b)

$$\text{Min } x^2+y^2+z^2$$

s.a.

$$xyz^2=2$$

$$L(x,y,z,\lambda) = x^2+y^2+z^2-\lambda(xyz^2-2)$$

$$\frac{\partial L}{\partial x} = 2x-\lambda yz^2 = 0 \Rightarrow \lambda = \frac{2x}{yz^2}$$

$$\frac{\partial L}{\partial y} = 2y-\lambda xz^2 = 0 \Rightarrow \lambda = \frac{2y}{xz^2}$$

$$\frac{\partial L}{\partial z} = 2z-2\lambda xyz = 0$$

$$\frac{\partial L}{\partial \lambda} = -(xyz^2-2) = 0$$

$x\neq 0$, $y\neq 0$, $z\neq 0$ pues de lo contrario $xyz^2\neq 2$

$$\Rightarrow \frac{2x}{yz^2} = \frac{2y}{xz^2} \Rightarrow \boxed{x^2=y^2}$$

$$\cancel{\Rightarrow \frac{2y}{xz^2} = \frac{z}{xy} \Rightarrow 2y^2 = z^3 \Rightarrow 2x^2 = 2y^2 = z^3}$$

Además $z^2 = \dfrac{2}{xy}$

$$\Rightarrow 2x - \lambda \cancel{y}\cdot\frac{2}{x\cancel{y}} = 0 \Rightarrow 2x - \frac{2\lambda}{x} = 0$$

$$\Rightarrow \cancel{2}x^2 - \cancel{2}\lambda = 0$$

$$x^2 = \lambda$$

$$\Rightarrow y^2 = \lambda$$

$$\Rightarrow \cancel{2}z = \cancel{2}\lambda \frac{2}{z^{\cancel{2}}}\cancel{z} \Rightarrow \boxed{z^2 = 2\lambda}$$

$$\Rightarrow 2\cancel{\sqrt{\lambda}} - \lambda\cancel{\sqrt{\lambda}}\cdot 2\lambda = 0$$

Sin pérdida de generalidad se puede tomar $x=\sqrt{\lambda}$, $y=\sqrt{\lambda}$

$$\Rightarrow 2\lambda^2 = 2 \Rightarrow \lambda = \pm 1 \Rightarrow \boxed{\lambda = 1}$$

$$\Rightarrow x=1,\ y=1\ \ z=\pm\sqrt{2}$$

Además $x=-1$, $y=-1$, $z=\pm\sqrt{2}$ satisfacen.