$$du\,dv = \left|\begin{matrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{matrix}\right| dx\,dy$$

$$= \left|\begin{matrix} 2x & 2y \\ 2x & -2y \end{matrix}\right| dx\,dy = (-4xy - 4xy)\,dx\,dy$$

$$= -8xy\,dx\,dy$$

$$\Rightarrow \boxed{du\,dv = 8xy\,dx\,dy}$$

además $x^2 = \frac{u+v}{2}$ $y^2 = \frac{u-v}{2}$

$$\iint_S x^3 y^3\,dA = \int_2^4 \int_1^2 \frac{(u+v)}{2} \cdot \frac{u-v}{2} \cdot \frac{1}{8}\,dv\,du$$

$$= \frac{1}{32} \int_2^4 \int_1^2 (u^2 - v^2)\,dv\,du$$

$$= \frac{1}{32}\left[\int_2^4 \int_1^2 u^2\,dv\,du - \int_2^4 \int_1^2 v^2\,dv\,du\right]$$

$$= \frac{1}{32}\left[\frac{u^3}{3}\Big|_2^4 - 2\,\frac{v^3}{3}\Big|_1^2\right]$$

$$= \frac{1}{32}\left(\frac{4^3}{3} - \frac{8}{3} - 2\,\frac{8}{3} + 2 \cdot \frac{1}{3}\right)$$

$$= \frac{1}{32}\left(\frac{64 - 8 - 16 + 2}{3}\right)$$

$$= \frac{14}{32}$$