r \rightarrow \infty$ podemos tomar:

$$\frac{T^2}{r^3} = \frac{m 4_{II}^2 r}{GMmr - \beta} \quad \left| : \frac{1/r}{1/r} \right.$$

$$\Rightarrow \frac{T^2}{r^3} = \frac{m 4_{II}^2}{GMm - \frac{\beta}{r}}$$

$$\Rightarrow \lim_{r \rightarrow \infty} \frac{m 4_{II}^2}{GMm - \frac{\beta}{r}} = \frac{4_{II}^2}{GM}$$

$$\Rightarrow \boxed{\frac{T^2}{r^3} = \frac{4_{II}^2}{GM}}$$

Para $r \rightarrow 0$

$$\frac{T^2}{r^4} = \frac{m 4_{II}^2}{GMmr - \beta}$$

$$\Rightarrow \boxed{\frac{T^2}{r^4} \approx \frac{m 4_{II}^2}{-\beta}}$$